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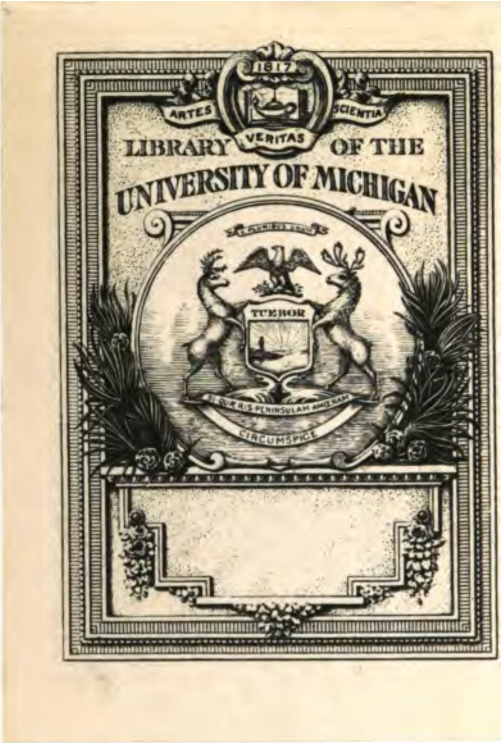

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THE

Arithmetician's GUIDE:

Being a New, Improved, and Compendious

SYSTEM OF PRACTICAL ARITHMETIC.

Deſigned either for the Uſe of Schools, or the benefit of private Perſons; and adapted to the Capacities of Beginners.

In THREE PARTS.

PART I.
Containing Definitions, Axioms, and all the Rules in Arithmetic of Whole Numbers.
PART II.
Containing Vulgar and Decimal Arithmetic, variouſly applied.
PART III.
Conſiſts of Geometrical Definitions and Menſuration.

By JOHN EADON,
Writing Maſter and Teacher of the Mathematics, in Sheffield.

SHEFFIELD:
Printed for W. Ward; and Sold by S. Crowder, and J. Wilkie, in *London*; and by the Author.

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TO THE WORTHY
TRUSTEES,
OF THE
Free Writing School,
IN
SHEFFIELD.

GENTLEMEN,

THE Experience I have had of your Zeal in the faithful diſcharge of your Truſt, the concern you ſeem to have for the Improvement of Youth placed at this SCHOOL *for* Education, *and your Diſpoſitions to promote every Deſign that tends to the Advancement of uſeful Literature, make me hope, that this Treatiſe, which aims that way, may venture to ſhelter itſelf under your Patronage.*

DEDICATION.

That a mutual Agreement and Harmony may long ſubſiſt amongſt you, that you may long continue an Honour to your Town, Encouragers of Science, Benefactors to Society, Happy in Yourſelves, your Families and every other circumſtance of Life, is the ſincere Wiſh of him, who with the deepeſt ſenſe of Gratitude, ſubſcribes himſelf,

Your Faithful, Obedient,

And moſt humble Servant,

J. EADON.

PREFACE.

HE muſt be little verſed in the common affairs of life, who does not know the great uſefulneſs of Arithmetic: as no buſineſs can be carried on without the help of numbers; no trade or commerce exerciſed without regular accompts. And as the utility of Arithmetic is thus extenſive, ſo there are extant many Treatiſes on the ſubject, ſome of which are excellent in their method. But in my opinion there is ſtill room for a general, conciſe, and rational piece, better adapted to the uſe of Schools, as well as of thoſe who have not the aſſiſtance of a Maſter: for all the Treatiſes that have occurred to me, have either all, or none of their examples wrought at large; which, in the firſt place, renders them unfit for a Maſter in his School; and in the ſecond place, not fit for thoſe who would acquire a competent ſkill in Arithmetic, without the aſſiſtance of a teacher. To remove this defect, is the intention of the following Treatiſe; and in what manner my deſign is executed, is wholly ſubmited to the impartial Reader.

In the courſe of this undertaking, I have exerted my utmoſt endeavours, to deliver the definitions and rules in as brief a manner as poſſible; to make them, at the ſame time general, and yet free from ſuperfluities, which commonly attend definitions of this kind; and have given ſuch notes after them, as deſcribe ſome particulars not flowing from the general rules themſelves, but tending to explain them, or facilitate the operations. To each rule alſo there is annexed, a great variety of the beſt choſen examples, with their anſwers, the greateſt part of which are new, and which the reader cannot fail of underſtanding, by means of the previous examples wrought at large. After the four firſt rules in ſimple and compound numbers, I have expreſſed the operations in algebraic ſigns, which in my opinion is far the beſt method, for it not only ſhortens the work, but the operation, when finiſhed, is itſelf a rule by inſpection.

And it may be noted, that, where fractions are in the anſwers to any of the examples in the rules preceding vulgar fractions, they are wrote down as they aroſe without any abbreviations; but all fractions in the anſwers afterwards are abbreviated as much as poſſible. I have treated the four fundamental rules, viz. Addition, Subtraction, Multiplication, and Diviſion, firſt in ſimple numbers, and afterwards in compound; for by teaching them in this order, difficulties do not ſo faſt ariſe. Yet I have delivered them in ſuch a manner, as to have little or no dependence on each other, that they may be taught in what order every Maſter chuſes.

I think it quite needleſs for the firſt ſimple rules to be copied out into the Scholars books; but if any Maſter chuſes to have it done, I would adviſe him to make his Scholars run over the examples firſt upon their ſlates; then begin the rules again, and write them, with a few examples to each, in their books; and thus he may with eaſe mix the ſimple and compound rules together. Tho' I ſaid before, there is nothing ſuperfluous in any of the problems or general rules; yet the judicious teacher may omit any notes, examples, or particular caſes he may think fit. And if every Scholar has a printed book, he may either write all the rules and examples in his accompt book, or may omit the Rules, Notes, and Tables, and write down only the operation of each example, eſpecially where the rules are long and tedious. This book may alſo be of peculiar ſervice to boys when taken from School, either in making a farther progreſs by their own application, or in the retainment of what they have already learned. I have, contrary to the practice of ſome Maſters, inſerted ſeveral wrought examples in each rule; having found by repeated experience, that a Scholar will ſooner get both the rule and its meaning, by ſeeing a few examples wrought, than by any other method. But any Maſter, if he pleaſes, may ſoon make new queſtions of thoſe that are ſolved, by only changing a figure in the data; and when his Scholars have gone through the operation, may make them invert the queſtion in order to prove the truth of their work; that is, make a new queſtion to prove the old one; and this will be no bad exerciſe for them.

This method of altering the data of a queſtion is of great utility in a School; for, as Mr. Dilworth obſerves, "ſome boys, lazily inclined, when they ſee another at "work upon the ſame queſtion, will be apt to make his "operation paſs for their own." But theſe little forgeries are ſoon detected by the vigilance of the Tutor. And this is beſt and moſt expeditiouſly performed by changing a figure in the data; for then the Scholar muſt of courſe go thro' the whole operation in order to obtain the ſolution, as alſo thro' a ſecond operation, in order to prove the truth of the firſt.

It is much the beſt way to enter upon Vulgar and Decimal Fractions, immediately after Reduction, if time will permit; but there are a great number of Scholars that have not time to learn Vulgar and Decimal Fractions with their various application, who yet of courſe, ought to learn the rule of Three, Practice, Tare and Tret, Intereſt, Rebate or Diſcount, and Equation of Payments, ſo eſſentially requiſite in the common affairs of Life; and for that reaſon, I have treated them in whole numbers: but the judicious Maſter may teach them in what order he pleaſes, regard being had to the capacity of his Pupil, and the time he has to ſpend therein. I have omitted Duodecimal Arithmetic, becauſe its place is better ſupplied, in all caſes, by Decimal Fractions.

In this Treatiſe are contained many uſeful and intereſting particulars, which might here be mentioned, but I rather choſe to refer to the Work, and to let the Book ſpeak for itſelf, only requeſting that the Reader will not too haſtily cenſure and condemn it; but that after impartially peruſing it, he will candidly excuſe ſuch defects as may occur to him, and have eſcaped my obſervation; and if he ſhould find a *tranſpoſition* of a *letter*, or a falſe figure, to excuſe it, as Errors of the Preſs will inevitably creep in, and ſome may have eſcaped my notice.

It is not in the leaſt pretended that all things treated of here are new; for I have collected ſome (ſuitable to my plan) from the Writings of my predeceſſors, and here make this public acknowledgement for the aſſiſtance.

N. B. Thoſe who may have occaſion to learn Merchants Accompts, I would recommend to their peruſal, ROOSE's BOOK-KEEPING, as being in my opinion, the beſt extant, both in Theory and Practice.

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ERRATA.

The third Queſtion in Double Fellowſhip, page 189, *read as follows.*

Six merchants trade after this manner. A puts in 50l. for 6 months, and 60l. for 4 months. B puts in 90l. for 8 months. C puts in 160l. for 5 months, and 190l. for 4 months. D puts in 200l. for 7 months, and 100l for 5 months. E puts in 300l. for 10 months, and 100l. for 2 months. F puts in 400l. for 3 months, 200l. for 4 months, and 200l. for 5 months. They gained 686,4l. what is the ſhare of each merchant.

Page	Line	for	read
8	3 b	490720	490740
18	4	1166	1186
29	9	10 years	100
35	10	21 poles	31 poles.
39	9 b	x×x	x×8
43	3 b	174l. 6s. $2\frac{1}{2}$d	874l. 6s. $2\frac{1}{2}$d.
51	13 b	1882471	1883471
76	6	4l. 10s.	14l 10s.
77	10	continued	contained
79	1 b	3l. 9s. $2\frac{1}{4}$d. Anſw.	83l. 9s. $2\frac{1}{4}$d. Anſw.
88	11	Anſ. 218l. 19s. $3\frac{1}{2}$d.	Anſw. 222l. 15s. $9\frac{1}{2}$d.
89	13	14l. 7s. 6d.	14l 17s. 6d.
97	10	3 cwt. Cloff.	1 cwt. Cloff.
100	5	13l. 13s.	13l. 16s.
128	10 b	334	336
	11 b	383 the com. Denom.	384 the com. Denom.
183	5	Anſw. $27\frac{2}{7}$	Anſw. $28\frac{2}{7}$
238	7&9 b	384,42 & for 384,3	384
244	6	24975	499500
245	8	4s.	3s.
252	9	$6144 \div 3 = 2072$	$6144 \div 3 = 2048$
253	2 b	$6144 - 3 \div 2$	$6144 - 3 \div 2 - 1$,
255	9	12285×048	12285×2048

Note, *b* denotes from the bottom.

The Reader is deſired to Correct theſe.

INTRODUCTION,

MATHEMATICS originally ſignified any diſcipline or learning, (Matheſis :) But now it is that ſcience which contemplates and treats of all kinds of quantities that are capable of being numbered or meaſured. That part which relates to number only, is called *Arithmetic*; and that which relates to meaſure in general, whether length, breadth, motion, force, &c. is called *Geometry*. When theſe two are converſant about multitude and magnitude abſtractedly conſidered, they are called pure or abſtract Mathematics, and are the foundation of all the other parts. When they are applied to particular ſubjects, they are called mixt Mathematics. Mathematics are alſo called ſpeculative, ſo far as they are concerned in finding out true propoſitions; and practical, as they relate to uſe, and are applied to practice.

QUANTITY is whatever will admit of augmentation and diminution; or is capable of any ſort of eſtimation or menſuration.

A PROPOSITION is ſomething propoſed to be proved or demonſtrated.

A THEOREM is a demonſtrable propoſition laid down as an acknowledged truth; and a ſet of ſuch theorems is called a Theory.

A PROBLEM is a queſtion requiring ſomething to be done. A limited Problem is that which has but one anſwer. An unlimitted Problem is that which has an infinite number of anſwers. A determinate Problem is that which has a certain number of anſwers.

SOLUTION of a Problem is the anſwer given to it. A numerical Solution is the anſwer in numbers. A geometrical Solution is an anſwer by the principles of Geometry. A mechanical Solution is one which is gained by trials.

A **LEMMA** ſignifies a propoſition, which ſerves previouſly to prepare the way for the more eaſy apprehenſion of the demonſtration of ſome Theorem.

A **CORROLLARY**, or *Conſectary*, is a conſequence drawn from a propoſition already demonſtrated.

A **SCHOLIUM** is a remark made on any propoſition, corollary or diſcourſe.

PRINCIPLES are the firſt grounds, rules, or foundations, of any Science; as Definitions, Axioms, Poſtulates, and Hypotheſes.

A **DEFINITION** is our explication of any word or term in any Science; which explication ought to be clear, and to contain no word or term, but what is already underſtood.

An **AXIOM**, or *Maxim*, is ſuch a common, plain, ſelf-evident propoſition, as nobody can deny without contradicting common ſenſe and reaſon. And nothing ought to be allowed for an Axiom but what is thus clear and ſelf evident: as, *that nothing can act where it is not; that a thing cannot be, and not be, at the ſame time; that the whole is greater than a part thereof; that no body can naturally go into nothing*: Out of an infinite number of ſelf evident truths that occur to the mind, men ſelect ſuch as are general, and of moſt uſe in demonſtrating any ſcience, and lay them up in ſtore to have recourſe to, as need requires. And tho' in their reaſoning they do not always mention ſuch and ſuch Axioms; yet the mind perceives the force of them, without ſtopping to hear them particularly repeated or named.

A POSTULATE, or *Petition*, is ſomething required to be granted, which is too evident to be denied.

An HYPOTHESIS is a ſuppoſition aſſumed to be true, by which a man is to argue, and build his reaſoning upon.

DEMONSTRATION, is a chain of arguments depending on one another, and founded primarily on firſt and ſelf evident Principles, or plain Propoſitions eſtabliſhed and proved from them, and at laſt ending in the invincible proof of the thing to be demonſtrated.

METHOD is the art of diſpoſing a train of arguments, in right order, either to find out the truth, or falſehood of a Propoſition; or to demonſtrate it to others when we have found it out. This is either analytical or ſynthetical.

ANALYSIS, or the *Analytic Method*, is the art of finding out the truth of a Propoſition, by ſuppoſing the thing to be done; and going back ſtep by ſtep, till we arrive at ſome known truth. This is called the *method of invention*, and is generally uſed in Algebra.

SYNTHESIS, or the *Synthetic Method*, is the ſearching out truth, by firſt laying down ſome ſimple and eaſy principles, and purſuing the conſequences till we come at the concluſion. This method begins at the moſt ſimple and eaſy things, and proceeds to the more compound and general. It is alſo called the method of compoſition, and is contrary to the Analytic method; as this proceeds from known principles to an unknown concluſion; whilſt the other goes in a retrograde order from the thing ſought, as if it was known, to ſome known principles. And therefore when any truth has been found out by the Analytic method; it may be demonſtrated in a backward order, by Syntheſis.

The Signification and Explanation of the Signs, or Characters, uſed in this Treatiſe.

CHARACTERS. SIGNIFICATION.

+ MORE, or to be added, being an affirmative ſign. Thus $9+6$ ſignifies 6 added to 9; and $a+b$ denotes a added to b, or the ſum of a and b. The like muſt be underſtood when ſeveral numbers are connected with the ſign + as $7+4\underline{+63+72}$. Letters with a line drawn over them, thus $a+b+c+d$ ſignifies the Sum of a, b, c, and d.

— LESS, or to be ſubtracted, being a negative ſign. Thus $9-6$ means 6 taken out of 9; and $a-b$ denotes the remainder when b is ſubtracted from a.

× MULTIPLIED by, as 9×6 ſignifies 6 times 9, or 9 multiplied by 6; alſo $a\times b$, or ab. is the product of a and b, multiplied together where note, if letters ſtand together like letters in a word, as $abcx$, they denote the product of a, b, c, and x. But when a number is prefixed to a letter, which is called its numeral coefficient, as $10x$, it denotes ten times x.

÷ Divided by, thus $9 \div 3$ ſignifies 9 divided by 3, alſo $3)9($ or $\frac{9}{3}$ ſignifies 9 divided by 3, and in general $a \div b$, or $b)a($, or $\frac{a}{b}$ is the quotient of a divided by b.

= Equality, or equal, as $9+6=15$, that is 9 and 6 equal 15; alſo $9-6=3$, that is 9 leſs by 6 equals 3; alſo $4 \times 3=12$, or 4 multiplied by 3 equals 12; alſo $12 \div 3=4$, or 12 divided by 3 is equal to 4.

: To, or *is to*, in the rule of *Three* or *Proportion*.

:: So is, or *what will*, a note in proportion, thus as $2:3::4:6$, ſignifies, as 2 is to 3, ſo is 4 to 6; and as $a:b::c:d$, that is as a is to b, ſo is c to d.

∺ Continual Proportion, $a:b:c:d$ ∺, or a, b, c, d, are in continual proportion.

§ Section, is put to diſtinguiſh the diviſion of a Chapter, as § IV. ſignifies Section Fourth.

Alſo for the ſake of brevity +ed is put for added, −ed for ſubtracted, ×ed for multiplied, × for multiply, ÷ for divide, and ÷ed for divided: likewiſe +ing for adding, −ing ſubtracting, ×ing multiplying, ×er multiplier, ÷ing for dividing, and ÷ſor for diviſor, in the following Treatiſe.

I would adviſe the Learner to make himſelf acquainted with the above Signs or Characters, with their ſignifications, which, being perfectly learnt, will help to contract the work, and is a ſhorter, better, and more ſignificant way of denoting what is to be done in moſt operations than can be expreſſed in words at length.

THE

Arithmeticians Guide.

ARITHMETIC.

DEFINITIONS.

ARITHMETIC is the art of numbering, or of performing calculations by a right management of numbers.

2. An *Unit* is the beginning of number, and is that by which every thing is called one.

3. *Number* is a collection of units: by this every thing is computed.

4. An *Integer* is a whole thing.

5. A *whole Number* is a preciſe number without any parts annexed.

6. A *mixt Number* is a whole number with ſome part of an unit annext; which part is called a fraction.

7. One number is ſaid to *meaſure* another, when it ſo exactly divides the other, that nothing remains: but if there be a remainder it is ſaid to divide the other, but not to meaſure it.

Corrollary. Any number is a meaſure to itſelf. And 1 is a meaſure to any number.

8. One number is ſaid to be the *multiple* of another when it contains it a preciſe number of times: as 30 is the multiple of 6, for 6 is contained 5 times in 30.

9. An *Aliquot part* is that which is contained a preciſe number of times in another. So 4 is an aliquot part of the numbers 8, 12, 16, 20, 24, 28.

Cor. Hence 1 is an aliquot part of any number: but a number cannot be called an aliquot part of itſelf.

10. An *aliquant* part is ſuch as is contained in another ſome number of times, with ſome part or parts over; as 3 and 4 with reſpect to 10.

11. An *even number* is that which may be divided into two equal parts; as 8 into 4 and 4.

12. An *odd number* is that which cannot be divided into two equal parts; as 3, 5, 7, 9, 11, &c.

Cor. The numbers 1, 2, 3, 4, &c. are alternately odd and even for ever.

13. A *Prime Number* is that which can only be meaſured by an Unit.

14. *Coprimes*, are ſuch numbers as are prime to each other, when only an Unit can meaſure them both.

Cor. Therefore 1 is prime to every number.

15. A *plain Number* is the product of two other numbers.

16. A *ſolid Number* is the product of three numbers.

17. A *ſquare Number* is the product of a number by it ſelf.

18. A *cube number* is the product of a number and its ſquare.

19. A *perfect Number* is that which is equal to the ſum of all its aliquot parts: as 6 is a perfect number, becauſe its aliquot parts 1, 2, 3, being added reſtore the ſame.

20. A *compoſite Number* is produced by multiplying ſeveral other numbers together, called Factors or Multipliers. Or it is that which other numbers beſides unity will meaſure, as 4 meaſures 8 by 2, and 2 meaſures 8 by 4.

21. *Denomination* is the name of any integer or thing. Thus pounds, ſhillings, pence, and farthings, are ſeveral denominations; where ſhillings are of a lower denomination than pounds, and higher than pence.

NOTATION.

THE *Characters* by which numbers are expreſſed, are either the ten *numerical Figures* (ſometimes called digits) of the *Arabians*, or the ſeven numerical Letters of the *Romans.* The Names of thoſe Characters, and the Numbers they ſtand for are here ſet down.

Arabian Figures.

0 nothing, or Cypher.
1 one, or an unit.
Then $1+1=2$ two.
$2+1=3$ three.
$3+1=4$ four.
$4+1=5$ five.
$5+1=6$ ſix.
$6+1=7$ ſeven.
$7+1=8$ eight.
$8+1=9$ nine.

Roman Letters.

I one
V five.
X ten.
L fifty.
C hundred.
D five hundred.
M. thouſand.

Then $9+1=$ ten, which has no ſingle character; and thus by the continual addition of 1, all numbers are generated.

The whole art of Arithmetic is performed by theſe ten figures, and their various application in the five fundamental Rules, viz. *Numeration, Addition, Subtraction, Multiplication,* and *Diviſion.*

AXIOMS.

1. IF to or from equal numbers, equal numbers be added or ſubtracted their ſum or remainder will be equal.

2. If equal numbers be ×ed or ÷ed by equal numbers; their product or quotient will be equal.

3. Unity or 1 neither multiplies nor divides; i. e. the product or quotient is ſtill the ſame.

4. If a number be compoſed of two numbers ×ed together; either of them meaſures it by the other.

5. If a number meaſures ſeveral other numbers, it likewiſe meaſures the ſum, or difference, of theſe numbers.

6. If a number meaſures another, it alſo meaſures every number which that other meaſures.

7. If a number meaſures the whole, and a part taken away, it alſo meaſures the reſidue.

8. The whole is equal to the ſum of all its parts taken together.

9. Thoſe things which are equal amongſt themſelves, are alſo equal to one another.

Practical.

Practical ARITHMETIC.

CHAP. I.

The fundamental RULES in Common ARITHMETIC.

§ I. Of NUMERATION.

NUMERATION teacheth to write any number named, and to read any number written. Obſerve therefore the places, names, and value of the figures in the Table.

NUMERATION TABLE.

&c.	Hundred Thouſands of Millions	Ten Thouſands of Millions	Thouſands of Millions	Hundreds of Millions	Tens of Millions	Millions	Hundreds of Thouſands	Tens of Thouſands	Thouſands	Hundreds	Tens	Units
	7	3	2	3	8	9	4	6	7	5	5	5

1. The value of any number does not depend on the figure or figures alone, but upon the figure or figures where they ſtand jointly. The order of places is backward from the right hand to the left: therefore any character

is increaſed in its value in a tenfold proportion as it is removed to the left of the other figures with which it is connected: as in the figures 555, the firſt 5 is five ones but the ſecond is 5 tens, or fifty, and the third is 5 five hundreds and ſo on. Therefore the numbers in the Table are read as follows.

Seven hundred and thirty two thouſands, three hundred and eighty nine millions, four hundred and ſixty ſeven thouſands, five hundred and fifty five.

2. A cypher of itſelf is nothing; yet when put on the right hand of other figures, it increaſes their value in the ſame tenfold proportion as above deſcribed. Thus 4 ſignifies 4 ones, but 40 ſignifies four tens, or forty, 400 ſignifies 4 hundred, and 4000 four thouſand, &c. Alſo 1 ſignifies one, 10 ten, 100 a hundred, 1000 a thouſand, and ſo on; and in general ten units make one ten, ten tens make one hundred, ten hundreds make 1 thouſand, &c.

3. To be able to read twelve figures is as much as any common buſineſs requires, but you may read as many as you pleaſe by the following Rule.

RULE. Begin at the units place and diſtinguiſh your figures into periods of ſix figures each, and half periods of three figures each. The firſt period to the right is units, the 2d millions, the 3d bi-millions, the 4th tri-millions, the 5th quadri-millions, the 6th, 7th, &c. quinti-millions, ſexti-millions, ſepti-millions, octi-millions, noni-millions, deci-millions, &c. The firſt half of any period is ſo many ones of it, but the laſt half is ſo many thouſands of it.

Example 1. Let it be required to read the following numbers,

Sexti-mil.	quinti-mil.	quadri-mil.	tri-mil.	Bi-mil.	Millions	units.
th. un.	th. un.	th. un.	th. un.	th. un.	th. un.	cxtcxu
673,874.	231,043	756,131.	736,730	567,311.	726,734.	967,361.

Six hundred ſeventy three thouſand, eight hundred ſeventy four ſexti-millions;

Two hundred thirty one thouſand, forty three quinti-mil.

Seven hundred fifty ſix thouſand, one hundred and thirty one quadri-millions;

Seven hundred thirty ſix thouſand, ſeven hundred and thirty tri-millions;

Five hundred ſixty ſeven thouſand, three hundred and eleven bi-millions;
Seven hundred twenty ſix thouſand, ſeven hundred thirty four millions;
Nine hundred ſixty ſeven thouſand, three hundred and ſixty one.

2. It is required to expreſs in words, 3700474.

3. Write down in words 72010346746701З.

4. Write down in figures, thirty ſeven thouſand bi-millions, three hundred forty thouſand, nine hundred and thirty four millions, two hundred five thouſand, five hundred twenty four.

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§ II. SIMPLE ADDITION.

SIMPLE ADDITION is a rule by which ſeveral numbers of the ſame denomination are collected together. The number ariſing from thoſe collections is called their Sum.

RULE.

1. Place the ſeveral numbers under each other ſo, that units may ſtand under units, tens under tens, hundreds under hundreds, &c. and draw a line underneath.

2. Begin at the place of units, add up all the figures in that row, and if their ſum be leſs than ten, ſet down that ſum ſtrait below; if their ſum be even ten or tens, ſet down a cypher; if above ten or tens, ſet down the overplus, and for every ten carry an unit to the next row.

3. Add up in like manner all the figures in the tens place together with the units you carried; ſet down the overplus above the even tens as before, carry the tens to the next row; and ſo on to the laſt; and the figures below the line will be the whole Sum.

To prove addition, begin at the top, add all the figures downwards, by the ſame rule as you added them upwards; if the ſum is the ſame the work is right.

Example 1. Let the numbers, 8967, 890, 103, and 76, be added together. When placed down according as the rule directs, they will ſtand thus;

8967
890
103
76

Sum 10036

Beginning at 6, ſay 6 and 3 is 9 and 7 is 16, ſet down 6 and carry 1. Then ſay 1 and 7 is 8 and 9 is 17 and 6 is 23, ſet down 3, and carry 2. Then 2 and 1 is 3 and 8 is 11 and 9 is 20, ſet down 0, and carry 2, laſtly, 2 and 8 is 10, which being the laſt, ſet down.

It is very eaſy to conceive the reaſon of carrying the tens to the next place; for the ſum of 6, 3 and 7 being 16, the 6 belongs to the units, and the 1 to the tens. Again, the ſum of 1, 7, 9 and 6 being 23, which are tens, the three belongs to the tens, and the 2 to the hundreds. Then the ſum of 2, 1, 8, and 9 being 20, which are 20 hundreds, the 0 belongs to that place, and the 2 to the next place which is thouſands. Laſtly the ſum of 2 and 8 is 10 i. e. 10 thouſand, that is 0 is in the place of thouſands, and 1 in the place of ten thouſands. Or in ſhort, thus.

The ſum of the row of units	16
The ſum of the row of tens	220
The ſum of the row of hundreds	1800
The ſum of the row of thouſands	8000
	10036 by axm 9.

Ex. 2.	Ex. 3.	Ex. 4.
674	407461	26
3723	78956	437
7673	4073	4188
91010	219	7610
701	11	10137
103781	490720	22398

5. A perſon dying left to his *Widow* 5000 pounds. He bequeathed to a *Charity* 864 pounds; to each of his five

nephews 1765 pounds: to each of his ſeven *nieces* 1053 pounds: to 10 poor Houſe-keepers 6 pounds each, and 563 pounds to his Executor; how much did he die poſſeſſed of? Anſwer 22683 pounds.

6. From the *Creation* to the *Flood* was 1656 years; thence to the building of *Solomon's Temple* 1336 years; thence to the birth of *Chriſt* 1008 years; in what year of the world was *Chriſt* born? Anſwer Anno M. 4000.

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§ III. Of *Simple* SUBTRACTION.

SUBTRACTION is the taking a leſs number, called the *ſubducend*, from a greater, called the *minuend*, in order to find a third number, called the *remainder*, or *difference*.

RULE.

1ſt. Place the *minuend* uppermoſt, and the *ſubducend* under, ſo as units may ſtand under units, tens under tens, &c. and under them draw a line.

2. Begin at the place of units, and take each lower figure from that which ſtands over it, ſetting the remainder ſtrait under them below the line, and all thoſe remainders together will be the required difference.

3. When the lower figure is greater than that which ſtands over it, conceive ten to be added to the upper, and take the lower from the ſum, ſet down the remainder, carrying 1 to be added to the next lower figure, take the ſum from the upper, ſet down the remainder; and ſo proceed from one row to another.

To prove ſubtraction, add the remainder to the *ſubducend*, and if the ſum be equal to the *minuend*, the work is right.

EXAMPLES.

1. From 3 8 0 4 2 6 7 8 the Minuend.
Take 5 1 2 7 4 3 5 the Subducend.

Rem. 3 2 9 1 5 2 4 3 or difference.

In operations of this ſort when the lower figure is greater than the upper, 10 is added to the upper; as for inſtance 7 is greater than 2, therefore one is borrowed from 4 to make 12; then 7 from 12, and there remains 5; then the next figure 2 ought to be taken from 3, inſtead of 4; but the reſult will be the ſame, whether you take 2 from 3, or add the 1. borrowed to 2, and take the ſum 3 from 4, in either caſe 1 remains. Again 1 from 0 you cannot take, but borrow 1 from 8 to make 10, from which take 1 and there remains 9, then one and 5 is 6, which taken from 8 leaves 2; laſtly nothing from 3 and 3 remains.

2. From 5 0 4 0 4 6 1 2 3 Minuend
Take 1 7 1 1 5 8 0 1 7 Subducend.

Rem. 3 3 2 8 8 8 1 0 6

3. Ralph was born in 1689, and died in 1765, how old was he? Anſwer 76 years.

4. Ralph died in 1765, his age was 76 years, in what year of the Lord was he born? Anſwer 1689.

5. In the year 1174, Henry II. conquered Ireland, and annexed it to the Title of the crown of England: how long is that ago, this year being 1765? Anſw. 591 years.

6. In what year of our Lord, did Henry II. conquer Ireland, it being 591 years ſince, and the preſent year 1765? Anſwer 1174 years.

7. What number added to 704376, will make it 800000? Anſwer 95624.

8. I have 3 Debtors A, B, and C. A, owes me 120000 pounds and B, 50000 pounds, A's debt exceeds C's by 94600 pounds, how much then is C's debt leſs than B's? Anſwer 24600 pounds.

9. Your *Grandfather* is 86 years of age, your *Father* is 64, you are not ſo old as your *Grandſire* by 68 years; what is the difference in years between your *Father* and *you*? Anſwer 46 years.

10. The building of *Soloman's Temple* was in the year of the World 3000, Troy was built 443 years before the

temple, and 260 years before London: Carthage was built 113 years before Rome, which was founded 744 years before Chriſt, who was born Anno Mundi 4000; is London or Carthage the ancienter city, and how much? Anſwer London by 326 years.

11. A is 13 years younger than B, and 17 years older than C, who in the year 1743 was known to be 24 years of age, how old was each of theſe perſons in 1765. Anſwer A, 63, B, 76, C, 46.

12. The ſemidiameter of the *Earth's Orbit*, or annual path round the *Sun*, in the *center* of the ſyſtem is about 81,000,000 of miles; that of *Venus*, 59,000,000; when they are both on the ſame ſide the *Sun*, they are in *Perigæo*; when on different ſides, in Apogæo: what is the difference of their *diſtances* in both thoſe *circumſtances*. Anſwer 118 millions of miles.

13. A is 19 years older than B. who was 27 years of age in the year of our Lord 1745: how old is C in 1765, who in the year 1763, was within 24 years of being as old as both, A and B together? Anſwer 87.

14. The mean diſtance between the Earth and Sun being 81 millions of miles, and between the Earth and Moon 240000: how far are theſe two luminaries aſunder in an eclipſe of the Sun, when the Moon is lineally between the Earth and Sun; and in another of the moon, when the Earth is in a line between her and him? Anſwer of the Sun 80760000 miles. Of the Moon 81240000.

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§ IV. Simple MULTIPLICATION.

MULTIPLICATION is the repeating a given number or quantity, called the multiplicand, ſo many times or parts of a time, as there are units in another given number, called the multiplier, and the reſult is called the product. Both multiplicand and multiplier are called Factors. Multiplication is a compendious method of addition, and is performed by help of the following Table, which muſt be got by heart.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3		9	12	15	18	21	24	27	30	33	36
4			16	20	24	28	32	36	40	44	48
5				25	30	35	40	45	50	55	60
6					36	42	48	54	60	66	72
7						49	56	63	70	77	84
8							64	72	80	88	96
9								81	90	99	108
10									100	110	120
11										121	132
12											144

The uſe of the Table is this; find one figure in the top column of the table, the other in the left hand column, and the ſquare where theſe two columns meet is the product. For inſtance, ſuppoſe you would multiply 8 by 9, find the 9 in the top, and 8 in the ſide column, and in the ſquare where they meet ſtands 72; thus the product of 6 and 7 is 42; of 8 by 8 is 64; of 6 times 12 is 72; of 3 times 9 is 27; of 12 times 12 is 144, and ſo of any other.

RULE.

1. Place the Multiplier under the Multiplicand, ſo that units may ſtand under units, &c. under them draw a line.

2. Multiply from the right hand to the left, thus: begin with the units in the Multiplier, by which multiply the units or firſt figure of the Multiplicand, and ſet down the overplus above the tens, carrying the tens in your mind. Then multiply the 2d figure of the Multiplicand by the ſame Multiplier, adding ſo many units, as you had tens to carry; ſet down the overplus, and carry the tens as before.

Do thus till you come to the laſt figure in the multiplicand, whoſe product ſet down intire.

3. Then take the ſecond figure of the Multiplier, and multiply by it as you did by the firſt; ſetting the firſt figure of the product under the figure you multiply with; and in like manner take each figure in its order, till all the figures of the multiplicand are multiplied by all the figures in the Multiplier; obſerving always to ſet the firſt figure of each product ſo many places towards the left hand, as the multiplying figure is diſtant from the place of units.

4. Laſtly, add all theſe products together, and their ſum will be the product of the two numbers given.

Multiplication may be proved, by making the multiplicand the multiplier, for then if your product comes out the ſame as before the work is right.

Some prove multiplication by caſting out the nines, but as that method is not infallible, I omit it.

The beſt and ſureſt way of proving multiplication, is by Diviſion.

EXAMPLES.

1. Multiply 8 multiplicand.
by 4 multiplier

Product 32. i.e. $8+8+8+8=8\times 4=4\times 8=32$

2. Multiply 7 0 5 3 4
by 8

Product 5 6 4 2 7 2

3. Mult. 9 6 0 8 9
by 7

Product 6 7 2 6 2 3

Explanation of the ſecond Example.

Say 8 times 4 is 32; ſet down 2 and carry 3; 8 times 3 is 24, and 3 you carry is 27; ſet down 7 and carry 2. 8 times 5 is 40 and 2 carried is 42; ſet down 2 and carry 4. 8 times 0 is 0 but 4 is 4; ſet down 4 and carry 0. 8 times 7 is 56 which ſet down, and you have 564272 for the Product.

Multiply

4. Multiply 70534.
by 5428.

564272
141068
282136
352670

Product 382858552

Demonſtration of the Rule.

In Ex. 2d. 8 multiplying 4 produces 32, of which the 2 belongs to the place of units, and the three to the tens. Then 8 multiplying 3 in the 2d place, or place of tens, produces 24, of which 4 belongs to the tens, to which the three carried, which are alſo tens, being added, makes 7 tens, and the 2 belongs to the 3d place, or hundreds. Then 8 multiplying 5 in the 3d place, produces 40, the 0 belongs to the 3d place to which add 2, which alſo belongs to the 3d place, and the ſum is 42, of which 2 belongs to the 3d place and 4 to the 4th place. Then 8 times 0 is 0, in the 4th place, but 4 added is 4. Laſtly 8 multiplying 7 produces 56, the 6 belongs to the 5th place, and the 5 to the 6th.

This may perhaps more evidently appear by ſetting down the particular products thus:

$$\left.\begin{matrix}4\\30\\500\\70000\end{matrix}\right\}\times 8=\left\{\begin{matrix}32\\240\\4000\\560000\end{matrix}\right.$$

$$70534 \times 8 = 564272 \quad \text{(axiom 8)}$$

And in Ex. 4. 8×4=32, the 2 falls in the place of units, and ſo on as in Ex. 2. Again 2×ing 4=8, which falls in the 2d place, becauſe the ×er is really 20. Again 4×ing 4=16, the 6 falls in the 3d place, becauſe the ×er is 400. Again 5×ing 4=20, the 0 falls in the 4th place, becauſe the ×er is 5000. Each figure being thus proceeded with as in Ex. 2d. the ſum of all their products is that required. (ax. 8)

Ex. 5th	36704674852		96745		3550993768556740
Ex. 6th.	8706543		54086		470902084698
Ex. 7th.	759837	×	90705	=	68921015085
Ex. 8th.	16358724		704006		11516639848344
Ex. 9th.	38015732		400700065		15232906283422580

CONTRACTIONS.

I. When either the multiplicand, or multiplier, or both, have cyphers towards the right hand, reject the cyphers, multiply the ſignificant figures as before, and to the product, annex as many cyphers as are in both factors.

Ex. 10th. 5968000 × 6 = 35808000

Ex. 11th. 5683 × 9000 = 51147000

$$\begin{array}{r} 5968000 \\ 6 \\ \hline 35808000 \end{array} \qquad \begin{array}{r} 5683 \\ 9000 \\ \hline 51147000 \end{array}$$

Ex 12. $97638000 \times 467000 = 455969460000000$

II. When the multiplier is an unit with cyphers to the right, as 10, 100, 1000, &c. the product will be equal to the multiplicand with ſo many cyphers annext as are in the multiplier.

Ex. 13. $3674 \times 10 = 36740$

Ex. 14. $3674 \times 10000 = 36740000$

III. When the multiplier is a compoſite number, multiply continually by the factors; i. e. multiply by one factor, and that product by the other &c. and the laſt product will give the anſwer.

Ex. 15th. Multiply 3604670 by $35, = 5 \times 7$,
$3604670 \times 7 = 25232690, \times 5 = 126163450$. Anſwer.

Ex. 16th. Multiply 3704604040 by $56, = 8 \times 7$.
$3704604040 \times 8 = 29636832320, \times 7 = 207457826240$.

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§ V. Simple DIVISION.

DIVISION is a compendious Subtraction; or the taking of one number, called the *Diviſor*, out of another, called the *Dividend*, as often as poſſible, in order to find a third number, called the *Quote* or *Quotient*; becauſe it ſhews (quoties) how often the Diviſor is contained in the Dividend, thus

Diviſor Dividend
4) 20 (5 Quote.

A General RULE.

1. Set down the Diviſor and Dividend in the form above, conſider if the Diviſor be leſs or equal to the ſame number of the left hand figures of the Dividend; if it is leſs or equal, ſet the figure in the quote, expreſſing the number of times it is contained in that part of the Dividend: but if not, take one place more of the dividend figures than are in the Diviſor, and ſet in the quote the number of times the Diviſor is contained in this part of the Dividend, as before.

2. Multiply the Diviſor by the quotient figure.

3. Subtract the product from the ſaid Dividend figures; and ſet down the remainder.

4. Make a prick under the next figure of the Dividend, in order to mark it, and bring it down to the right of the remainder, then this number call the Dividual.

5. Seek how oft the Diviſor can be had in the dividual, and ſet the figure expreſſing the number of times in the quote, multiply the Diviſor thereby, ſubtract the product as before, and in this manner the operation muſt be continued till all the figures in the dividend are brought down one by one. And *note*, for every figure brought down, a figure muſt be placed in the quote, except when the Dividual is leſs than the Diviſor, and then write a cypher in the quote.

Note 1. If the product of any quotient figure and diviſor exceeds the dividual, the quotient figure belonging to ſuch product muſt be leſſened till the product is equal to, or leſs, than its dividual: again, if after ſubtracting the product from its dividual, the remainder is equal to, or exceeds the Diviſor; the quotient figure muſt be increaſed till the remainder be leſs than the Diviſor.

2d. If there be a *Remainder* after diviſion is finiſhed, annex it to the quote with the diviſor under it, with a ſmall line drawn between: then the quote will be a mixt number.

To *prove Diviſion*, multiply the Diviſor and quote together, adding the remainder, if any; and the *Product* will be equal to the *Dividend* if the work is right.

EXAMPLES.

Diviſor Dividend Quote
1ſt. 3) 8 1 2 4 (2 7 0 8.
6 · · ·
——
21
21
——
· · 24
24
——
· ·

Explanation.

Firſt aſk how oft 3 in 8, which is 2 times, then place 2 in the Quotient, and multiply 3, (the diviſor) by 2, and the product is 6; which ſubducted from 8 leaves 2; then prick under 1 and bring it down to the right of 2. which is then 21 for a dividual; then aſk how oft 3 in 21, the anſwer is 7, which place in the quotient; then 7 times 3 is 21, which ſubducted from 21 the dividual, leaves 0. Then prick and bring down the next figure 2, and aſk how oft 3 in two, the anſwer is 0, which place in the quotient; then prick and bring down 4 to the right of 2, and the dividual is 24, then aſk how oft 3 in 24, the anſwer is 8, which place in the quotient; then 8 times 3 is 24; which ſubducted from 24, (the dividual) leaves nothing.

Then 2708 is the quote. Proved thus: 2708×3=8124 the dividend.

Ex. 2d. Divide 18972584 by 6023.

6023) 18972584 (3150 $\frac{134}{6023}$ quote
18069 · · ·
——
9035
6023
——
30128
30115
——
134 the remainder.

Proof 3150×6023,+134,=18972584.

Demonſtration of the RULE.

In Ex. 1. Since 3 is contained twice in 8 therefore it is contained 2000 times in 8124, i. e. 2 muſt be in the 4th place. And ſince 3 is contained 7 times in 21, it is contained 700 times in the whole remainder 2124, and therefore 7 muſt occupy the third place in the quote; and as 3 cannot be had once in 2, it cannot be had 10 times in the whole remainder 24, therefore a cypher muſt occupy the tens place of the quote; but ſince 3 is contained 8 times in 24, and 4 poſſeſſes the units place of the Dividend, 8 muſt conſequently poſſeſs the units place of the quote. Therefore the Diviſor is contained in the whole dividend 2708 times.

And in Ex. 2. ſince 6023 is contained 3 times in 18972 it is contained 3000 times in 18972584; and 100 times in the remainder 903584; and 50 times in the next remainder 301284; and 0 times in the laſt remainder 134. Or the Diviſor is contained in the whole Dividend 3150 times.

CONTRACTIONS.

I. When your Diviſor is 12 or leſs than 12, diviſion may be expeditiouſly performed by multiplying and ſubtracting mentally, and writing down only the quote below the Dividend.

EXAMPLES.

5)7367347

Quote 1473469 $\frac{2}{5}$

7)1967469

Quote 281067

12)1646749

Quote 137229 $\frac{1}{12}$

8)7674674

Quote 959334 $\frac{2}{8}$

II. If the Diviſor hath cyphers to the right of it, cut them off, then cut off ſo many of the right hand places of the Dividend as there are cyphers in the Diviſor, which annex to the remainder, when the operation is finiſhed.

EXAMPLES.

1. Divide 9850716 by 8300

Contracted	At Large.
83 | 00)98507 | 16(1186 $\frac{6916}{8300}$	8300)9850716(1166 $\frac{6916}{8300}$
83 . . .	8300 . . .
155	15507
83	8300
720	72071
664	66400
567	56716
498	49800
6916 Rem.	6916 Rem.

III. To divide by an unit with cyphers as 10, 100, 1000, &c. cut off from the Dividend ſo many places as the Diviſor has cyphers, and the figures ſo cut off on the left is the quote, and thoſe on the right the remainder.

Ex. Divide 7217367 by 100. The Quote is 72173 and 67 remainder.

IV. When the Diviſor is a compoſite number, i. e. when it is the product of two or more ſmall numbers, it is much eaſier to divide continually by thoſe numbers, than by the whole diviſor at once, i. e. divide the dividend by one of thoſe numbers, and that quotient by the other, and ſo proceed.

Note. If there be any remainders after ſuch diviſions, multiply the laſt remainder by the preceding Diviſor, and to the product add the remainder belonging to the ſame Diviſor; then multiply the ſum by the next preceding Diviſor, and to the product add its correſponding remainder: and ſo proceed through all the Diviſors and Remainders, and the laſt ſum will be the true remainder as if the Diviſion had been performed at once.

EX.

EXAMPLES:

1ſt. Divide 563746673 by 35=7×5.

                7)
5)563746673(112749334(16107047 $\frac{28}{35}$ quote

1ſt rem. 3   2d rem 5

then 5x5=25,+3=28 the true remainder.

2d. Divide 67467367 by 72,=8x9

                9)
8)67467367(8433420(937046 $\frac{55}{72}$ quote

rem. 7   rem. 6

then 6x8,+7=55 the true remainder.

3d. Divide 7386989 by 144=6x6x4

                6)   6)
4)7386989(1846747(307791(51298 $\frac{77}{144}$ quote

rem. 1   rem. 1   rem. 3

then 3x6,=18,+1=19,x4=76,+1=77 true remainder.

V. When you have a large Dividend, and your Diviſor is often repeated; make a table of all the products of the Diviſor and the 9 digits. By this table Diviſion may be wrought by inſpection, by help of Addition and Subtraction only. For you have no more to do, but only to take out of the table the number always next leſs than (or equal to) each dividual, and ſubtract it as in the general rule, remembering always to place in the quote the figure in the ſmall column, which is oppoſite the Product made uſe of.

EXAMPLES.

1. Divide 403779820571 by 35016.

TABLE.

1	35016
2	70032
3	105048
4	140064
5	175080
6	210196
7	245112
8	280128
9	315144

```
35016)403779820571)11531294.
      35016 .......
      -----
       53619
       35016
       ------
       186038
       175080
       ------
        109582
        105048
        ------
         45340
         35016
         ------
         103245
          70032
         ------
          332137
          315144
          ------
           169931
           140064
           ------
            29867 remain.
```

2d.	3550993768556740		96745		36704674852
3d.	3550993768556740		36704674852		96745
4th.	470902084698		54086		8706543
5th.	11516639848344	÷	704006	=	16358724
6th.	11516639848344		16358724		704006
7th.	152332906283422580		409700065		8016732
8th.	152332906283422580		380157328		400700065

§ VI. *Tables of Money, Weights, and Meaſures.*

1. *Of* MONEY.

q.	d.	s.	£.
4 =	1		
48	12 =	1	
960	240	20 =	1.

Note 1. L. denotes pounds, S. ſhillings, D. pence, and Q. farthings. Which are the firſt letters of their latin names libræ, ſolidi, denarii, quadrantes.

2. The letters at top denote the names of all the numbers ſtrait below them. All the numbers alſo on the ſame line, from left to right are of equal value: thus in the laſt line of this table, 960 farth. 240 pence, 20 ſhill. and 1 pound are all equal to each other: and ſo in all the following tables.

3. ¼ denotes 1 farthing, or 1 quarter of any thing.
½ - - - - - a half-penny, or a half of any thing.
¾ - - - - - - 3 farthings, or three quarters of any thing.

2. *Of* TROY WEIGHT.

Grains gr.	penny weights dwt.	ounces oz.	pound lb.
24 =	1		
480	20 =	1	
5760	240	12 =	1

Note. By this Weight gold, ſilver, jewels, amber, bread, corn, liquors, &c. are weighed. By this alſo the proportion of Gravity in Philoſophical Experiments, which any two bodies have to each other, are tryed; as ſuppoſe gold to ſilver, &c.

3. *Of Apothecaries* WEIGHT.

Grains	Scruples.	Drams.	Ounces.	Pound.
20 =	1			
60	3 =	1		
480	24	8 =	1	
5760	288	96	12 =	1

Note. By this weight Apothecaries compound all their medicines; tho' they buy and ſell their drugs by Avoirdupois Weight. Apothecaries is the ſame as Troy Weight, having only ſome different diviſions.

4. Of

4 *Of Avoirdupois* WEIGHT.

Drams dr.	Ounces oz.	pounds lb.	Quarters qr.	Hundreds C.	Ton
16 =	1				
256	16 =	1			
7168	448	28 =	1		
28672	1792	112	4 =	1	
573440	35840	2240	80	20 =	1

Note. By this weight are weighed all phyſical drugs, butter, cheeſe, fleſh, grocery, roſin, wax, pitch, tar, tallow, ſoap, hemp, &c. i. e. all things of a droſſy nature, and all metals, except gold and ſilver.

5. *Of* LONG MEASURE.

Inches	Feet	Yards	Poles	Furlongs	Mile
12 =	1				
36	3 =	1			
198	16½	5½ =	1		
7920	660	220	40 =	1	
63360	5280	1760	320	8 =	1

Note. An inch is ſuppoſed equal to 3 barley corns in length. 4 inches equal to 1 hand. 6 feet equal to 1 fathom. 3 miles equal to 1 league. 60 geographical miles equal to 1 degree, or 69 and a half ſtatute miles nearly.

Alſo 360 degrees, or 25000 miles nearly, is the circumference of the Earth.

6. *Of Cloth* MEASURE.

Inches	Nails	Quar.	Yard
2¼ =	1		
9	4 =	1	
36	16	4 =	1

Note. 3 qrs. are equal to an ell Flemiſh.
5 ——— ——— Engliſh.
6 ——— ——— French.
4 qr. 1 ⅕th of an inch —— Scots.

7. *Of Square or Land* MEASURE.

Square Inch	ſqr. feet	ſqr. Yds.	ſqr Poles	Roods	Acre
144 =	1				
1296	9 =	1			
39204	272¼	30¼ =	1		
1568160	10890	1210	40 =	1	
6272640	43560	4840	160	4 =	1

Note. The Land Chain now in uſe is 22 yards long, and is divided into 100 equal parts called Links; 10 ſuch chains in length and 1 in breadth make 100000 ſquare Links (that is 100×100×10) or 1 ſquare acre; therefore if any number of ſquare Links be divided by 100000 the quotient will be acres.

8. *Of Wine* MEASURE.

Pints	Gallons	Tierces	Hogſh.	Punch.	Pipe or But	Tun
8 =	1					
336	42 =	1				
504	63	1½ =	1			
672	84	2	1⅓ =	1		
1008	126	3	2	1½ =	1	
2016	252	6	4	3	2 =	1

Note. 231 cubic, or ſolid inches are equal to 1 gallon, 10 gallons equal to 1 anchor. 18 gall. equal to 1 rundlet. 31 gall. and a half equal to 1 barrel.

By this meaſure all wines, brandies, ſpirits, ſtrong waters, mead, perry, cyder, vinegar, oil, honey, &c. are meaſured.

9. *Of Ale and Beer* MEASURE.

Pints	Gall.	Firkins	Kilderkins	Barrels	Hhds
8 =	1				
68	8½ =	1			
136	17	2 =	1		
272	34	4	2 =	1	
408	51	6	3	1½ =	1

Note. The Ale gall. contains 282 cubic inches. In London, the ale firkin contains 8 gall. and the beer firkin 9; the other meaſures above it decreaſe and increaſe in the ſame proportion.

Of

10. *Of Dry* MEASURE.

Pints	Galls.	Pecks	Buſhels	Combs	Quar.	Weys	Laſt
8	= 1						
16	2	= 1					
64	8	4	= 1				
256	32	16	4	= 1			
512	64	32	8	2	= 1		
2560	320	160	40	10	5	= 1	
5120	640	320	80	20	10	2	= 1

Note. The gall. dry meaſure, contains $268\frac{4}{5}$ cubic inches. At London 36 buſhels of coals make a chaldron. A buſhel of water meaſure is 5 pecks.

By this meaſure all dry wares, ſuch as corn, ſeeds, fruits, roots, ſand, ſalt, coals, oyſters, muſcles, cockles, &c. are meaſured.

11. *Of* TIME.

Minutes	Hours	Days	Weeks	Month
60	= 1			
1440	24	= 1		
10080	168	7	= 1	
40320	672	28	4	= 1.

Note. 1. The minute is divided into 60 equal parts called ſeconds, the ſeconds into 60 equal parts called thirds, and theſe again into 60 fourths, &c.

2. The Julian year conſiſts of 13 months 1 day 6 hours, or 52 weeks 1 day 6 hours, or 365 days 6 hours, nearly.

3. An equal month (mentioned above) is ſo called becauſe it always conſiſts of 28 days. On the contrary, a kalendar month is unequal, becauſe it conſiſts of an unequal number of days. The year is divided into 12 kalendar months, viz. January February, March, April, May, June, July, Auguſt, September, October, November, December.

To know the days in each month obſerve.

Thirty days hath September,
April, June, and November,
February hath twenty-eight alone,
And all the reſt have thirty-one;
Except in leap-year, and then's the time
February's days are twenty and nine.

4. The exact ſolar year conſiſts but of 365 days, 5 hours, 48 minutes, and 57 ſeconds; tho' the common or Julian year is ſaid to contain 365 days 6 hours: which odd 6 hours every fourth year making a day, that day is added to February, which has then 29 days; and the year is called Biſſextile or Leap-year. Therefore all that are born on the 29th of February, have their birth day but every fourth year.

12. *Of a Circle's Circumference, how Divided.*

Minutes		Degrees		Signs		Circles Circum.
60	=	1				
1800		30	=	1		
21600		360		12	=	1

Note 1. The Minute is divided into 60 Seconds, &c. as in time.

2. By this Aſtronomers meaſure diſtances in the Heavens, Geographers meaſure this Terraqueous Globe; and Mathematicians meaſure any propoſed angle.

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§ VII. *ADDITION of different Denominations.*

RULE.

1. PLACE all numbers, of the ſame denomination, directly under each other, and under them draw a line.

2. Begin at the leaſt denomination, and add up all the figures in that row, as in ſimple addition.

3. Find how many ones of the next ſuperior denomination are contained in the ſum, which you may do, by dividing it by ſo many of this name as makes one of the next, or any other way.

4. Set down the remainder or overplus under the added row, and carry the quote to the figures of the next denomination, whoſe ſum you muſt find and proceed with as before; and ſo of the reſt. In the laſt denomination, add

them up as integers, whoſe ſum ſet down, which, together with the ſeveral remainders, will expreſs the required ſum.

Note. Addition of money may be performed either by the above rule, or by help of the following tables.

PENCE TABLE.

d.		s.	d.	
20	—	1	8	
30	—	2	6	is half a Crown.
40	—	3	4	
50	—	4	2	
60	—	5	0	a Crown.
70	—	5	10	
80	—	6	8	a Noble.
90	—	7	6	
100	—	8	4	
110	—	9	2	
120	—	10	0	an Angel.
160	—	13	4	a Mark.

SHILLING TABLE.

s.		l.	s.	
21	—	1	1	is a Guinea.
23	—	1	3	a Jacobus.
25	—	1	5	a Carolus.
27	—	1	7	a Moidore.
40	—	2	0	
60	—	3	0	
80	—	4	0	
100	—	5	0	
120	—	6	0	
140	—	7	0	
160	—	8	0	
180	—	9	0	

Examples of MONEY.

EXAMPLE 1. EXPLANATION.

l.	s.	d.
56	14	7½
67	15	6½
45	13	5¾
74	12	8¼
67	14	9¼
89	11	4¼
93	10	2¼
495	12	7¾

Begin at the farthings, add them up and their ſum is 11, or 2 pence 3 farthings; put down ¾ under the farthings, and carry 2 to the pence place; and the ſum of the pence is 43 or 3 ſhillings and 7 pence; place the 7 under the pence and carry 3 to the ſhillings, and the ſum is 92 ſhillings, or 4 pounds 12 ſhillings; place down the 12 ſhillings; and carry 4 to the firſt row of the pounds, and the ſum of it will be 45; then place down the 5, and carry 4 to the laſt row, and the ſum is 49, which place down, and the work is done. The anſwer being 495l. 12s. 7¾d.

2d.			3d.			4th.		
l.	s.	d.	l.	s.	d.	l.	s.	d.
87	16	7¾	74	14	0½	7	17	3½
96	18	8	7	9	4	76	13	4
77	17	6½	19	10	11¾	67	10	0¼
65	15	5	80	14	9½	49	8	7
31	11	3½	73	19	0	76	12	11¾
43	13	4	8	0	1½	47	16	1
24	14	9½	2	3	10	76	0	0½
428	7	8¼						

5th. A of Rotherham is debtor to B of Sheffield. For 367 groſs of penknives, 467l. 17s. 6d. For 264 groſs of table knives and forks, 567l. 13s. 9½d. For 129 groſs of razors, 96l. 14s. 7¾d. For 8 dozen of ſciſſors, 3l. 0s. 10½d. For 10 tons of iron, 179l. 17s. 6¾d. For 48 tons of copper, 1176l. 9s. 3½d. For 10 tons of ſteel, 250l. 10s. 6d. For his acceptance of a bill drawn, 76l. 19s. 6¼d. For carriage, and incidental charges 4l. 13s. 6½d. For what ſum muſt B draw to clear the accompt?

Anſ. 2823l. 17s. 2¾d.

		l.	s.	d.
1	Penknives, value,	467	17	6
2	Table knives and forks, ditto.	567	13	9½
3	Razors, ditto.	96	14	7¾
4	Sciſſors, ditto.	3	0	10½
5	Iron, ditto.	179	17	6¾
6	Copper, ditto.	1176	9	3½
7	Steel, ditto.	250	10	6
8	A Bill, ditto.	76	19	6¼
9	Incidental expences, ditto.	4	13	6½

6th. Suppoſe A is indebted to B 57l. 6s. 8d. To C 127l. 14s. To D 9s. 6½d. To E 17l. 0s. 3¾d. To F 1044l. 19s. and to G 1l. 7s. 2d. What is A's whole debt? Anſ. 1248l. 16s. 8¼d.

7th. A Nobleman, going out of Town, is informed by his Steward, that his corn chandler's bill comes to 123l. 19s. His brewer's to 41l. 10s. His butcher's to 212l. 0s. 6d. That to his Lordſhip's baker is owing 24l. To his tallow chandler, 13l. 8s. To his taylor, 137l. 9s. 9d. To his draper, 74l. 13s. 6d. That his coach-maker's demand was 214l. 16s. 6d. His wine merchant's 68l. 12s. His confectioner's 16l. 2s. That his rent was 82 guineas, and his ſervants wages 46l. 5s. What money muſt he ſend to his banker for, in caſe he would carry with him 50l. to defray his expences on the Road? Anſ. 1108l. 18s. 3d.

Examples of Weights and Meaſures.

	1. TROY WEIGHT.				2. APOTHECARIES WT.				
	lb.	*oz.*	*dwts.*	*grs.*	*lb.*	*oz.*	*dr.*	*ſc.*	*gr.*
	47	11	15	14	27	5	7	1	19
	14	9	13	15	43	6	5	2	18
	71	10	10	10	34	4	6	2	17
	67	10	15	17	75	3	6	1	16
	81	9	10	18	36	2	4	2	17
Sum	284	4	6	2	217	0	0	0	7

In the firſt Exam: the Grains are 74, or 3 penny weights and 2 grains; put down 2 under the grains, and carry 3 to the penny weights row; and the ſum of the penny weights is 66, or 3 ounces and 6 dwts.; place the 6 under the dwts, and carry 3 to the ounces, and the ſum is 52 ounces, or 4 pounds 4 ounces; place down the 4 ounces; and carry 4 to the pounds; then proceed as in the examples of integers, and you will find the ſum to be 284 lb. 4 oz. 6 dwts. 2 grs.

3d. AVOUR-

3d. AVOIRDUPOISE WT.

T.	*c.*	*q.*	*lb.*	*oz.*	*dr.*
12	15	2	24	12	15
17	10	3	21	15	10
0	18	1	14	11	5
1	19	3	27	15	12
Sum 33	5	0	5	7	10

4th. LONG MEASURE.

deg.	*l.*	*m.*	*f.*	*yd.*	*ft.*	*i.*	*bc.*
76	19	2	7	84	2	11	2
60	10	1	6	76	1	10	1
30	3	0	2	61	0	6	0
10	0	2	5	30	1	3	2
Sum 177	14	1	5	33	0	7	2

5. Cloth Meaſure.

Ells Eng.	*Q.*	*N.*
70	4	3
61	3	2
16	2	1
10	1	0
3	0	2
162	2	0

6. Land Meaſure.

A.	*R.*	*P.*
3	3	37
72	1	4
21	2	16
76	1	0
9	0	3
183	0	20

7. Wine Meaſure.

Hhds.	*Gall.*	*Q.*	*P.*
614	14	2	1
721	37	1	0
361	56	3	1
123	45	0	1
102	17	1	0
1923	45	0	1

8. Ale & Beer Meaſure.

hhds.	*k.*	*g.*	*p.*
29	2	16	7
17	1	11	2
12	2	14	5
9	0	12	6
69	2	4	4

9. Dry Meaſure.

bu.	*p.*	*g.*	*p.*
7	3	1	7
19	1	0	6
8	2	1	2
6	1	1	4
42	1	1	3

10. Time.

yrs.	*m.*	*w.*	*d.*	*h.*	*m.*
70	12	3	6	23	59
16	9	1	6	16	16
10	6	2	1	17	30
2	0	1	6	10	10
10	3	2	0	19	55

11. In a Gentleman's ſervice of plate there are diſhes 12, weighing 16lb. 1 oz. 13 dwts: plates 32, weighing 35lb. 1 oz. 11 dwts: ſpoons 5 dozen, weighing 8lb. 8 oz. 6 dwts: ſalts ſix, weighing 2lb. 8 oz: knives and forks weighing 11lb. 9 oz. 6 dwts: preſenters ſix, weighing 9lb. 5 oz. 4 dwts: mugs, tumblers, beakers, and other odd pieces, weighing 23lb. 8 oz 18 dwts: a ſilver tea kettle

and lamp, weighing 10lb. 6 oz. 9 dwts; and the reſt of that equipage 7lb. 9 oz. 2 dwts. What quantity of plate had the Butler under his care? Anſ. 125 . . 0 oz. 9dwts.

12. An Apothecary made a compoſition of ſix ingredients, the 1ſt of which weighed 17lb 9 oz. 6dr. 2ſcr. 19gr.; the 2d, 9lb. 7 oz. 5dr. 1ſcr. 14gr.; the 3d 3lb. 6dr. 1ſcr. 12gr.; the 4th 24lb. 3 oz. 1 dr 2ſcr. the 5th 11 oz. 15gr.; and the 6th 1lb. 5 oz. 4dr. 1ſcr. 19gr. What was the weight of the whole?

Anſw. 57lb. 2 oz. 1dr. 1ſcr. 19gr.

13. A Grocer bought of a Merchant, ſugars weighing 7cwt. 3qr. 7lb. teas, 1cwt. 2qr. 17lb. 13 oz. coffee 3qr. 17lb 15 oz, ſpices, 4cwt. 3qr. 27lb. 9 oz. ſix casks of currants, each 3cwt. 2qr. 17lb. 12 oz. five casks of raiſins, each 3cwt. 3qr. 26lb. 14 oz and three bags of hops, each 2cwt. 1qr, 13 oz What is the weight of the whole? Anſw. 64cwt. 5lb. 10 oz.

14. From the city A to B is 8 miles, 6 fur. 36 poles; from B to C is 5m. 7fur. 23p. from C to D is 36m. 19p from D to E 163m. 2fur. and from E to F is 7fur. 21p. What is the diſtance between A and F. Anſw. 215 miles, 19 poles.

5. A Mercer bought five pieces of cloth, the firſt of which contained 49 yds. 2qrs. the ſecond, 34 yds. 3qrs. 3 nls. the third, 17 yds. 3 nls. the fourth, 19 yds. and the fifth, 143 yds. 2 nls. How many yards did he buy in all? Anſ. 263 yards 3 qr.

16. There are 4 pieces of land, the 1ſt of which meaſures 42ac. 2r. 17p. the 2d 53ac. 3r. the 3d 9ac. 17p. and the 4th 160ac. 3r. What is the ſum of their meaſures?

Anſw. 266ac. 34 poles.

17. A Gentleman bought of a Wine Merchant, of port wine 6 tuns, 3hhds. 46gal. of Bourdeaux claret 2hhds. 56 gal. 7pts. of Madeira 3hhds, 24gal. 5pts. and of Mountain 1tun, 37gal. What quantity did he buy in all?

Anſw. 9 tuns, 2hhds, 38gal. 4pts.

18. An Inn-keeper bought ale of a wholeſale Brewer as follows, viz. at one time 4hhds, 34gal, at another 3hhds. 19gal. 2qrts. at another 47gal. 1pt. and at another 8hhds. How much did he buy in all?

Anſw. 16hhds. 49gal. 2qrts. 1pt.

19. A Corn Merchant bought, of wheat 15 laſts, 1 wey, 3qr. 6buſh. of rye 4qrs. 7buſh. 3pks. of oats 30 laſts, 3qr. of beans 1 wey, 2qr. 2pks. and of peas 9qr. How much did he buy in all? Anſ. 48 laſts, 2qr. 6buſh. 1pk.

20. When B was born, A was 4 ys. 11 ms. 3 ws. old; when C was born, B was 7 ys. 9 ms. 2 ws. 5 ds. old; when D was born, C's age was 13 ys. 2 ws. 6 ds. 17 hrs. and when E was born D's age was 9 years. What was A's age when E was born? Anſ. 34 years 9 months, 4 days, 17 hours, reckoning 13 months to the year.

21. England was conquered by *William* I. Oct. 4, 1066; his ſon *William* II. came to the Crown Sept. 9, 1087, and left it Aug. 2, 1100; *William* III. received it February 3, 1689, and died March 8, 1701: How many Days did each of theſe Princes govern ſeparately, and how many together, reſpect being had to the intercalary Days (added to February, every leap year) as they roſe in the courſe of time? Anſ. Will. I. 7645 Days, Will. II. 4710 Days, Will. III. 4416 Days, and altogether 16771 Days

Note. Every fourth year being Leap-Year, to find which are ſuch, divide the year of our Lord by 4, and when nothing remains, thoſe are the Leap-Years; to ſuch therefore you add one Day to 365.

§ VIII. *SUBTRACTION of different Denominations.*

RULE.

1. PLACE the leſs number under the greater, as directed in addition of different denominations, and under them draw a line.

2. Begin at the leaſt denomination, take the under numbers from the upper, and ſet down their reſpective remainders below them.

3. But if the under number in any denomination happens to be greater than the upper, borrow one, i. e. add as many to the upper number as makes one of the next

ſuperior denomination, and ſubtract as before, and ſo on to the laſt or greateſt denomination, which ſubtract as integers.

EXAMPLES of MONEY.

	Ex. 1. L.	s.	d.	Ex. 2. L.	s.	d.	
From	767	14	7½	772	12	3¼	Minuend
Take	213	7	2¼	189	13	7¾	Subducend
Rem.	554	7	5¼	582	18	7½	
Proof	767	14	7½	772	12	3¼	

3. What is the difference between 136l. 13s. 1½d. and 98l. 16s. 9¾d.? Anſ. 37l. 16s. 3¾d.

4. A Gentleman bought an eſtate, valued at 34761l. 13s. 4d. his caſh only amounted to 21967l. 17s. 7½d. how much muſt he ſend for to his Banker, to enable him to pay for it?

	£	s.	d.
Value of the Eſtate — —	34761	13	4
His ready money — —	21967	17	7½
He muſt ſend to his Banker for	12793	15	8½

5. What ſum when added to 1367l. 14s. 7¾ will make it 2000l.? Anſ. 632l. 5s 4¼d.

6 What 5 different ſums of money added togather will make 3674l. 13s. 9¼.?

To ſolve this Queſtion, ſet down at pleaſure any four of the numbers, take their ſum for a ſubducend, make the required number a minuend, and the remainder will be the 5th number?

	L.	s.	d.		L.	s.	d.	
The 4 Nos. taken at pleaſure.	267	13	4½	Minuend	3674	13	9¼	
	300	19	10¾	Subducend	1490	19	9½	
	401	14	7¼					
	520	11	11	Rem.	2183	14	0¼	5th No.
their ſum	1490	19	9½	Proof	3674	13	9¼	

7. A lent B 404l. 13s. 6d. and ſome time after 260l. 17s. more; he alſo ſold him a horſe on truſt valued at 10 guineas. B paid him at three ſeveral times 80 guineas each, at another time gave him a note upon C, for 100l. and ſold him goods to the value of 69l. 17s. 4d¼. I demand how the balance ſtands between them?

A's BILL againſt B.

	L.	s.	d.
1. Caſh lent	404	13	6
2. Ditto	260	17	0
3. Horſe on Truſt	10	10	0
B's whole debt,	676	0	6
A's whole debt,	421	17	4¼
Balance due to A	254	3	1¾

B's BILL againſt A.

	L.	s.	d.
1. Guineas at three ſeveral payments 80 each.	252	10	0
2. Note upon C, value	100	0	0
3. Goods valued at	69	17	4¼
A's whole debt	421	17	4¼
Balance due to A	254	3	1¾
Proof	676	0	6

8. A Steward received Rents from ſeveral perſons as follow: from A 120l. 10s. 7½d. from B 210l. 13s. 2d. from C 314l. 14s. 11d. and from D 96l. 12s. 7¾d. He remitted ~~to his Lord~~ in caſh 200l. in Bank notes 156l. he paid for repairs done to the eſtate 26l. 10s. 6d. his own ſalary ~~for a quarter~~ was 20l. 10s. — What remains in his hands due to the Lord? Anſ. 339l. 10s. 9¼d.

9. A Trader failing was indebted to A 71l. 12s. 6d. To B 34l. 9s. 9d. To C 16l. 8s. 8d. To D 44l. To E 66l. 7s. 6d. To F 11l. 2s. 3d. To G 19l. 10s, and to H a fine of thirty marks. At the time of this diſaſter he had by him in caſh 3l. 13s. 6d. in *commodities* 23l. 10s. in *houſhold furniture* 13l. 8s. 6d. in plate 7l 18s. 5d. in a *tenement* 56l. 15s. in recoverable book debts, 87l. 13s. 10d. Suppoſing theſe things faithfully ſurrendered to his creditors, what will they then loſe by him? Anſ. 9[illegible]l. 5d.

10. A *chaiſe, horſe,* and *harneſs,* were together valued at 100l. the horſe and harneſs was worth 36l. 14s. 6d. the *chaiſe* and harneſs were eſteemed at 79l. 19s. 9d. Their ſeveral values are required. Anſ. 79l. 19s 9d + 36l. 14s 6d. —100 = 16l. 14s. 3d. value of the harneſs; hence the value of the *horſe* is 20l. 3d. and of the *chaiſe* 63l. 5s. 6d.

11. A gave a *bond* for 114l. 10s. the *intereſt* came to 19l. He then paid off 40 guineas, and gave a freſh bond for what was behind. By that time there was 13l. 4s 8d. due on the ſecond for *intereſt*, he paid off 37l. 14s. 2d. more, took up the *old bond*, and ſigned a *new one* for the *reſidue*. The *principal* again ran on till there was 9l. 11s 3d. more due, and then he determined to take the new bond up. What money had his *creditor* to receive? Anſ. 76l. 11s. 9d.

12. A *Merchant* at his out-ſetting in *trade* owed 280l. He had in caſh 1764l. 14s. 6d. in wares 567l. He cleared the firſt year 316l. What at the years end was his neat *balance*? Anſ. 2367l. 14s. 6d.

Examples of Weights, Meaſures, &c.

1. *Troy Weight.*

	lb.	oz.	dwts.	gr.
From	74	7	17	14
Take	55	9	13	15
Rem.	18	10	3	23
Proof	74	7	17	14

2. *Apothecaries Weight*

lb.	oz.	dr.	ſc.	gr.
96	7	2	1	13
69	9	5	0	14
26	9	5	0	19
96	7	2	1	13

3 *Avoir-*

3. *Avoirdupoiſe Weight.*

	tons	c.	qr.	lb.	oz.	dr.
From	27	12	1	15	13	14
Take	18	16	2	13	15	15
Rem.	8	15	3	1	13	15

4 *Cloth Meaſure.*

yds.	qr.	nls.
371	1	2
186	3	3
184	1	3

5. *Land Meaſure.*

	A.	R.	P.
From	261	1	21
Take	192	2	39
Rem.	68	2	32

6. *Long Meaſure.*

deg.	lea	m.	fur.	pl.	yds.	ft.	in.	b.c.
39	14	1	3	17	2	1	9	1
19	17	1	5	21	3	1	9	2
19	16	2	5	35	3½	2	11	2

7. *Wine Meaſure.*

	tons	hhds.	gal.	pts.
From	36	2	32	3
Take	29	3	47	7
Rem.	6	2	47	4

8. *Ale and Beer Meaſure.*

hhds	k.	gal.	pts.
76	1	16	5
67	2	16	6
8	1	16	7

9. *Dry Meaſure.*

	laſt	qr.	buſh.	p.	g.
From	7	5	5	3	0
Take	6	7	6	3	1
Rem.	0	7	6	3	1

10. *Time.*

mon.	ws.	ds.	hrs.	m.
11	2	3	13	19
9	3	5	16	39
1	2	4	20	40

11. A *Silver-ſmith* bought 5 *ingots* of ſilver each weighing 5 lb. 9 oz. 17 dwts. 19 gr. Out of which he made and ſold 3 *tankards*, each weighing 1 lb. 7 oz. 19 dts. ſix *cups*, each 9 oz. 16 dwts. 13 gr. eight ſalts, each 4 oz. 18 dwts. twelve ſnuff boxes, each 3 oz. 23 gr. and 16 ſpoons, each 1 oz. How much ſilver had he left? Anſ. 11 lb. 6 oz. 17 dwts. 5 gr.

12. An *Apothecary* made a compoſition of ſeveral drugs weighing 13 lb. 6 oz. 5 dr. He at one time ſold 7 lb. 7 dr. 1 ſcr. 14 gr. and at another time 3 lb. 9 oz. How much of the compoſition had he left? Anſ. 2 lb. 8 oz. 5 dr. 1 ſcr 6 gr.

13. Received in lieu of two golden repeaters, ſent to Jamaica, in 1763, the 5 *cheſts* of *indigo* following; and on a like adventure, in 1765, the ſubſequent five cheſts: The *queſtion* is, how much indigo I had leſs the ſecond time than the firſt?

Anno 1763.	C.	qr.	lb.		lb.	Anno 1765.	C.	qr.	lb.		lb.
No. 1	2	3	16	Tare	24	No. 1	1	3	27	Tare	19
2	4	1	17	—	27	2	2	1	27	—	26
3	7	2	10	—	25	3	3	0	18	—	25
4	3	0	16	—	19	4	5	1	0	—	23
5	6	1	0	—	26	5	2	3	11	—	18

Anſ. 8 c. 2 qr. 4 lb. neat.

14. The diſtance of the Towns A and B is 19 miles 7 furlongs, 126 yards; alſo of A and C, 100 miles, What is the diſtance of B and C? Anſ. 80 miles, 94 yards.

15. There is 39 gal. 3 qts. 1 pt. of wine, in a hogſhead; how much put to it will fill up the hogſhead? Anſ. 23 gal. 1 pt.

16. A Mercer bought 3 pieces of cloth each containing 27 yds. 3 qr. 2 nls. he at one time ſold 44 yds. 1 qr. 3 nls. and at another 13 yds. 3 qr. 2 nls. How much has he left? Anſ. 25 yds. 1 qr. 1 nl.

17. There is a field whoſe content is 18 ac. 1 r. 17 pls. of which 9 ac. 2 r. 27 pls is ſown with wheat and the reſt with rye. What quantity then is ſown with rye? Anſ. 8 ac. 2 r. 30 p.

18. What quantity of Ale when added to 17 hhds. 27 gal. 7 pts. will make it 20 hogſheads? Anſ. 2 hhds. 33 gal 1 pt.

19 There are two quantities of wheat which when added together make 16 laſts, 7 qr. 7 buſh. 3 pks. 1 gal. the leſs quantity is 6 laſts, 5 qr. 6 buſh. What is the greater? Anſ. 10 laſts, 2 qr. 1 buſh. 3 pks. 1 gal.

§ IX. MULTIPLICATION of different Denominations.

RULE.

1. PLACE the ×er under the leaſt denomination of the multiplicand.

2. Multiply the leaſt denomination by the ×er, ſee how many ones of the next ſuperior denomination are contained in the product, as in addition of different denominations.

3. Set down the odds, and carry the ones to the product of the next ſuperior denomination, with it proceed as before; and in like manner with all the other denominations to the greateſt.

Note, Multiplication and Diviſion of different denominations, muſt be learned together, becauſe they reciprocally prove each other: therefore the learner is deſired to turn to § X. and learn it along with § IX.

EXAMPLES of MONEY.

1. what is the value of 9 yds. of velvet

	l.	s.	d.
at	2	12	7½
per yd.			9
Anſ	23	13	7½

2. Tea, 3lb at

		s.	d.
		8	9¾
			3
Anſ.	£ 1	6	5¼

3. Knives 5 doz. at

		s.	d.
		9	6½
			5
Anſ.	£	17	8½

4. Sugar 7 lb. at 8¾d. anſ. 5s. 1¼d

5. Tobacco 8lb. at 1s. 7½d anſ. 13s.

6. Wine 10 gal. at 9s. 6d. anſ. 4l. 15s.

7 Sugar 9C. at 4l. 17s. 11¾d anſ. 44l. 1s. 9¾d.

8. Beef 7 ſtone at 3s 4½d anſ. 1l. 3s. 7½d

If the ×er be a large compoſite number made up of ſeveral factors ×ed together, it is commonly the beſt way to × ſucceſſively by the factors inſtead of the whole ×er. But note, for the ſake of eaſe, never × by any number above 10.

9. What is the value of 45lb. of tea at

	l.	s.	d.
at		7	8½
			9
price of 9lb.	£ 3	9	4½
			by 5
price of 45lb.	17	6	10½

10. Raiſins 100 lb. at 7½

			7½
			10
price of 10lb.		6	3
			10
of 100lb.	3	2	6

11. Tobacco, 80lb. at 1s. 8½d. anſ. 6l. 16s 8d

12. Cloth, 35 yds. at 8s. 6d anſ. 14l. 7s. 6d.

13. Silver 81 oz. at 6s. 8½d. anſ. 27l. 3s. 4½d.

14. Razors 90 groſs at 3l. 13s. 7½. anſ. 331l. 6s. 3d.

15 Knives 70 groſs at 5l. 10s. 5½d. anſ. 386l. 12s. 1d.

If the ×er is not compoſed of ſmall numbers ×ed together; find two or more numbers whoſe product comes neareſt: then × as before, and add what is wanting, or ſubtract what is over.

When the given ×er conſiſts of ¼, ½, or ¾; take ¼, ½, or ¾, of the multiplicand and add it to the product. But in proving it by diviſion, ſubtract the value of the ¼, ½, or ¾ from the given ſum, and divide the remainder by the integers.

16. Knives 37 doz. l. s d
at 2 5 $7\frac{1}{4}$
6

price of 6 doz. is 13 13 $10\frac{1}{2}$
6

price of 36 doz. is 82 3 3
+ price of 1 doz. 2 5 $7\frac{1}{4}$

price of 37 doz. 84 8 $10\frac{1}{4}$

17. Sciſſars 47 groſs
at 3 10 $5\frac{1}{4}$
8

price of 8 groſs is 28 3 6
6

price of 48 groſs 169 1 0
—price of 1 groſs 3 10 $5\frac{1}{4}$

price of 47 groſs 165 10 $6\frac{3}{4}$

In the following queſtions x is put to repreſent the given price.

l. s. d.
18 Gold $67\frac{1}{2}$ oz. at 3 10 $7\frac{1}{2}$
8

$8\times x=8x$ 28 5 0
8

$8x\times 8=64x$ = 226 0 0
$x\times 3=3x$ = 10 11 $10\frac{1}{2}$
$x\div 2=\frac{1}{2}x$ = 1 15 $3\frac{3}{4}$

Anſ. $67\frac{1}{2}x$ 238 7 $2\frac{1}{4}$

19. Coffee $134\frac{3}{4}$ oz. at
$x=$ 0 $5\frac{3}{4}$
10

$x\times 10=10x=$ 4 $9\frac{1}{2}$
10

$10x\times 10=100x=$ 2 7 11
$10x\times 3=30x=$ 0 14 $4\frac{1}{2}$
$x\times 4=4x=$ 0 1 11
$x\div 2=\frac{1}{2}x=$ 0 0 $2\frac{3}{4}$
$\frac{1}{2}x\div 2=\frac{1}{4}x=$ 0 0 $1\frac{1}{4}$

Anſ. $134\frac{3}{4}x=$ 3 4 $6\frac{1}{2}$

20. 6742 at $x=$ 8 6
10

$x\times 10=10x=$ 4 5 0
$10x\times 10=100x=$ 42 10 0
10

$100x 10=1000x=$ 425 0 0
6

$1000x\times 6$ $6000x=$ 2550 0 0
$100x\times 7=700x=$ 297 10 0
$10x\times 4=40x=$ 17 0 0
$x\times 2=2x=$ 0 17 0

Anſ. $6742x=$ 2865 7 0

21. Cloth $789\frac{1}{2}$ yds at 1s $7\frac{1}{2}$d Anſ. 64l. 2s. $11\frac{1}{4}$d.

22. Snuff-Boxes $103\frac{1}{2}$ doz. at 17s. $6\frac{1}{2}$d. Anſ. 90l. 15s. $6\frac{3}{4}$d.

23. Salts $376\frac{1}{2}$ doz. at 1l. 17s. $8\frac{3}{4}$d. Anſ. 710l. 5s. $0\frac{1}{2}$d.

24. $1367\frac{3}{4}$ at 12s. $9\frac{1}{2}$d. Anſ. 874l. 15s. $9\frac{1}{2}$d.

25. 7098 at $1\frac{3}{4}$d. Anſ. 51l. 15s. $1\frac{1}{2}$d.

26. 14087 at 16s $8\frac{1}{2}$d. Anſ. 11768l. 10s. $3\frac{1}{2}$d.

Examples of Weights, Meaſures, &c.

lb. oz dwts gr.
1. Multply 36 11 16 13
by 496
product 18344 10 4 16

lb. oz. dr. ſc. gr.
2. mul. 16 5 6 1 12
by 45
prod. 741 9 6

C. qr. lb.
3. mult. 15 3 12 by 17
prod. 269 2 8

miles fur: pols
4. mult: 23 5 25 by 9
prod. 213 2 25

yd: qrs nls. by 27.
5. mult: 48 3 3
prod. 1321 1 1

Tuns hhds: gal pts.
6. mul. 16 3 15 5 by 18
prod. 302 2 29 2

acres roods p:
7. mult. 34 2 16 by 5
prod. 173 acres.

hhds. gal. pts.
8 mult. 45 32 7 by 27
prod. 1232 20 5

c. qr. lb. oz. drs.
9. 76 3 17 14 13 by 464
prod. 35686 c. 1 qr. 1l 9 oz

la: w: qr. buſh:
10. mul 1 1 3 5 by 32
prod. 59 1 1

m. w. d. h. m.
11. mul 7 3 4 22 14 by 7
prod. 55 1 6 11 38

Degrees
12. mult: 5 17 10 20 by 42
prod. 216 44 3 40

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§ X. DIVISION *of different Denominations.*

RULE.

1. PLACE the Diviſor and Dividend as in ſimple Diviſion.

2. Begin at the higheſt denomination and divide each by the Diviſor, placing the anſwer in the Quote, which anſwer, will be of the ſame denomination with that divided:

3. If there be a remainder, after ÷ing any of the denominations except the laſt; × that remainder by ſo many ones of that denomination as makes one of the next leſs; adding thoſe of the ſaid leſs denomination to the product, if ſuch there be, and divide as before.

Thus proceed thro' all the denominations.

EXAMPLES.

1. If 9 yards of velvet coſt 23l. 13s. 7½ what will 1 yard coſt?

```
        l.   s.   d.
   9)  23   13   7½ (2l.
       18
       ——
Rem.    5l.
       20  + 13s.
       ——
   9) 113       (12s.
        9
       ——
       23
       18
       ——
rem.    5s.
       12  + 7d.
       ——
   9(  67       (7d.
       63
       ——
        4
        4  + 2 qrs.
       ——
   9)  18       (2 q.
       18
       ——
        0
```

EXPLANATION.

Say how oft 9 in 23 2 times, which ſet in the quote; then twice 9 is 18, which —ed from 23, leaves 5l. which × by

20, +ing 13s. to the product, and the ſum is 113s. Then ſay how oft 9 in 113? Anſ. 12 times, with a remainder of 5. This remainder × by 12 +ing 7 to the product and the ſum is 67d. in which 9 may be had 7 times, with a remainder of 4. This remainder × by 4, +ing two to the product and the ſum is 18, in which 9 may be had twice without a remainder. So the quote is 2l. 12 . 7½ which proves the firſt ſum in compound multiplication. And ſo of all the reſt.

2. What is tea per lb. if 3lb. coſt 1. 6s. 5¼d. ? Anſ. 8s. 9¾d.

3. What is the 5th part of 17s. 8½d. Anſ. 3s. 6½d.

4. What is the 7th part of 5s. 1½d. ? Anſ. 8¾d.

5. What is the 8th part of 13s. ? Anſ. 1s. 7½d.

6. What is wine per gal. if 10 gals. coſt 4l. 15s. ? Anſ. 9s. 6d.

7. If 44l. 1s. 9¾d. be equally ÷ed amongſt 9 perſons, what is the ſhare of each ? Anſ. 4l. 17s. 11¾d.

8. What is beef per ſtone, if 7 ſtone coſt 1l. 3s. 7½d Anſ. 3s. 4½d.

If the ÷ſor exceed 12, and is a compoſite number, it is beſt to divide continually by the factors, and ſet the quotes under the dividends, as in ſimple diviſion.

But if the ÷ſor be not a compoſite number, you muſt divide by it after the manner of long diviſion.

9. Divide 17l. 6s. 10½d. by 45

	l.	s.	d.
9)	17	6	10½
5)	1	18	6½
Anſ.	0	7	8½

10. Divide 3l. 2s. 6d. by 100.

	l.	s.	d.
10)	3	2	6
10)	0	6	3
Anſ.	0	0	7½

11. If 6l. 16s. 8d. be divided amongſt 80 perſons what is each man's ſhare? Anſ. 1s. 8½d.

12. Divide 14l. 17. 6d. by 35.

	l.	s.	d.
7)	14	17	6
5)	2	2	6
Anſ.	0	8	6

13. Divide 27l. 3s. 4½d. by 81.

	l.	s.	d.
9)	27	3	4½
9)	3	0	4½
Anſ.	0	6	8½

14. Divide 331l. 6s. 3d. by 90. Anſ. 3l. 13s. 7½d

15. Divide 386l. 12s. 1d. by 70. Anſ. 5l. 10s. 5½d.

16. Divide 84l. 8s. 10¾d. by 37. Anſ. 2l. 5s. 7¼d.

17. Divide 165l. 10s 6¾d. by 47. Anſ. 3l. 10s. 5¼d.

18. If 67 oz. of ſilver coſt 236l. 11s. 10½d what is it per oz.? Anſ. 3l. 10s. 7½d.

19. If 134 oz. of coffee coſt 3l. 4s. 2½d. what is it per oz.? Anſ. 5¾d.

20. Divide 2865l. 7s. by 6742. Anſ. 8s. 6d.

21. Divide 64l. 2s. 1½d. by 789. Anſ. 1s. 7½d.

22. Divide 90l. 6s. 9½ by 103. Anſ. 17s. 6½d.

23. Divide 709l. 6s. 2d. by 376. Anſ. 1l. 17s. 8¾d

24. Divide 174l. 6s. 2½d. by 1367. Anſ. 12s. 9½d.

25. Divide 51l. 15s. 1½d. by 7098. Anſ. 1¾.d.

26. Divide 11768l. 10s. 3½d. by 14087. Anſ. 16s. 8½.

Examples of Weights, Meaſures, &c.

1. Divide 18344 lb. 10 oz. 4 dwts. 16 gr. by 496.

lb. oz. dwts. gr.
496) 18344 10 4 16 (36lb.
1488
3464
2976
Rem. 488
12 + 10
496) 5866 (11
5456
Rem. 410
20, + 4
496) 8204 (16
496
3244
2976
Rem. 268
24, + 16
496) 6448 (13
496
1488
1488 Anſ. 36 lb. 11 oz. 16 dwts. 13 gr.

2. Divide 741 lb. 9 oz. 6 dr. by 45.

	lb.	oz.	dr.	ſc.	gr.
9)	741	9	6	0	0
5)	82	5	0	2	0
Anſ.	16	5	6	1	12

3. Divide 269 C. 2 qr. 12 lb. by 17. Anſ. 15 C. 3 qr. 12 lb.

4. Divide 213 m. 2 f. 25 p. by 9. Anſ. 23 m. 5 f. 25 p.

5. Divide 1321 yds. 1 qr. 1 nl. by 27. Anſ. 48 yds. 3 qr. 3 nls.

6. Divide 302 tuns, 2 hhds. 29 gal. 2 pts. by 18. Anſ. 16 tuns, 3 hhds. 15 gal. 5 pts.

7. Divide 173 acres, by 5. Anſ. 34 acr. 2 r. 16 poles.

8. Divide 1232 hhds. 20 gal. 5 pts. by 27. Anſ. 45 hhds. 32 gal. 7 pts.

9. Divide 35686 C. 1 qr. 1 lb. 9 oz. by 464. Anſ. 76 C. 3 qr. 17 lb. 14 oz. 13 dr.

10. Divide 59 la. 1 w. 1 qr. by 32. Anſ. 1 la. 1 w. 3 qr. 5 buſh.

11. Divide 55 m. 1 w. 6 dys. 11 h. 38 min. by 7. Anſ. 7 m. 3 w. 4 dys. 22 hrs. 14 m.

12. Divide 216 degrees 44 m. 3 ſ. 40 t. by 41. Anſ. 5 degrees 17 m. 10 ſ. 20 t.

13. My purſe and my Money, quoth Jack, is worth 17l. 6s. 10½d. but the money is worth 44 of the purſe: pray what was there in it? Anſ. 16l. 19s. 2d.

14. A Gentleman bought a horſe and harniſh for 52l. 8s. 6d. the horſe coſt 8 times as much as the harniſh. I demand the value of the horſe and harniſh ſeparately?

Anſ. { The horſe coſt 46 l. 12 s.
{ The harniſh 5 l. 16 s. 6 d.

The ſum and difference of any two numbers being given, the numbers themſelves may be found thus: Add the given ſum and difference together, and half that ſum will be the greater number; then from the given ſum ſubtract the given difference, and half that remainder will be the leſs number.

EXAMPLES.

1. There are two different ſums of money, which both together make 7642l. 16s 4½d. the leſs quantity taken from the greater leaves 762l. 9s. 4d. What are the ſums ſeparately?

	l.	s.	d.		l.	s.	d.
Firſt	7642	16	4½	Secondly from	7642	16	4½
Added to	762	9	4	take	762	9	4
2)	8405	5	8½	2)	6880	7	0½
Greater ſum	4202	12	10¼	Leſs ſum	3440	3	6¼

2. A Dealer bought two lots of ſnuff, that together weighed 9 cwt. 100 lb. for 97l. 17s. 6d. Their difference in point of weight was 1 cwt. 72 lb. and of price 8l. 13s. 3d. Their reſpective weights and values are required.

		cwt.	lb.		l.	s.	d.
Anſ.	Lot 1ſt.	5	86	Coſt	53	5	4½
	Lot 2d	4	14		44	12	1½

3. Says A to B, give me your money and I ſhall have 787l. 19s. 11½d. nay replied B, but if you give me ſo much of your money as I now have, you will but have 246l 10s. left. The queſtion is, how much money had each?

		l.	s.	d.
Anſ	A's money	517	4	11¾
	B's money	270	14	11¾

§ XI. *Of* REDUCTION.

REDUCTION is not properly any Rule of Arithmetic different from Multiplication and Diviſion already taught, but rather an application of theſe two rules, in converting numbers of one denomination into another, but ſtill retaining the ſame value; and is generally performed by three *caſes* in the following rule.

RULE.

CASE I. If numbers of a greater denomination are to be reduced to their equivalents of a leſs, multiply continually by all the denominations from the given one to that ſought; adding to each product, by the way, thoſe of the ſame denomination with itſelf, if ſuch there be, and the laſt product will be the anſwer. This is called *reduction deſcending*.

CASE II. If numbers of a leſs denomination are to be reduced to their equivalents of a greater, divide continually by all the denominations from the given one to that ſought, and the laſt quote, with the ſeveral remainders (if any), will be the anſwer. This is called *reduction aſcending*.

CASE III. If numbers of any propoſed *denomination* cannot be reduced to that required by Multiplication or Diviſion alone, both rules are to be uſed promiſcuouſly; for numbers are frequently reduced to ſome convenient denomination by the one rule, in order to be reduced to the required one by the other.

Note, When in reduction aſcending, you have any remainders after ÷ing, they will always be of the ſame denomination with their reſpective dividends.

Examples of MONEY.

1. In 36l. how many farthings ?

L.

By caſe 1. 36

20

720 s.

12

8640 d.

4

Anſ. 34560 farthings.

2. In 34560 farthings how many pounds ?

By caſe ſecond.

qr: 12) 2 | 0s) anſ.

4) 34560 (8640d. (72 | 0 (36l:

3. In 32l. 14s. 6¼d. how many farthings ?

l. s. d.

32 14 6¼

20

654 s. + ing 14.

12

7854 d. + ing 6.

4

Anſ. 31419 qr. + ing 3.

4. In 31419 farthings how many pounds ?

qr. 12) d. 2 | 0)s.

4) 31419 (7854 (65 | 4 (32l.

rem. 3 rem. 6 rem. 14.

Anſ. 32l 14s. 6¼d.

By this time the young Student is ſuppoſed to be well acquainted with the characters in the Introduction, in which characters will now be inſerted the method of working moſt of the queſtions, that are ſolved.

5. In 460 guineas, and 15s. how many crowns? By caſe 3d. 460 guineas × 21, + 15s. = 9675s. ÷ 5 = 1935 crowns.

The above operation is to be thus underſtood; 460 multiplied by 21, and added to 15 is equal to 9675s. which divided by 5 is equal to 1935 crowns.

6. In 1935 crowns how many guineas? By caſe 3d. 1935 × 5 = 9675s. ÷ 21 = 460 guineas, with a remainder of 15s.

7. In 460 guineas, how many farthings? Anſ. 463680 qr.

8. In 540 dollars, at 4s. 4d. per dollar, how many pounds? Anſ. 117l.

9. In 460 marks, how many nobles, groats, and farthings? Anſ. 920 nobles, 18400 groats, and 294400 farthings.

10. In 340l. how many angels, crowns, half crowns, ſixpences, and pence? 340 × 2 = 680 ang. × 2 = 1360 cr. × 2 = 2720 half cr. × 5 = 13600 ſixpences, × 6 = 81600 pence.

11. In 81600 d. how many ſix-pences, half crowns, crowns, angels, and pounds? 81600d. ÷ 6 = 13600 ſixpences ÷ 5 = 2720 half cr. ÷ 2 = 1360 cr. ÷ 2 = 680 ang. ÷ 2 = 340l.

12. If A lends B 1296 guineas when they were valued at 1l. 1s. 6d. a piece: how many muſt B pay A when they are valued at 1l. 1s. a piece? Anſ. 1326 guineas, 18 ſhill.

13. Reduce 987654320 farthings into pounds, ſhillings, and pence. Anſ. 987654320 q. = 246913580 d. = 20576131s. 8d. = 1028806l. 11s. 8d.

14. Reduce 270 moidores into ſevenpence, half-penny pieces. Anſ. 270 moi. = 7290s. = 174960 halfpences = 11664 ſevenpence, halfpence.

15. How many pieces of 4d. of 6d. and of 8d. each of an equal number may be had in exchange for 360l. 16s 6d? By caſe 3 thus, 4+6+8=18d. the diviſor, and 360l. 16s. 6d.=86598d. the dividend; then 86598÷18=4811, the number of pieces of each ſort, viz. of 4d. of 6d. and 8d. which may be proved, thus;

	l.	s.	d.
4 pences 4811 ÷ 3 ÷ 20 =	80	3	8
6 pences 4811 ÷ 2 ÷ 20 =	120	5	6
8 pences 4811 × 2 ÷ 3 ÷ 20 =	160	7	4
Proof	360	16	6

Note, this queſtion is no more than if I had ſaid, in 360l. 16s. 6d. how many 18d. pieces: ſince the ſum of all the parts is equal to the whole. (Ax. 8)

16. Seven perſons viz. A. B. C. D. E. F. and G. have 99999l. 2s. amongſt them, and to be thus divided, viz. as often as A takes a guinea, B is to take a mark, C an Angel, D a noble, E a crown, F half a crown: and G a ſix-pence: Each perſons particular ſhare is required.

Firſt, a guinea, a mark, an angel, a noble, a crown, a half crown, and a ſix pence, when +ed together, make 59s. for a diviſor, and 99999l. 2s.=1999982s. for a dividend, whence 1999982÷59=33898 the number each takes of his reſpective piece; which is proved thus

		l.	s.	d.	
33898 guineas	=	35592	18	0	A's ſhare.
33898 marks	=	22598	13	4	B's ſhare.
33898 angels	=	16949	0	0	C's ſhare.
33898 nobles	=	11299	6	8	D's ſhare.
33898 crowns	=	8474	10	0	E's ſhare.
33898 half crowns	=	4237	5	0	F's ſhare.
33898 ſix-pences	=	847	9	0	G's ſhare
Proof		99999	2	0	(axiom 8)

17. A General of an army diſtributes 47l. 17s. 7½d. among 4 captains, 5 lieutenants, and 60 common ſoldiers,

in the manner following; every captain is to have 3 times as much as a lieutenant, and every lieutenant twice as much as a common ſoldier: I demand their ſeveral ſhares.

The ſhare of a common ſoldier,		10s.	2½d.
of a lieutenant	1l.	0	4½
of a captain	3	1	1½

18. In 254l. 11s. 2½d. how many French crowns, at 4s. 6½d. per crown? Anſ. 1121 crowns.

19. In 1121 French crowns, at 4s. 6½d each, how much ſterling money? Anſ. 254l. 11s. 2½d.

20. In 4260 dollars, at 4s. 6d. each, how many milreas each 6s. 6d.? Anſ. $2949\frac{18}{78}$ milreas.

21. In $2949\frac{18}{78}$ milreas, each 6s. 6d. how many dollars, each 4s. 6d.? 6s. 6d. = 78d. the ×er, and 4s. 6d. = 54d. the ÷for, hence 2949×78, +18 = 230040d. ÷54 = 4260 dollars. The Anſ.

Examples of Weights, Meaſures, &c.

1. In 326lb. 11oz. 17 dwt. 23 gr. Troy wt. how many grains? 326 lb.×12+11 = 3923 oz.×20+17 = 78477 dwt.×24+23 = 1882471 gr. The Anſ.

2. In 1883471 grains troy, how many pounds, ounces and pennyweights? 1883471 gr. = 78477 dwts. 23 gr. = 3923oz. 17 dwts. 23 gr. = 326lb. 11 oz. 17 dwts. 23 gr. Anſ.

3. Reduce 367 lb. 11 oz. 7 dr. 2 ſcr. 17 gr into ounces, drams, ſcruples and grains? 367 lb. 11 oz. = 4415, oz,×8+7 = 35327 dr.×3+2 = 105983 ſrc.×20+17 = 2119677 grains.

4. Reduce 2119677 gr. apothecaries weight into pounds. 2119677 gr. = 105783 ſcr. 17 gr. = 35327 dr. 2 ſcr. 17. gr. = 4415 oz. 7 dr. 2 ſcr. 17 gr. = 367lb. 11 oz. 7 dr 2 ſcr. 17 gr.

5. In 36 tons, 17 c. 2 qr. 23 lb. 13 oz. 14 dr, how many drams? Anſ. 21151710 drams.

6. In 21151710 drams, how many tons?
21151710 ÷ 16=1338981 oz, ÷ 16=82623 lb. ÷ 28=
rem. 14 rem. 13 rem. 23
2950 qr ÷4=737 c. ÷20=36 tons.
rem. 2 rem. 17
The Anſ. 36 tons 17 c. 2 qrs 23lb. 13 oz. 14 dr.

7. How many barley corns will reach from Sheffield to London, the diſtance being 162 miles?
162 m ×8=1296 fur. ×40=51840 poles, ×11=570240 ½ yds. ×18=10264320 in. ×3=30792960 barley corns.

8. If 30792960 barley corns will reach from Sheffield to London, how many miles are theſe places aſunder?
30792960÷3=10264320 inches, ÷ 18=570240, ½ yds. ÷11=51840 poles, ÷40=1296 fur. ÷8=162 miles. Anſ.

9. If the circumference of the earth contains 360 degrees, and each degree 70 Engliſh miles; how many barley corns will reach round the earth? Anſ. 4790016000.

10. In 3 pieces of cloth, each containing 46 yds. 3 qr. 2 nls. how many nls.? Anſ. 2250.

11 In 35364 ells Engliſh, how many ells Flemiſh?
35364×5=176820 qr.÷3=58940 ells Flemiſh. Anſ.

12. In 370 ac. 2 r. 30 p. of land, how many poles? Anſ. 59310.

13. In 32 tuns, 3 hhds. 54 gal. 7 pts of wine, how many pts? 32×4+3=131 hhds. ×63+54=8307 gal. ×8 +7=66463 pints. Anſ.

14. In 66463 pints of wine, how many tuns?
66463÷8=8307 gal. ÷63=131 hhds. ÷4=32 tuns.
rem. 7 rem. 54 rem. 3
Anſ. 32 tuns, 3 hhds. 54 gal. 7 pts.

15. In 36 hhds of ale, how many gallons, pints and cubic inches? Anſ. 36 hhds=1836 gal.=14688 pts. and 1836 gal.×282=517752 cubic inches.

16. In 517752 cubic inches, how many hogſheads, gallons, and quarts of ale? 517752 c in.=1836 gal.=7344 qrts.=36 hogſheads.

17. In 36 chald. 3 qr. 7 buſh. 3 p. 1 gal. of wheat, how many pints? 36×4+3=147 qr.×8+7=1183 buſh.×4+3=4735 pks.×2+1=9471 gal,×8=75768 pints.

18. In 75768 pints, how many chaldrons of wheat? Anſ. 36 chal. 3 qr. 7 buſh. 3 pks. 1 gal.

19. How many days, hours, and minutes, in 1765 Julian years?

Firſt, 1765÷4=441 days 6 hours, produced by the odd hours in every year. Therefore 1765×365,+441=644666 days,×24+6=15471990 hrs, ×60=928319400 minutes. The Anſ.

20. In 1765 months how many ſeconds? 1765×28×24×60×60=4269888000 ſeconds.

21. In 4269888000 ſeconds, how many months, weeks, and days? 4269888000 ſeconds=49420 days, ÷7=7060 weeks÷4=1765, months.

22. How many ſeconds in a ſolar year? Anſ 31556937 ſeconds.

23. How many thirds in 29 days 12 hours 44 minutes 1 ſecond, 45 thirds, or one lunar month? Anſ. 153086505 thirds.

24. The mean time of a lunation, i. e. from new moon to new moon, is 29 days, 12 hours 44 minutes, 1 ſecond, and 45 thirds: I demand then how many lunations are contained in 1765 Julian Years? Firſt 1765 Julianyears =334194984000 thirds, which ÷ed by 153086505 (the thirds in 1 lunation,) is=21830 lunations, 13 days, 18 hours, 43 minutes, 17 ſeconds 30 thirds. Anſ.

25. How many minutes, ſeconds, and thirds, in the 12 ſigns of the Zodiac? Anſ. 12 × 30 = 360 degrees, = 21600 min. = 1296000 ſec. = 77760000 thirds.

26. A cubic foot of water weighs 76lb. troy; and air is 860 times lighter then water: What then is the weight of a cubic foot of air? Anſ. 1 oz. 1 dwt $5\frac{70}{860}$ grs.

27. There is a ſquare field, the length of whoſe ſide is 760 links, how many acres doth it contain? Anſ. 5 acres, 3 roods, 4 poles.

760
760
5 | 77600
4
3 | 10400
40
4 | 16000

Note. to find the area of a ſquare, 1ſt multiply the ſide into itſelf.

2d. Having multiplied the ſide into itſelf, cut off five figures from the right of the product.

3d. The five figures ſo cut off multiply by 4 and from the right of the product cut off five figures more.

4th. Multiply the five figures laſt cut off by 40, and cut off other five figures as before: thoſe figures to the left are acres, roods, and poles.

The reaſon for cutting off five figures from the right hand of the product is obvious; becauſe your diviſor is 100000, Vide table 7 § VI.

28. There is an oblong field whoſe length is 4620, and breadth 370 yards, what is its area in acres?

Note, to find the area of an oblong, × the length into the breadth. 4620 × 370 = 1709400 ſquare yards, which ÷ed by 4840, the ſquare yards in one acre, gives 353 acres, and 880 yards remain.

29. In a triangular field, whoſe baſe is 9 chains, 64 links, or 964 links, and the perpendicular 5 chains, 56 links, how many acres are contained?

Note. To find the area of a triangle, multiply half the baſe into the perpendicular, or half the perpendicular into the baſe, or multiply the baſe and perpendicular together and take half the product; any of theſe ways equally gives the area.

964÷2=482
556

2 | 67992
4

2 | 71968
40

28 | 78720

	A.	R.	P.
Anſ.	2	2	28

30. In a Trapezium whoſe Diagonal is 1660 links, one perpendicular is 702 links, and the other perpendicular is 712 links, what is the area in acres ?

Note, To find the area of a Trapezium, multiply half the ſum of the perpendiculars into the Diagonal, or half the Diagonal into the ſum of the perpendiculars, or multiply the ſum of the perpendiculars into the Diagonal and take half the product, any of theſe ways gives the area.

702+712÷2=1660
707

11620
116200

11 | 73620
4

2 | 94480
40

37 | 79200

	A.	R.	P.
Anſ.	11	2	37.

31. Suppoſe a curve whoſe length is 340 links, with breadths taken at five places as follows, viz. the 1ſt 33 links, the 2d. 48, the 3d 62, the 4th 43, and the 5th 24: What is the area ?

Note, To find the area of a curve, or ſegment in a field, meaſure in a right line from one corner of the figure to the other, and at right angles to this line take as many breadths as you chuſe, but eſpecially the greateſt and leaſt breadths, add all theſe breadths together and divide the ſum by the number of them which gives the mean breadth, this ×ed by the length gives the area.

$33+48+62+43+24=210$ the ſum of the breadths, then $210\div5=42$ the mean breadth, and $340\times42=14280$ ſquare links the area.

N. B. The ſquare, the oblong, the triangle, the trapezium, and curve, are all the figures that are uſed in the practice of Land Meaſure.

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§ XII. The RULE of THREE DIRECT.

THIS Rule is called the Rule of Three, becauſe it teaches by three numbers given, called terms, to find a fourth that ſhall have the ſame proportion to ſome one of the given numbers, as is expreſſed by the other two. For this reaſon it is ſometimes named the *Rule of Proportion*. And becauſe of its excellent and extenſive uſe, it is often called the *Golden Rule*.

RULE.

In this Rule three things are uſually required, viz. *preparation*, *diſpoſition*, and *operation*.

1. As to the preparation, it muſt be obſerved that of the three numbers given in the queſtion, 2 will always be of the ſame kind, and muſt be reduced to the ſame denomination; if they be not ſo already; and if the remaining number conſiſts of ſeveral denominations, that alſo muſt be reduced to ſome ſimple one.

2. To ſtate the queſtion, diſpoſe the prepared numbers ſo, that the firſt and third may be thoſe of the ſame denomination; place that number the third which aſks the queſtion, or contains the demand, and the remaining number will of courſe be the ſecond term.

3. Having thus ſtated the queſtion, for the operation, multiply the ſecond and third terms together; divide the product by the firſt, and the quote thence ariſing will be the fourth term ſought; which fourth term will always be of the ſame name, kind, or ſpecies with the ſecond.

Note 1. When it can conveniently be done, multiply or divide as in multiplication and diviſion of different denominations.

2. When there happens to be a remainder, it will either make a fractional part, or it muſt be reduced to the name next below the laſt quotient, and divided as before; ſo ſhall the quote be ſo many of the ſaid next name; do this ſo long as there is any remainder, till you have reduced it to the leaſt name, and all the quotes together will be the anſwer.

EXAMPLES.

1. If 3lb. of tea coſt 18 ſhillings, what will 12lb. coſt at the ſame rate?

lb. s. lb.
If 3 : 18 :: 12
12

s. l. s.
3) 216 (72 = 3 12 the Anſ.
21

6
6

0

Note. 12lb. being the term which contains the demand, it muſt occupy the third place, 3lb. being of the ſame kind and denomination, muſt be put in the firſt place; and 18s. which is of the ſame kind with the anſwer, muſt be in the ſecond place.

2. If 12lb. of tea coſt 3l. 12s. what will 3lb. coſt? If 12lb. : 72s. :: 3lb. 72×3÷12=18s. the Anſ.

3. If 18s. will buy 3lb. of tea, how much will 72s. buy at that rate? If 18s. : 3lb. :: 72s. : 72×3÷18=12lb. the Anſ.

4. If 72s. will buy 12lb. of tea, how much will 18s. buy at the ſame rate? If 72s. : 12lb. :: 18s : 18×12÷72=3lb. the Anſ.

Note. The three laſt queſtions are only the firſt varied: by ſuch variations may all the queſtions in the Rule of Three be proved. Or thus, in the Rule of Three Direct, the product of the firſt and fourth terms, will always be equal to the product of the ſecond and third, when the work is right.

5. What will 3 quarters of a yard of velvet coſt, when 21½ yards is worth 22l. 10s. 6d?

Here 22l. 10s. 6d.=901 ſixpences; and 21½yds=86 quarters.

```
              qr.    ſix d.    qr.
    Then if   86  :   901  ::   3
                        3
                    ------   ſix d.   s.   d.
               86)  2703     (31   =  15 : 6
                    258
                    ----
                     123
                      86
                    ----
                      37
                       6
                    ----
                 86) 222 (2d.
                     172
                     ---
                      50
                       4
                     ---
                 86) 200 (2
                     172
                     ---
                      28
                     ---
                       s.   d.   qr.
            Anſ.      15    8    2 28/86
```

qr. l. s. d. qr.
Or thus, if 86 : 22 10 6 :: 3
3

86) 67 11 6 (0l.
20

86) 1351 (15s.
86

491
430

61 Rem.
12

86) 738 (8d.
688

50 Rem.
4

86) 200 (2 qr.
172

28 Rem.

6. What will $21\frac{1}{2}$ yards of velvet coſt, when 3 quarters is worth 15s. 8d. $2\frac{28}{86}$q.?

Here 15s. 8d. $2\frac{28}{86} = 754\frac{28}{86}$q. and $21\frac{1}{2} = 86$qr. Then if 3qr. : $754\frac{28}{86}$q : : 86 : $754 \times 86 + 28 \div 3 = 21624$ q. = 22l. 10s. 6d. the Anſ.

7. If $21\frac{1}{2}$ yds.=(86qr.) of cloth coſt 22l. 10s. 6d.= 21624q.) how many yards may be bought for 15s. 8d. $2\frac{28}{86}$q = $754\frac{28}{86}$q. ?

If 21624 q. : 86 :: $754\frac{28}{86}$q : $754 \times 86 + 28 \div 21624 =$ 3 qr. Anſ.

8. Silver at 5s. 8d. per ounce; What's the value of 4 tankards whoſe weights are as follows? viz.

	lb.	oz.	dwts	grs.
1ſt weighs	2	11	19	23
2d ditto	3	7	17	7
3d ditto	3	8	14	4
4th ditto	4	0	7	9
The weight of them all	14	4	18	19 =

83011 grains. 1 oz.=480 grains; and 5s. 8d. = 68d.

Then as 480 gr. : 68d. :: 83011 gr. 48l. 19s. 11d. 3 q. $\frac{272}{480}$ the Anſ.

Note. When four terms are placed as in the form above, they are to be underſtood and read thus: as the 1ſt term is to the 2d, ſo is the 3d to the 4th. That is the product of the 2d and 3d terms divided by the 1ſt produces the 4th term, of the ſame denomination with the 2d, which 4th term, when reduced to the required denomination is wrote in the 4th place for the anſwer.

9. If 83011 grains of ſilver coſt 48l. 19s. 11d. 3q. $\frac{272}{480}$ =(47039$\frac{272}{480}$ q what is valued at per ounce?

As 13011 gr. : 47039$\frac{272}{480}$q :: 480 gr. : 5s. 8d. the Anſ.

10. A Merchant bought an hhd. of wine for 32l. how much per gallon muſt he ſell it for to gain 4l. 10s. by it? It muſt all be ſold for 32l. +4l. 10s.=36l. 10s.=730s.

Then as 63 gal. : 730s :: 1 gal. : 11s. 7$\frac{3}{63}$ d. the Anſ.

11. If 1 gallon of wine coſt 11s. 7$\frac{3}{63}$ d. what will a hhd. coſt at that rate? Anſ. 36l. 10s.

12. If 1 be advanced to 2, and 2 be advanced to 3. To how much will 20 in ſuch proportion advanced be?

1+2=3 the ſum before advanced; 2+3=5 their advanced ſum: then as 3 : 5 :: 20 : 33$\frac{1}{3}$ the Anſ.

13. If 2 be diminiſhed to 1, and 3 be diminiſhed to 2; what will $33\frac{1}{3}$ be diminiſhed to in the ſame proportion?

As 5 : 3 :: $33\frac{1}{3}$: $33\times3+1\div5=20$ the Anſ.

14. There is a ciſtern with two evacuating cocks viz. A and B; the cock A can empty it in 56 hours, and B in 42; in how many hours can they empty the ſame, if they run together?

Firſt $56+42=98$; then ſay, as 98hrs : 56hrs :: 42hrs : 24h. the Anſ.

Note. The work of ſome ſtatings in the Rule of Three Direct, may be much ſhortened when they can be done by any of the three following contractions.

I. Divide the 2d term by the 1ſt, ×ing the quote into the third; and the product is the anſwer.

15. If 7 yards coſt 56s. what will 20 coſt?
As 7 yds. : 56s. :: 20yds : 160s $=$ 8l. the Anſ.
Thus $56\div7=8$, and $20\times8=160$s.

II. Divide the 3d term by the 1ſt, ×ing the quote into the 2d, and the product is the anſwer.

16. If 8lb. of ſugar coſt 6s. what will 1cwt. coſt?
As 8lb. : 6s. :: 112lb. : 84s. $=$ 4l. 4s. the Anſ.
Thus $112\div8=14$, and $14\times6=84$s.

III. When the firſt term and either of the other two, can be exactly ÷ed by ſome common ÷for; then ÷ them, and take the quotes inſtead of theſe terms; proceeding thus as oft as you can.

17. If 9 yards coſt 12s. what will 24 yds. coſt?

yds. s. yds.
3) 9 : 12 :: 24
3) 3 : 12 :: 8 contracted by ÷ing the 1ſt and 3d by 3.
1 : 4 :: 8 con. by ÷ing the 1ſt and 2d by 3.
4
———
32s. the Anſ.

18. If a hhd of wine coſt 49l. 14s. what will 72 gallons coſt? Anſ. 56l. 16s.

gal. l. s. gal.
63 : 49 14 : : 72
Contr. 7 : 49 14 : : 8 by ÷ing the 1ſt and 3d terms by 9.
Contr. 1 : 7 2 : : 8 by ÷ing the 1ſt and 2d by 7.
8
———
56 16

19. If $\frac{4}{11}$ of an ounce of gold coſt 28s what will $\frac{7}{11}$ oz. coſt?

oz. s. oz.
$\frac{4}{11} : 28 : : \frac{7}{11}$
4 : 28 : : 7 cont. by rejecting the ÷ for 11
1 : 7 : : 7 : 49s. the Anſ

20. A certain ſteeple projected upon level ground a ſhadow to the diſtance of 57 yards, when a four foot ſtaff perpendicularly erected caſt a ſhadow of 5 feet 4 inches: what was the height of the ſteeple? Here 5 feet 4 inches =64 inches, and 57 yards=2052 inches. Then as 64 in. : 4 feet : : 2052 : 42 yds. 2 f. 3 inches the Anſ.

21. If a wall 6 feet thick, 9 feet high, and 432 feet long, coſt 720l. in building, what will be the price of a wall of the ſame materials, that is 12 feet thick, 18 feet high, and 576 feet long?
9×6×432=23328 cubic feet: and 12×18×576=124416 cubic feet. Then as 23328 cu. feet : 720l. : : 124416 cu. feet : 3840l. Anſ.

In the mint of England a pound of gold, that is 11 ounces fine and 1 allay, is at this time coined into 44 guineas and a half: I demand how much ſterling a pound of pure gold is worth obſerving that the allay is valued at nothing? Anſ. 50l. 19s. 5d. 1$\frac{9}{11}$q.

23. What is the annual intereſt of 987l. 6s 3d. at the rate of 6 per cent.? Anſ. 59l. 4s. 9¼d.

24. If 1000 French feet be equivalent to 1068 Engliſh feet, and the circumference of the earth according to the French menſuration be 123249600 French feet: I demand the ſame in Engliſh miles. Anſ. 24930 miles, 57 yards 1 foot 9 inches $\frac{18}{10}$ barley corns.

25. Suppoſe all things as in the foregoing queſtion, I demand how long a ſound will be in paſſing from pole to pole, upon a ſuppoſition that a ſound paſſes over 1142 feet in a ſecond of time? Anſ. 16 hours, 32 ſeconds 28 thirds.

26. A Draper bought four bales of cloth each containing 6 pieces, and each piece 27 yards, at 16l. 4s a piece; I demand the price of the whole and the rate per yard? Anſ. the whole coſt 388l. 16s. and 12s. per yard.

27. If the daily expences of all the people in London be 65660l. ſterling: how much will keep them a year and a half? Anſ. 35948850l.

28. If I give 9d. for a ſkin, how much muſt I pay for 19 ſcore, 6 doz. and 8 skins? Anſ. 17l. 5s.

29. A Trader bought 92lb. of tea at 12s. 10d. per lb. and not anſwering in goodneſs, is willing to loſe 40s. by the whole, at what rate muſt he ſell it by the ounce? Anſ. 9d. 3 $\frac{288}{1472}$ qr.

30. A Gentleman ſpends 1l. 6s. 8d. a day and lays up, at the years end 598l. what is his yearly income? Anſ. 1084l. 13s. 4d.

31. I have 820l. 12s. 6d. to diſtribute among 32 poor men at 8½d. and 70 poor women at 5¾d. and 40 children at 3¼ each per day; how long will the ſame adminiſter to their relief. Anſ. 244 $\frac{168}{3218}$ days.

32. Bought 30 pieces of broad cloth for 130l. and ſold them again for 150l. Now, if they had coſt 150l. how muſt they have been ſold to have yielded gain at the ſame rate? Anſ. 173l. 1s. 6d. $1\frac{12}{13}$ q.

33. A ſhip ſailed 48 leagues in 24 hours, how many leagues will ſhe ſail in a week at the ſame rate? Anſ. 336 leagues.

34. A Merchant hath owing him 968l. and his debtor agrees to pay him 13s. for every pound; the queſtion is what muſt he pay him for the whole debt, and what muſt the Merchant loſe by compoſition or abatement?

Anſ. The debtor muſt pay 629l. 4s. The Merchant muſt loſe 338l. 16s.

35. As I was beating on the foreſt grounds,
Up ſtarts a hare before my two greyhounds:
The dogs being light of foot, did fairly run
Unto her fifteen rods juſt twenty-one:
The diſtance that ſhe ſtarted up before
Was four ſcore, ſixteen rods juſt, and no more:
Now this I'd have you unto me declare,
How far they ran before they caught the hare.

Anſ. 336 rods. For 21−15=6 rods the dogs gain at the hare in running 21 rods; but the queſtion is how far muſt they run to gain 96 rods at her, i. e. to overtake her, which is thus ſolved. As 6 : 21 : : 96 : 336.

XIII. *The Rule of Three Inverſe.*

INVERSE, or reciprocal, proportion is, when of four numbers the 3d bears the ſame ratio or proportion to the firſt as the 2d. does to the 4th.

Therefore the leſs the 3d. term is, in reſpect to the firſt, the greater will the 4th term be, in reſpect to the ſecond.

The *preparation* and *diſpoſition* of the terms differ nothing here from the Rule of Three Direct, only the *operation is inverſe*. The only difficulty is in knowing when a queſtion belongs to one rule, and when to the other; in order to which obſerve the following directions.

Having ſtated the queſtion, obſerve whether the 3d term, being more than the 1ſt, requires more, or being leſs requires leſs; if ſo, the queſtion belongs to the direct rule: but if the 3d term being more than the 1ſt, requires leſs, or being leſs, requires more, it then belongs to the Rule of Three Inverſe, and in ſuch caſe, the fourth term muſt be found by an *inverſe operation*, i. e. by ×ing the firſt and ſecond terms together, and ÷ing the product by the third term.

EXAMPLES.

1. If 10 yards of cloth, half yard wide, will make a garment, how many yards 5 quarters wide, will make a lining for the ſaid garment?

Here it is manifeſt, that by how much more this lining has of breadth, ſo much leſs muſt it have in length: therefore the third term being more than the firſt, and requiring the fourth to be leſs, this is more requires leſs.

qr. yds. qr.
If 2 : 10 :: 5 : 10×2÷5=4 yds. the Anſ.

2: If 4 yards of cloth 5 quarters wide, will make a garment; how many yards, half yard wide, will line the ſame garment?

Here it is obvious, that by how much leſs the lining has of breadth, ſo much more it muſt have of length: therefore this is leſs requires more.

qr. yds. qr.
If 5 : 4ds :: 2: 5×4÷2=10yds. the Anſ.

This rule may be proved by inverting this queſtion as above, or thus, the product of the firſt and ſecond terms, will always be equal to the product of the third and fourth, when the work is right. Thus in queſt. 1ſt 10×2=5×4.

CON-

CONTRACTIONS.

I. Divide the ſecond term by the third, ×ing the quote into the firſt, and the product is the anſwer.

3. If 24 men in 27 days do a piece of work, in how long time will 9 men do the like?

men days men days
If 24 : 27 :: 9 : 72 the Anſ.
Thus 27÷9=3, and 24×3=72 the Anſ. as before.

II. Divide the firſt term by the third, ×ing the ſecond by the quote and the product is the Anſwer.

4. If a friend lend me 2800l for 8 months, how long ſhould I lend him 700l. to requite him?

l. m. l. m.
If 2800 : 8 :: 700 : 32 the Anſ.
Thus 2800÷700=4 and 8×4=32 months the Anſ.

III. When the third term and either of the other two can be exactly ÷ed by ſome common ÷for; then ÷ them and take the quotes inſtead of theſe terms; proceeding thus as oft as you can.

5. If 378 oxen will eat up the graſs of a field in 36 days, how long will 18 oxen be eating up the ſame?

oxen days oxen days
378 : 36 :: 18 : 756 the Anſ.
contracted 63 : 36 :: 3 by ÷ing the 1ſt and 3d by 6
contracted 63 : 12 :: 1 by ÷ing the 2d and 3d by 3.
12

Anſ. 756 days; by ×ing 1ſt and 2d terms together.

6. If the penny white loaf ought to weigh 9 ounces troy, when wheat is at 4s. 6d. a buſhel, what ought it to weigh when wheat is at 6s. 9d. a buſhel. Anſ. 6 ounces

7. Suppoſe 3 cocks, or 4 hens, will eat up a certain quantity of corn in 56 days, how long will 1 cock and 1 hen be in eating up the ſame ?

Becauſe of 3 cocks, or 4 hens, ſome number muſt be found that is diviſible by 3 and by 4 without remainder, ſuch is the number 12; (for $3 \times 4 = 12$) make then 3 cocks or 4 hens equivalent to 12 chickens, and you will have 1 cock equivalent to 4 chickens, 1 hen to 3 chickens, and 1 cock and 1 hen to 7 chickens; in which caſe the queſtion will ſtand thus: If *12 chickens will eat up a certain quantity of corn in 56 days, how long will 7 chickens be in eating up the ſame?* Anſ. 96 days.

8. There are 2400 ſoldiers in a garriſon in a beſieged town, who have only proviſions for 9 months; how many muſt be diſbanded that the proviſions may ſerve 16 months ? Anſ. 1050.

9. At what price per buſhel is wheat, when the penny white loaf weighs 5 ounces, 8 dwts. if it weighs 9 ounces when wheat is at 4s. 6d. a buſhel ? Anſ. 7s. 6d.

10. If 1l. 2s. worth of wine will ſuffice a club of 12 men when the wine is ſold after the rate of 25l. 4s. a hhd; how many men will 1l. 2s. worth ſerve, when the wine is ſold after the rate of 18l. 18s. a hhd ? Anſ. 16 men.

Here follows a collection of queſtions put down promiſcuouſly in both rules; wherein the learner is left to himſelf to diſtinguiſh the particular rule, whether Direct or Inverſe, which each queſtion belongs to.

1. If the expence in houſe-keeping, during 6 weeks amount to 9l. 3s. 6d. how long will 100l. laſt at that rate? Anſ. 65 weeks, 2 days, $18\frac{138}{367}$ hours.

2. If 736 dollars, at 4s. 6d. each, were given in exchange for 144 Jacobuſes, what was the value of 1 Jacobus? Anſ. 23s.

3. If 5 oxen, or 7 colts, will eat up a cloſe in 87 days, in what time will 2 oxen and 3 colts eat up the ſame? (vide Saunderſon's Algebra)

Becauſe of the 5 oxen, or 7 colts, ſome number muſt be found that can be ÷ed by 5 and by 7 without remainder; ſuch is the number $35=7\times5$; make then 5 oxen or 7 colts equivalent to 35 heifers, and you'll have 1 ox equivalent to 7 heifers, 1 colt to 5 heifers, 2 oxen $=14$ heifers, 3 colts $=15$ heifers, and 2 oxen $+$ 3 colts $=14+15=29$ heifers, and the queſtion will ſtand thus: *If 35 heifers will eat up a cloſe in 87 days, in what time will 29 heifers eat up the ſame?* Anſ. in 105 days.

4 If 4lb of tea coſt 1l. 13s. 4d. what will 4 parcels coſt, each weighing 2 c. 3 qr. 21 lb? Anſ. 548l. 6s. 8d.

5. If a field will graze 18 horſes for ſeven weeks, how long will it graze 42 horſes? Anſ. 3 weeks.

6. If 2 c. 3 qr 21 lb. of ſugar coſt 6l. 1s. 8d. what will 12 c. 2 qr. coſt at the ſame rate? Anſ. 25l. 17s. 8d. $3\frac{3}{47}$q.

7. If 60 gallons of water in one hour's time fall into a ciſtern, containing 200 gallons, and by a pipe in the ſame ciſtern there runs out 45 gallons in one hour; in how many hours will the ciſtern be filled, if both the cocks keep running? Anſ. 13 hours 20 min.

8. If a piece of ordnance ſhoot $14\frac{3}{4}$lb. of powder at a time; how many pounds will diſcharge it 68 times? and what will the powder come to, at $11\frac{1}{4}$d. per lb? Anſ. 1003lb. of powder. Value 47l. $3\frac{3}{4}$d.

9. If a ſquare pipe $4\frac{1}{2}$ inches wide, will diſcharge a certain quantity of water in one hours time; in what time will another ſquare pipe, $2\frac{1}{2}$ inches wide, diſcharge the ſame quantity from the ſame current?

The diſcharge of water through pipes, all other things being the ſame, is as the area of their orifices. The orifice of a ſquare pipe $4\frac{1}{2}$ inches wide contains 81 ſquare half inches, and the orifice of a pipe $2\frac{1}{2}$ inches wide contains 25 ſquare half inches. Say then : *If an orifice of 81 ſquare half inches will diſcharge a certain quantity of water in 1 hour, in what time will an orifice of 25 ſquare half inches diſcharge the ſame?* Anſ. 3 hours, 14 ſeconds, 24 thirds.

10. A Butcher bought an ox for 10l. 10s. which weighed, beſides ſkin and legs, 108 ſtone. There were in him $8\frac{1}{2}$ ſtone of fat, which he ſold for $4\frac{3}{4}$d. per lb. the ſkin and legs he ſold for 1l. 5s. 9d how much does 1 ſtone of beef ſtand him in? Anſ. 1s. 4d. $2\frac{32}{199}$q.

11. a Plummer bought 134 fother of lead at $1\frac{1}{4}$d. per pound, what did the whole coſt him? Anſ. 1524l. 5s.

Note, A fother of lead is 19 cwt. 2 qr.

12. A man bought 120 eggs at 2 a penny, and 120 more at 3 a penny, he mixt both parcels together and ſold them out at 5 for two-pence, whether did he gain or loſe by the bargain, and how much? Anſ. he loſt 4d.

13. A Draper laid out 120l. as follows, viz. one third of it in callicoes, at 20s. 5d. per piece, one third in cambricks at 32s. 9d per piece, and the reſt in holland at 5s. 9d: per ell, how much had he of each ſort?

$$\text{Anſ.}\left\{\begin{array}{l}\text{Callicoes } 39\frac{45}{245} \text{ pieces.}\\ \text{Cambricks } 24\frac{168}{393} \text{ pieces.}\\ \text{Hollands } 139\frac{9}{69} \text{ ells.}\end{array}\right.$$

§ XIV. *Compound Proportion.*

COMPOUND Proportion (or the double Rule of Three) is, when five numbers are given to find a ſixth proportional, and is commonly performed by two operations in the ſingle *Rule of Three*, either *Direct* or *Inverſe* as the queſtion requires.

The three firſt terms contain a ſuppoſition, and the two laſt a demand.

RULE.

1. Of the two numbers which contain the demand, one muſt be the third term in the firſt ſtating, and the other the third-term in the ſecond ſtating.

2. The fourth term, found by the firſt operation, muſt always be put for the middle term in the ſecond ſtating, and the fourth term found by this ſecond operation, is the anſwer required.

EXAMPLES.

1. If 100l. principal in 12 months, gain 6l. intereſt; what will 250l. principal gain in 9 months.

If 100l. : 6l. :: 250l. : 250l×6÷100=15l.

This 4th number, viz. 15l. is the intereſt of 250l. for 12 months, and the queſtion is reduced to this: If 12 months gain 15l. what will 9 months gain?

m. l. m l. s.
If 12 : 15 :: 9 : 15×9÷12=11. 5 the Anſ.

2. If 250l. principal in 9 months gain 11l. 5s.=225s. intereſt, what will 100l. principal gain in 12 months?

l. s. l. s.
If 250 : 225 :: 100 : 90 fourth term.

m. s m s.
If 9 : 90 :: 12 : 120=6l. the Anſ.

3. If 100l. principal in 12 months gain 6l. in what time will 250l. gain 11l. 5s.=225s ?

s. m. s. m.
6l.=120s. If 120 : 12 :: 225 : 22½ This is direct.
22½m.=45 half months : then ſay,

l. ½m. l. ½m.
If 100 : 45 :: 250 : 18=9 mon. This is inverſe.

4. If the carriage of 100lb. weight 30 miles coſt 2s; what will the carriage of 500lb. coſt for 200 miles ?

lb. s. lb. s.
If 100 : 2 :: 500 : 10
If 30 : 10 :: 200 : 66s. 8d. the Anſwer.

All queſtions in Compound Proportion, may be ſolved with greater expedition by the following Rule.

2 RULE.

1. Here as in the ſingle Rule of Three, put that term in the ſecond place which is of the ſame kind and denomination with the anſwer, or term ſought; and the terms of ſuppoſition one above another in the firſt place; alſo the terms of demand in the ſame order, one above another, in the third place. But note, the firſt and third term of every row muſt be of one name or denomination.

2. Take the three terms in the firſt row, and of them form a queſtion in your mind as in the ſingle Rule of Three, marking which term would be the ×er, viz. the firſt or the third. Proceed thus with each row as ſo many ſeparate queſtions in the ſingle Rule of Three, uſing the ſecond term in common with each of them, and always marking which extream would be the ×er. That is, in any row ſay : If the firſt term gives the ſecond, does the third require more, or leſs ? If more, mark the greater extream, if leſs, the leſſer, for a ×er.

3. Multiply the middle term and all theſe ×ers together for a dividend; and the reſt of the terms together for a diviſor: the quote thence ariſing will be the anſwer, and of the ſame name with the middle number.

CONTRACTIONS.

I. When the ſame numbers are concerned in both the dividend and ÷ior, throw them out of both.

II. When it can be done, ÷ any numbers by ſome common ÷for, and take the quotes inſtead of them.

Ex. 5. If a regiment of 936 ſoldiers eat 351 quarters of wheat in 168 days, how many quarters will ſuffice an army of 11232 ſoldiers 56 days?

Soldiers qrs. Soldiers contracted
12) 936 : 351 :: 11232 ×er 78 : 351 :: 936 ×er.
days days
7) 168 : —— :: 56 ×er 24 : —— :: 8 ×er.
by contraction.

$$\begin{array}{l}\text{Dividend}\\ \text{Diviſor}\end{array}\ \frac{351\times936\times8}{78\times24}=\frac{351\times936\times1}{78\times3}=1404\text{ qr. the Anſ.}$$

6. If a regiment of 936 ſoldiers eat 351 quarters of wheat in 168 days, how many ſoldiers will 1404 quarters ſuffice 56 days?

qr. Soldiers qr.
351 : 936 :: 1404 ×
× 168 days : — :: 56 days.

$$\text{Then}\ \frac{936\times1404\times168}{351\times56}=11232\text{ ſoldiers the Anſ.}$$

EXPLANATION.

Ex. 5th. Say, if 936 ſoldiers eat up 351 quarters, 11232 ſoldiers will eat up more quarters, therefore mark the greater extream 11232 for a ×er.

Again, ſay if 168 days require 351 quarters of wheat to ſuffice an army; 56 days will require fewer quarters to ſuffice the ſame army; ſo mark the leſs extream 56 for a ×er.

Then contract the terms by firſt ÷ing by 12 and then by 7. Again you have 351×936×8 for a dividend, and 78×24 for a ÷for; which being contracted further by ÷ing both dividend and ÷for by 8, you have 351×936=328536 for a dividend, and 78×3=234 for a ÷for, and the quote 1404 quarters for the Anſwer.

Ex. 6. Say if 351 quarters ſerve 936 men, 1404 quarters will ſerve more; therefore 1404 muſt be a ×er.

Again ſay, if 168 days require 936 men to eat up any quantity, 56 days will require more men to eat up the ſame, mark therefore the greater extream 168 for a ×er. Then having 936×1404×168 for a dividend, and 351×56 for a ÷for, the quote ariſing is 11232 men the Anſwer.

7. If the carriage of 240 feet of wood, that weighs 7 ſtone a foot, comes to 6l. for 50 miles, how much will the carriage of 70 feet of free ſtone, that weighs 10ſt. a foot, coſt for 300 miles?

240 ft. : 120s. :: 70 ft. ×
7 ſt. : ——— :: 10 ſt. ×
50 m. : ——— :: 300 m. ×

Then $\frac{120\times70\times10\times300}{240\times7\times50}=300=15$l. the Anſ.

8. If 240 men, in 6 days of 12 hours each, dig a trench of 8 degrees of hardneſs, and 230 yards long, 3 wide, and 2 deep; in how many days of 10 hours, will 24 men dig a trench, of 5 degrees of hardneſs, 320 yards long, 5 wide, and 3 deep?

× 240 m. : 6 days :: 24 m.
× 12 hrs : ——— :: 10 hrs.
8 deg : ——— :: 5 deg. ×
230 long : ——— :: 320 long ×
3 w. : ——— :: 5 w. ×
2 deep : ——— :: 3 deep ×

$$\frac{\text{Dividend } 6\times240\times12\times5\times320\times5\times3}{\div\text{ſor } 24\times10\times8\times230\times3\times2}=$$

(by contraction) $\frac{6\times12\times5\times32\times5}{8\times23\times2}=$

$\frac{3\times3\times5\times16\times5}{23}=156\frac{12}{23}$ days the Anſ.

9. If 54 men can build a wall in 18 days, when the day is 17 hours long, in how many days will 68 men build the ſame when the day is but 9 hours long?

men days men
× 54 : 18 :: 68
× 17hr : — :: 9hr

$$\left.\right\}\frac{54\times17\times18}{68\times9}=\frac{27\times17\times2}{34}=27$$

days the Anſwer.

10. If there muſt be 5400 bricks, 6 inches long, and 3 broad, to pave a hall; how many bricks will it require to pave the ſame hall that are 9 inches long and 4 broad?

bricks
× 6 l. : 5400 :: 9 l.
× 3 b. : ——— :: 4 b.

$\left.\right\}\frac{6\times3\times5400}{9\times4}=2700$ bricks Anſ.

Note, The two ſingle compoſing analogies whereby the two laſt queſtions are ſolved, are both of them Inverſe; though it hath been aſſerted by ſome, that it never happens that both analogies are Inverſe.

11. A Scrivener lent 800l. upon intereſt the 17th of Auguſt 1760; and took it in on the 13th of January 1764, what did it amount to at 5l. per cent. per annum?

From the 17th of Auguſt 1760 to the 12th of January 64 are 1244 days. The day on which money is t out muſt be reckoned, but not the day on which it is taken in.

$$\begin{array}{rcccl} & \text{p.} & \text{int.} & & \text{p.} \\ & 100\text{l.}: & 5\text{l.} & :: & 800\text{l.}\times \\ \text{days } 365 & : & — & :: & 1244 \text{ days.}\times \end{array}$$

$$\text{Theorem}\ \frac{800\times5\times1244}{365\times100}=136\text{l. }6\text{s. }6\text{d. }3\frac{225}{365}\text{q. Int.}$$

800l. + 136l. 6s. $6\frac{1}{4}$d. = 936l. 6s. $6\frac{1}{4}$d. the Amount.

The above Theorem furniſheth us with a general rule for finding the intereſt of any ſum of money for any number of days, which will be better explained hereafter.

12. If a regiment of 936 ſoldiers eat 351 quarters of wheat in 168 days, how many days will 1404 quarters ſuffice an army of 11232 ſoldiers? Anſ. 56 days.

13. If the carriage of 5 c. 3qr. weight 150 miles, coſt 3l. 7s. 4d. what muſt be paid for the carriage of 7 c. 2 qrs. 25 lb. weight 64 miles at the ſame rate? Anſ. 1l. 18s. 8d.

14. A Gentleman lent a certain ſum of money at 6l. per cent. per annum for 5 years and 6 months, and at

the end of that term, received for intereſt thereof 478l. 10s. what was the ſum of money lent ? Anſ. 1450l.

15. If 8 men in 6 days dig 24 cubic fathoms of earth, how many fathoms will 12 men dig in 3 days at the ſame rate of working ? Anſ. 18 fathoms.

16. If the carriage of 60 cwt. 20 miles coſt 4l. 10s. what weight ought to be carried 30 miles, for 5l. 8s. 9d. at that rate ? Anſ. 15 cwt.

17. If 2 penny white loaves will ſuffice 3 men, when wheat is at 4s. 6d. a buſhel, how many ſuch loaves will ſuffice 9 men, when wheat is at 9s a buſhel ? Anſ. 12 penny loaves.

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§ XV. PRACTICE.

THE Rule of Practice is a certain compendious way of working ſuch queſtions in the Rule of Three as have an unit for the firſt term, and is of great uſe among Merchants, Tradeſmen, &c in the quick diſpatch of buſineſs. It is performed by conſidering what aliquot part of a pound ſterling the given price is, or is reduceable to ; and taking ſuch part or parts accordingly.

A TABLE of the aliquot parts of a pound and a ſhilling ſterling, to be got by heart.

s.	d.		d.		
10	0	is half a l.	6	is the 40th of a L.	or = ½ of a ſhill.
6	8	3d	5	48th	
5	0	4th	4	60th	= 3d
4	0	5th	3¾	64th	
3	4	6th	3	80th	= 4th
2	6	8th	2½	96th	
2	0	10th	2	120th	= 6th
1	8	12th	1½	160th	= 8th
1	4	15th	1¼	192d	
1	3	16th	1	240th	= 12th
1	0	20th	0¾	320th	= 16th
0	10	24th	0½	480th	= 24th
0	8	30th	0¼	960th	= 48th
0	7½	32d.			

For the better delineating the meaning of any operation,
put $x =$ the quantity given, or number propoſed,
$a =$ the firſt operation,
$b =$ the ſecond,
$c =$ the third, and ſo on alphabetically,
$r =$ the rate or price of C. wt. yd. &c.

RULE 1.

When the given price of 1 is an aliquot part of a penny, ſhilling, or pound, take the ſame part of x by ÷ing it by ſo many as the given price of 1 is continued in a penny, ſhilling, or pound, for the anſwer in pence, ſhillings, or pounds reſpectively.

EXAMPLES.

Yards per yd.
$x = 3672$ at 10s.
$x \div 2 = 1836$l. Anſ.

$x = 4361$ at 6s. 8d.
$x \div 3 = 1453$l. 13s. 4d. Anſ.

$x = 566$ at 5s.
$x \div 4 = 141$l. 10s. Anſ.

$x = 6732$ at 4s.
$x \div 5 = 1346$l. 8s. Anſ.

$x = 5121$ at 3s. 4d.
$x \div 6 = 853$l. 10s. Anſ.

Yards per yd.
$x = 8347$ at 2s. 6d.
$x \div 8 = 1043$l. 7s. 6d. Anſ.

$x = 4267$ at 2s.
$x \div 10 = 426$l. 14s. Anſ.

$x = 5283$ at 1s. 8d.
$x \div 12 = 440$l. 5s. Anſ.

$x = 6473$ at 1s. 4d.
$x \div 5 = 1294\ 12 = a$
$a \div 3 = 431$l. 10s 8d.

$x = 7386$ at 6d.
$x \div 40 = 184$l. 13s. Anſ.

$x =$

lb. per lb.

x =7284 at 1s. 3d.

x÷8= 910 10 =a

a÷2=455l 5s. Anſ.

x =3267 at 1s.

x÷20=163l. 7s. Anſ.

x =7413 at 10d.

x÷4=1853 5=a

a÷6= 308l. 17s. 6d. Anſ.

x =5432 at 8d.

x÷30=181l. 1s. 4d.

x =6215 at 7½d.

x÷8= 776 17 6d=a

a÷4= 194l. 4s. 4½ Anſ.

x = 8437 at 5d.

x÷6= 1406 l. 3s. 4d.=a

a÷8= 175l. 15s. 5d Anſ.

x =8347 at 4d.

x÷60=139l. 2s. 4d. Anſ.

lb. per lb.

x =4915 at 3¾d

x÷8= 614l. 7s. 6d.=a

a÷8=76l. 15s. 11¼d Anſ.

x =7284 at 3d.

x÷80= 91l. 1s. Anſ.

x =5473 at 2½d.

x÷12=456l. 1s. 8d.=a

a÷8= 57l 0s. 2½d. Anſ.

x =7432 at 2d.

x÷120=61l. 18s 8d. Anſ.

x =5743 at 1½d.

x÷80=71l. 15s. 9d.=a

a÷2= 35l. 17s 10½d Anſ.

x =6578 at 1¼d.

x÷12=548l 3s. 4d.=a

a÷4= 137l. 0s. 10d.=b

b÷4= 34l 5s. 2½d. Anſ.

x =4327 at 1d.

x÷120= 36l. 1s. 2d=a

a÷2 = 18l. 0s. 7d. Anſ.

x =

lb. per lb.
x =8274 at ¾d.
x÷80= 103l. 8s. 6=a
a÷4=25l. 17s. 1½d. Anſ.

x =7438 at ½d.
x÷120=61l. 19s. 8d.=a
a÷4= 15l. 9s. 11d. Anſ.

lb. per lb.
x =8347 at ¼d.
x÷120=69l. 11s. 2d.=a
a÷8= 8l 13s. 10¾d. Anſ.

proof of the laſt example.
x = 8347 at ¼d.
x÷80= 104l. 6s. 9d =a
a÷12=8l. 13s. 10¾d Anſ.

The proof of this rule is to work the queſtion by different methods, or by the Rule of Three.

RULE II.

When the given price of 1 is not an aliquot part of a penny, ſhilling, or pound, divide it into ſeveral aliquot parts; then work for each by Rule I. and their ſum will be the anſwer. Or it may often be ſo divided, that the leſs will be aliquot parts of the greater; then take the ſame parts of the prices found for the greater.

EXAMPLES.

lb. per lb.
x =3987 at 2¼d.
x÷120=33l. 4s. 6d. =a
a÷8 = 4l. 3s. 0¾ =b
a+b = 37l. 7s. 6¼d. Anſ.

lb. per lb.
x =4713 at 4¼d.
x÷120= 39l. 5s. 6d.=a
x÷120= 39l. 5s. 6d.=a
a÷8 = 4l 18s. 2¼d=b
2a+b=3l. 9s. 2¼d. Anſ.

x =

lb. per lb.
$x = 6395$ at $3\frac{1}{2}$d.

$x \div 80$ is 79l. 18s. 9d $= a$
$a \div 6$ is 13l. 6s. $5\frac{1}{2}$d is b

$a + b$ is 93l. 5s. $2\frac{1}{2}$d. Anſ.

$x = 4325$ at $4\frac{1}{2}$d.

$x \div 80$ is 54l. 1s. 3d. $= a$
$a \div 2$ is 27l. 0s. $7\frac{1}{2}$d is b

$a + b$ is 81l. 1s. $10\frac{1}{2}$d.

$x = 3891$ at $4\frac{3}{4}$d.

$x \div 80$ is 48l. 12s. 9d $= a$
$a \div 2$ is 24l. 6s. $4\frac{1}{2}$ is b
$b \div 6$ is 4l. 1s $0\frac{3}{4}$d is c

$a + b + c$ is 77l. 0s. $2\frac{1}{4}$d Anſ

$x = 6793$ at $5\frac{3}{4}$d.

$x \div 80$ is 84l. 18s. 3d. $= a$
$x \div 120$ is 56l: 12s. 2d. is b
$a \div 4$ is 21l. 4s. $6\frac{3}{4}$d. is c

$a + b + c$ is 162l. 14s. $11\frac{3}{4}$d.
Anſ.

$x = 534$ at $7\frac{1}{2}$d

$x \div 40$ is 13l. 7s. 0d. $= a$
$a \div 6$ is 2l. 4s. 6d is b
$b \div 4$ is 0l. 11s. $1\frac{1}{2}$d is c

$a + b + c$ is 16l. 2s. $7\frac{1}{2}$d Anſ.

lb. per lb.
$x = 4443$ at $7\frac{3}{4}$d.

$x \div 40$ is 111l. 1s. 6d. $= a$
$a \div 4$ is 27l. 15s. $4\frac{1}{2}$d. is b
$b \div 6$ is 4l. 12s. $6\frac{3}{4}$d. is c

$a + b + c$ is 143l. 9s. $5\frac{1}{4}$d.
Anſ.

Ells. per Ell.
$x = 4846$ at $8\frac{1}{2}$d.

$x \div 60$ is 80l. 15s. 4d. $= a$
$x \div 60$ is 80l. 15s. 4d. is a
$a \div 8$ is 10l. 1s. 11d. is b

$2a + b$ is 171l. 12s. 7d.

$x = 7501$ at $8\frac{3}{4}$d.

$x \div 120$ is 62l. 10s. 2d. $= a$
$a \times 3$ is 187l. 10s. 6d. is b
$b \div 8$ is 23l. 8s $9\frac{3}{4}$d. is c

$a + b + c$ is 273l. 9s $5\frac{3}{4}$d. d

$x = 4591$ at $9\frac{1}{4}$d.

$x \div 40$ is 114l. 15s. 6d. $= a$
$a \div 2$ is 57l. 7s. 9d $= b$
$b \div 12$ is 4l. 15s. $7\frac{3}{4}$d. $= c$

$a + b + c$ is 176l. 18s. $10\frac{3}{4}$d
Anſ.

Ells per Ell.
x =4065 at 9d.

x÷80 is 50l. 16s. 3d.=a

a×3 is 152l. 8s. 9d. Anſ.

x =4416 at $9\frac{1}{2}$d.

x÷40= 110l. 8s. =a
a÷2 is 55l. 4s. is b
b÷6 is 9l. 4s. is c

a+b+c is 174l. 16s. Anſ.

x =5559 at $11\frac{3}{4}$d.

x÷30= 185l. 6s. 0d.=a
x÷80 is 69l. 9s. 9d. is b
b÷4 is 17l. 7s. $5\frac{1}{4}$d is c

a+b+c is 272l. 3s. $2\frac{1}{4}$d. Anſ.

x =3956 at 1s. $0\frac{3}{4}$d.

x÷40= 98l. 18s. 0d=a
x÷40 is 98l. 18s. 0d. is a
a÷8 is 12l. 7s. 3d is b

2a+b is 210l. 3s. 3d. Anſ.

x =6756 at 1s. $1\frac{1}{4}$d.

x÷20= 337l. 16s. 0d=a.
a÷12 is 28l. 3s. 0d. is b
b÷4 is 7l. 0s. 9d. is c

a+b+c is 372l 19s 9d Anſ

Ells per Ell.
x =9479 at 1s. $1\frac{3}{4}$d.

x÷20 is 473l. 19s. 0d.=a
a÷8 is 59l. 4s. $10\frac{1}{2}$ is b
b÷6 is 9l. 17s. $5\frac{3}{4}$d is c

a+b+c is 543l 1s $4\frac{1}{4}$d Anſ.

x =7128 at 1s. $2\frac{1}{4}$d.

x÷20 is 356l. 8s. 0d.=a
a÷6 is 59l. 8s. 0d. is b
b÷8 is 7l. 8s. 6d. is c

a+b+c is 423l. 4s. 6d.

x =9124 at 1s. $3\frac{1}{4}$d.

x÷80 is 114l. 1s. 0d.=a

a×5 is 570l. 5s. 0d. is b
a÷12 is 9l. 10s. 1d is c

b+c is 579l. 15s. 1d. Anſ.

x =8397 at 1s. $3\frac{3}{4}$d.

x÷20 is 419l. 17s. 0d.=a
a÷4 is 104l. 19s. 3d. is b
b÷4 is 26l. 4s. $9\frac{3}{4}$d. is c

a+b+c is 551l 1s $0\frac{3}{4}$d Anſ

x =4597 at 1s. 4d.

x÷30 is 153l. 4s. 8d =a

a×2 is 306l. 9s. 4d. Anſ.

Ells per Ell.
x =2597 at 1s. 4¼d.

x÷120 is 21l. 12s. 10d.=a

a×8 is 173l. 2s. 8 is b
a÷8 is 2l. 14s. 1¼ is c

b+c is 175l. 16s. 9¼d.

x =8090 at 1s. 4½d.

x÷20 is 404l. 10s. od=a

a÷8 is 50l. 11s. 3d. is b

b×11 is 556l. 3s. 9d. Anſ.

Ells per Ell.
x =7948 at 1s. 4¾d

x÷20 is 397l. 8s. od.=a
a÷4 is 99l. 7s. od. is b
b÷2 is 49l. 13s. 6d. is c
c÷6 is 8l. 5s. 7d is d

a+b+c+d=554l 14s 1d.
Anſ

x =3798 at 1s. 6¾d.

x÷20 is 189l. 18s. od=a
a÷2 is 94l. 19s. od. is b
b÷8 is 11l. 17s. 4½d is c

a+b+c 296l 14s 4½d Anſ.

RULE III.

Sometimes the value may be eaſily found by reckoning the price of ſome even number above what is given, and then taking ſome aliquot part for what is above, and ſubtracting it from the former.

EXAMPLES.

lb. per lb.
x =3987 at 1¾d

x÷120 is 33l. 4s. 6d. =a
a÷8 is 4l. 3s. 0¾d is b

a—b is 29l. 1s. 5¼d. Anſ.

lb. per lb.
x =112 at 11d.

x÷20 is 5l. 12s. od=a
a÷12 is 0l. 9s 4d. is b

a—b is 5l. 2s. 8d. Anſ.

lb. per lb.
x =2958 at 1s. 5¼d.

x÷40 is 73l. 19s. 0d.=a

a×3 is 221l. 17s 0d is b
a÷8 is 9l. 4s. 10½d. is c

b—c is 212l. 12s. 1½d Anſ

---

x =4721 at 1s. 5d.

x÷12 is 393l. 8s. 4d.=a
x÷80 is 59l. 0s. 3d. is b

a—b is 334l. 8s. 1d. Anſ.

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dozens per doz.
x =3219 at 5½d.

x÷40 is 80l. 9s. 6d. =a
a÷12 is 6l. 14s. 1½d is b

a—b is 73l. 15s. 4½d. Anſ.

dozens per doz.
x =8746 at 5¼d

x÷40 is 218l. 13s. 0d.=a
a÷8 is 27l. 6s. 7½d is b

a—b is 191l. 6s. 4½d Anſ.

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x =7698 at 1s. 2¼d

x÷80 is 96l. 4s. 6d.=a

a×5 is 481l. 2s. 6d. is b
a÷12 is 8l 0s. 4½d is c

b—c is 473l. 2s. 1½d. Anſ.

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x =4793 at 1s. 5½d

x÷12 is 399l. 8s. 4d=a
a÷8 is 49l. 18s. 6½d is b

a—b is 349l 9s. 9½d Anſ.

Queſtions to exerciſe the foregoing RULES.

Queſtions lb.		s.	d.		Anſwers l.	s.	d.
4350	at	1	7¼	is	348	18	1½
6987	at	1	7¾		574	19	5¼
4329	at	1	8¾		374	5	6¾
4859	at	1	9¾		440	6	11¼
8696	at	0	7		253	12	8
6748	at	0	9¾		274	2	9
5349	at	0	10½		234	0	4½
8107	at	0	11½		388	9	2½

7100

Queſtions		s.	d.		Anſwers l.	s.	d.
7100	at	5	10½		2085	12	6
702	at	11	4½		399	5	3
853	at	13	9		586	8	9
184	at	15	4		141	1	4
571	at	19	0¾		544	4	[illegible]¼
9271	at	1	10½		869	3	1½
3578	at	1	10¾		339	3	3½
6986	at	1	11		669	9	10
3583	at	1	11¾		354	11	4¼
3543	at	0	8¼	is	121	15	9¾
959	at	0	10¼		40	19	1¾
5056	at	0	10¾		226	9	4
3500	at	0	11¼		164	1	3
579	at	3	2		91	13	6
1504	at	9	2		689	6	8
847	at	12	7½		534	13	4½
714	at	14	11¼		533	5	4½
544	at	18	3¾		498	2	0
970	at	19	8		953	16	8

RULE IV.

If there are Pounds in the Price, multiply the given quantity by the number of them, and if there be alſo ſome odd money, find its produce by the former Rules and add them together.

EXAMPLES.

x = 7642 at 6l.

x × 6 is 45852l. Anſ.

x = 347 at 16l.

x × 16 is 5552l. Anſ.

x = 369 at 3l. 6s. 8d.

x × 3 is 1107l. = a
x ÷ 3 is 123l. is b

a + b is 1230l. Anſ.

x = 261 at 2l. 10s. 6d.

x × 2 is 522l. 0s. 0d. = a
x ÷ 2 is 130l. 10s. 0d is b.
b ÷ 20 is 6l. 10s. 6d. is c

a + b + c is 659l 0s 6d Anſ.

Questions		l.	s.	d.		Anſwers l.	s.	d.
407	at	1	13	5	is	680	0	7
941	at	7	0	4		6602	13	8
950	at	4	17	8		4639	3	4
4613	at	1	5	7		5900	15	11
318	at	4	6	10		1380	13	0
563	at	11	14	6		6601	3	6

RULE V.

When the price is an even number of ſhillings, multiply the quantity by half their number, doubling the units, or firſt figure of the product, for ſhillings; the reſt are pounds.

EXAMPLES.

x = 7423 at 6s.

x × 3 is 2226l. 18s. Anſ.

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x = 7412 at 18s.

x × 9 is 6670l. 16s. Anſ.

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x = 8324 at 2l. 6s.

x × 2 is 16648l. 0s. = a
x × 3 is 2497l. 4s. is b

a + b is 19145l. 4s. Anſ.

x = 8327 at 1l. 8s.
x × 4 is 3330l. 16s. = a

x + a is 11657l. 16s.

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x = 8327 at 1 s.

x × 7 is 5828l. 18s. Anſ.

---

x = 83 at 5l. 18s.

x × 5 is 415l. 0s. = a
x × 9 is 74l. 14s. is b

a + b is 489l. 14s. Anſ.

Queſtions.

Queſtions s.		Anſwers. l.	s.	d.
345 at 12	is	207	0	0
456 at 14		319	4	0
567 at 18		510	6	0
979 at 8		391	12	0

RULE VI.

When the rate is an odd number of ſhillings, work for the greateſt even number contained in it by the laſt Rule; and for the other odd ſhilling, take one 20th of the given quantity. Or × the quantity by the number of ſhillings, and ÷ the product by 20; the quote will be pounds, and the remainder ſhillings.

EXAMPLES.

x =682 at 7s.

x × 3 is 204l. 12s. = a
x ÷ 20 is 34l. 2s. is b

a + b is 238l. 14s. Anſ.

x =678 at 7s.

x × 7 is 4746s = a

a ÷ 20 is 237l. 6s. Anſ.

x =238 at 2l. 17s.

x × 2 is 476l. 0s. 0d. = a
x × 8 is 190l. 8s. is b.
x ÷ 20 is 11l. 18s. is c

a + b + c is 678l. 6s. Anſ.

x =7622 at 3s.

x × 1 is 762l. 4s. = a
x ÷ 20 is 381l. 2s. is b

a + b is 1143l. 6s. Anſ.

x =789 at 11s.

x × 11 is 8679s. = a

a ÷ 20 is 433l. 19s. Anſ.

Queſtions.		Anſwers l.	s.
898 at 17s.	=	763	6
796 at 13s.		517	8
652 at 15s.		489	0

RULE VII.

When the quantity, whoſe price is required, conſiſts, of ſeveral ſubordinate denominations whereof only the higheſt is rated; work for the rated denomination ſingly, by ſome of the foregoing rules; and for the lower denominations, add ſuch parts of the rate, as thoſe lower denominations are parts of the rated denomination.

A Table of aliquot parts of a cwt.

lb.	cwt.	lb.	cwt.
56	is ½	7	is the 16th
28	— 4th	4	— 28th
16	— 7th	3½	— 32d
14	— 8th	2	— 56th
8	— 14th	1¾	— 64th
		1	— 112th

Note. Aliquot parts of moſt other things are eaſily found.

EXAMPLES.

What's the value of 123 C. 2 qr. 14lb. at 1l. 13s. 4d. = r. per C.

		l.	s.	d.	
123 C. at 1l. 0s. 0d.	=	123	0	0	= a
123 C. at 0 6 8	is	41	0	0	is b
123 C. at Ditto	is	41	0	0	is b
r ÷ 2, for 2 qrs.	is	0	16	8	is c
c ÷ 4, for 14lb	is	0	4	2	is d
+2b+c+d	is	206	0	10	Anſ.

What's the value of 34 tons 5 C. 2 qr. at 25l. 10s. =r per ton.

Tons l.		l.	s.	d.	
34×25	=	850	0	0	=a
34÷2	is	17	0	0	is b
r÷4, for 5 C.	is	6	7	6	is c
c÷10, for 2 qr.	is	0	12	9	is d
a+b+c+d	is	874	0	3	Anſ.

What's the value of 37 C. 3 qrs. 14 lb. at 5l. 17s. 7½d. per cwt.? Anſ. 218l. 19s. 3½d.

What's the value of 483¾ yards at 17s. 10½d per yard? Anſ. 432l. 7s. 0¾d.

What's the value of 7 C. 2 qr. 15½lb. at 3l. 7d. per cwt.? Anſ. 23l. 2s. 9d.

What's the value of 14 acres, 3 ro. 5 pls. at 2l. 12s. 10d. per acre? Anſ 39l. 11¼d.

What's the value of 3 C. 22¾lb. at 13s. 5¾d. per C.? Anſ. 2l. 3s. 2d.

What come 12 gallons, 5 pints to at 5s. 8d. per gallon? Anſ. 3l. 11s. 6½d.

Note 1. If the rated denomination be not the higheſt denomination of the quantity propoſed, reduce the higheſt to the rated denomination.

What is the value of 4 tons, 5 cwt. 1 qr. 7 lb. at 2l. 5s. =r per C.?

ton	C.	qr.	lb.
4	5	1	7
20			

at 1l.	=	85l.	0s.	0d.	=a
2	is	85	0	0	is a
a÷4	is	21	5	0	is b.
r÷4	is	0	11	3	is c for 1 qr.
c÷4	is	0	2	9¾	is d for 7 lb.
a+b+c+d	=	191	19	0¾	Anſ.

I

I ſhall give ſome examples wherein the given quantity is but the part of a integer or whole thing.

3 lb. at 7l. = r per C.

		l.	s.	d.		lb.
r÷8	=	0	17	6	= a, for	14
a÷7	is	0	2	6	is b, for	2
b÷2	is	0	1	3	is c,	1
b+c	is	0	3	9	for	3

12lb at 14l. 7s. 6d = r per C.

r÷7	=	2	2	6	= a for	16lb.
a÷4	is	0	10	$7\frac{1}{2}$	is b for	4
a−b	is	1	11	$10\frac{1}{2}$	for	12

49lb at 9l. 19s: 10d. = r per C.

r÷4	=	2	9	$11\frac{1}{2}$	= a for	28lb.
a÷2	is	1	4	$11\frac{3}{4}$	is b for	14
b÷2	is	0	12	$5\frac{3}{4}$ $\frac{1}{2}$	is c for	7
a+b+c	is	4	7	5 − $\frac{1}{2}$	for	49

90lb. at 12l. 17s. 3d. = r per C.

r÷2	=	6l.	8s.	$7\frac{1}{2}$d.	= a for	56lb.
a÷2	is	3	4	$3\frac{1}{4}$	is b	28
b÷7	is	0	9	$2\frac{1}{4}$	is c	4
c÷2	is	0	4	7 − $\frac{1}{2}$	is d	2
a+b+c+d	is	10l.	6s.	$8\frac{1}{2}$d. $\frac{1}{2}$	for	90

97lb at 9l. 17s. 11d. = r per C.

r÷2 =	4	18	11½	= a for	56lb.
a÷2 is	2	9	5¾	is b	28
b÷4 is	0	12	4¼ ¾	is c	7
b÷7 is	0	7	0¾	is d	4
d÷2 is	0	3	6¼ ½	is e	2
a+b and c is	8	11	4¾ ¼	for	97

Note 2. Tons, hundreds, and quarters are reduced to the form of l. s. d. by multiplying only the quarters by 3. Then work by ſome of the foregoing rules.

What's the value of 33 tons, 3 C. 3 qrs. at 33l. 3s. 4d. per ton ?

	l.	s.	d.			l.	s.	d.
Let r =	33	3	9	thrice the qrs.				
			10					
r×10 =	331	17	6	= a				
			3					
a×3 is	995	12	6	is b }	at	33	0	0
r×3 is	99	11	3	is c }				
r÷6 is	5	10	7½	is d	at	0	3	4
b+c+d is	1100	11	4½	Anſ.	at	3	33	4

RULE VIII.

TROY WEIGHT.

The ounces and penny weights are in the proportion of l. s. but the grains are as half-pence ; therefore take half the grains, and the whole is reduced to the form of l. s. d.

What comes 67 oz. 8 dwt. 9 gr. of Gold to, at 4l. 2s. per oz.

l. s. d.
Let r = 67 8 4½ half the grains.
4
l. s.
r × 4 is 269 13 6 = a at 4 0
r ÷ 10 is 6 14 10 is b at 0 2
a + b is 276 8 4 Anſ. at 4 2

What comes 45 oz. 6 dwt. 7 gr. of plate to, at 5s. 10d. per oz.

l. s. d.
Let r = 45 6 3½ half the grains.
s. d.
r ÷ 4 is 11 6 6¾ ½ = a at 5 10
a ÷ 6 is 1 17 9—½ ⅙ = b at 0 10
a + b is 13 4 4 ⅙ Anſ. at 5 10

What comes 36lb. 11 oz. 12dwts. 12gr. to at 8s 4d per oz.

Firſt 36 × 12 + 11 = 443 oz. and 12 grs. ÷ 2 = 6 then we have

l. s. d.
443 12 6 = x, at 8s. 4d.
x ÷ 3 = 147 17 6 is a for 6 8
a ÷ 4 is 36 19 4½ is b for 1 8
a + b is 184 16 10½ for 8 4 Anſ.

What comes 736lb 11 oz. 12 dwts. 13 gr. to at 23l. 13s. 4d. per lb.? Anſ. 17441l. 11s. 11½d. nearly.

§ 171.

§ XVI. TARE and TRET.

GROSS WEIGHT of any commodity, is its own weight together with that of its package, whether caſk, cheſt, or whatever elſe.

TARE is the weight of the package, or an allowance made inſtead thereof. What remains after the Tare is taken from the Groſs, may be called *Tare-ſuttle*, if there are more deductions.

TRET is an allowance of 4lb. upon every 104lb. of Tare-ſuttle on account of duſt or other waſte. What remains after Tret is deducted, may be called Tret-ſuttle, if there be any following deductions.

CLOFF is an allowance of 2lb. for every 3 cwt. and ſome ſay for every 100lb. of Tret-ſuttle, to make the weight hold good when ſold by retail.

When all the deductions are made, the laſt remainder is called Neat or Net weight.

Note. The Tret being 4 to 104, or 1 to 26, will be found by taking the 26th part of the Tare-ſuttle.

CASE I.

If the Tare be at ſo much the whole; ſubtract the Tare from the Groſs weight, and the remainder is Neat.

EXAMPLE I.

Suppoſe the Groſs weight 261 cwt. 2 qr. 18 lb. 13 oz. Tare to be allowed 9 cwt. 3 qr. 19 lb. 14 oz. What's the Neat weight?

	c.	qr.	lb.	oz.	
From	261	2	18	13	Groſs.
Take	9	3	19	14	Tare.
Anſw.	251	2	26	15	Neat.

EXAMPLE II.

If the Groſs weight be 323 cwt. 2 qr. 17 lb. Tare 11 cwt. 1 qr. 18 lb. what is the Neat weight?

Anſw. 312 cwt. 27 lb.

CASE II.

When the allowance for Tare is at ſo much per frail, cheſt, bag, &c. then × the Tare of one frail by the number of frails, and ſubtract the product from the Groſs weight.

EXAMPLE I.

What is the Neat weight of 4 frails of raiſins, weighing Groſs as follows, Tare 18 lb. per frail?

	cwt.	qr.	lb.	
No. 1 Groſs weight	3	1	27	
2	3	2	18	
3	3	1	13	
4	3	3	0	
	14	1	2	whole Groſs weight, = x
18 × 4 = 72 lb. =	0	2	16	whole Tare = a
x − a =	13	2	14	Neat.

EXAMPLE II.

In 16 hogſheads of tobacco, each 5 cwt. 1 qr. 19 lb. Groſs, Tare per hhd. 100 lb. how much Neat weight?

	cwt.	qr.	lb.	
	5	1	19 = x	
			4	
x × 4 =	21	2	20 = a	
			4	
a × 4 =	86	2	24 = b	Groſs.
16 × 100 = 1600 lb. =	14	1	4 = c	Tare.
b − c =	72	1	20	Neat.

EXAM-

EXAMPLE III.

In 70 bales of Smyrna ſilk, each weighing 317lb. Groſs, Tare at 16lb. per bale, what is the Neat weight? Anſw. 21070lb.

EXAMPLE IV.

What is the Neat weight of 30 bales of Cyprus ſilk, each bale weighing 249lb. Groſs, Tare per bale 14lb? Anſw. 7050lb.

CASE III.

When the Tare is an aliquot part or parts of an hundred weight, work by the rules in Practice, i. e. take ſuch part or parts of the Groſs weight as the Tare is of (112lb.) an hundred weight.

EXAMPLE I.

What is the Neat weight of 44 cwt. 1 qr. 17 lb. Groſs, when 16 lb. per cwt is allowed for Tare?

	cwt.	qr.	lb.	
$x=$	44	1	17	Groſs
$x\div 7=$	6	1	$10\frac{3}{7}$	$=a$, Tare.
$x-a=$	38	0	$6\frac{4}{7}$	Neat.

EXAMPLE II.

What's the Neat of 32 cwt. 2 qr. Tare at 14 lb. per cwt?

	cwt.	qr.	lb.	
$x=$	32	2	0	Groſs.
$x\div 8=$	4	0	7	Tare$=b$.
$x-b=$	28	1	21	Neat.

	cwt.	qr.	lb.	
Groſs $x=$	124	3	0	Tare 18lb. per cwt.
$x\div 7=$	17	3	$8=a$,	for 16 lb.
$a\div 8=$	2	0	$25\frac{1}{2}=b$,	for 2 lb.
$a+b=$	20	0	$5\frac{1}{2}=c$,	for 18 lb.
$x-c=$	104	2	$22\frac{1}{2}$	Neat.

	cwt.	qr.	lb.	
Groſs $x=$	289	2	0	Tare 22 per cwt.
$x\div 8=$	36	0	$21=a$,	for 14lb.
$a\div 2=$	18	0	$10\frac{1}{2}=b$,	for 7lb.
$b\div 7=$	2	2	$9\frac{1}{2}=c$,	for 1lb.
$a+b+c=$	56	3	$13=d$	Tare for 22lb.
$x-d=$	232	2	15	Neat.

	cwt.	qr.	lb.	
Groſs $x=$	246	1	4	Tare 21lb. per cwt.
$x\div 8=$	30	3	$4=a$,	for 14lb.
$a\div 2=$	15	1	$16=b$,	for 7lb.
$a+b=$	46	0	$20=c$,	Tare for 21lb.
$x-c=$	200	0	12	Neat.

What's the Neat of 29 cwt. 3 qr. 24 lb. Tare 12 lb. per cwt? Anſw. 26 cwt. 3 qr. ½lb.

What's the Neat of 39 cwt. 1 qr. 14 lb. Tare at 18 lb. per cwt? Anſw. 33 cwt. 0 qr. 5½ lb.

What's the Neat of 363 cwt. 3 qr. 14 lb. Groſs, when 24 lb. per cwt. is allowed for Tare? Anſw. 285 cwt. 3 qr. 15½lb.

CASE IV.

When Tret is allowed with Tare, find the Tare-ſuttle according to the foregoing rules; and the Tret will be found by taking the 26th part of the Tare-ſuttle; alſo when Cloff is allowed, take it from the Tret-ſuttle, and the remainder is Neat.

EXAMPLE I.

Groſs 17 cwt. 3 qr. 14 lb. Tare 12lb. per cwt. Tret 4 lb. to 104 lb. or 1 to 26; and Cloff 2 to 100, or 1 to 50; how much Neat weight?

	cwt.	qr.	lb.	oz.	
x=	17	3	14	0	
x÷8 is	2	0	26	4	=a, for 14lb.
a÷7 is	0	1	7	12	is b, for 2lb.
a—b is	1	3	18	8	is c, Tare, 12lb.
x—c is	15	3	23	8	is d, Tare-ſuttle
d÷26 is	0	2	12	12	is e, Tret.
d—e is	15	1	10	12	is f, Tret-ſuttle
f÷50 is	0	1	6	6	is g, Cloff.
f—g is	15	0	4	6	Neat.

EXAMPLE II.

Bought 3 hhds, of ſugar whoſe Groſs weights are as under,

		cwt.	qr.	lb.
Tare of all 1 cwt.	No. 1 Groſs weight	11	2	15
1 qr. 2 lb. Tret	2	15	1	17
4 lb. to 104. at	3	17	3	14

2l. 16s. per cwt. Neat, what come they to? Anſw. 117l. 10s.

EXAMPLE III.

What is the Neat weight of 3 hhds of tobacco, weighing 15 cwt. 3 qr. 20 lb. Groſs, Tare per cent. 7 lb, Tret 4 lb. to 104, Cloff 2 lb. for 3 cwt? Anſw. 14 cwt. 1 qr. 3 lb. nearly.

EXAMPLE IV.

In 7 hhds of tobacco, each weighing 5 cwt. 2 qr. 7 lb. Groſs; how much Neat weight, allowing 8 lb. per cent for Tare, 4 lb. per 104 lb. for Tret, and 2 lb. per 3 cwt. for Cloff? Anſw. 34 cwt. 0 qr. 7 lb. 14 oz.

EXAMPLE V.

In 4 frails of raiſins, each 7 cwt. 1 qr. 12 lb. Groſs, Tare 2 qr. 10 lb. per frail, Tret 4 lb. per 104 lb. and Cloff 2 lb. per 100 lb; how much Neat? Anſw. 25 cwt. 2 qr. $1\frac{1}{4}$ lb.

Note. In calculating oil, if you reduce the Neat weight into half pounds, and divide them by 15, the quote will be the Neat gallons; becauſe 7 lb. and a half Neat are allowed to the gallon.

EXAMPLE I.

If 14 lb. per cwt. be allowed for Tare, how many gallons Neat are there in 5 casks of oil, weighing as follows:

	cwt.	qr.	lb.
No. 1	4	3	26
2	7	1	18
3	6	2	20
4	5	3	16
5	7	2	4

$x = 32$ 2 0 Groſs

$x \div 8$ is 4 0 7 $= a$ Tare.

$x - a$ is 28 1 21 is $6370 = b$, $\frac{1}{2}$lbs. Neat.

$b \div 15$ is 424 $\frac{10}{15}$ gallons the Anſw.

EXAM-

EXAMPLE II.

In 7 caſks of oil, each weighing 3 cwt. 1 qr. Groſs; how many Neat gallons, allowing 20lb. per cwt. Tare, and $7\frac{1}{2}$lb. per gallon? Anſw. $279\frac{1}{15}$ gallons.

EXAMPLE III.

In 289cwt. 2qr. Groſs weight of oil, how many gallons Neat at $7\frac{1}{2}$lb. per gallon, allowing 22lb. per cwt. for Tare? Anſw. 3474 gallons.

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§ XVII. OF INTEREST.

INTEREST is either *ſimple* or *compound*: and is the premium, or reward, allowed for the loan of money by the borrower to the lender.

Firſt of Simple Intereſt: in which the four following things are particularly to be regarded.

1. The money lent, called the *principal*, which put $=$p.
2. The intereſt of 100l. for one year, called the rate per cent. *per annum*, or *rate* of *intereſt*: which put $=$r.
3. The *time* for which the ſaid money, or *principal*, is lent: which put $=$n, whether years, half years, quarters, months, weeks, or days.
4. The amount ($=$m,) or the ſum of principal and intereſt: which, when the other three are given, may be found as follows.

A GENERAL RULE.

Multiply the principal by the rate per cent. and that product by the time, whether in years, half years, quarters, months, weeks, or days, for a dividend; then multiply 100 by one year for a diviſor, taking the year in the ſame denomination as the time propoſed in the queſtion, and the quotient thence ariſing will be the intereſt, in the ſame denomination with the principal; to which add the principal, and the ſum will be the amount. That is, $\frac{p\times r\times n}{100\times t}$ = the intereſt, And $p+\frac{p\times r\times n}{100+t}=m$, the amount. t being put = one year, or the months, weeks, or days in one year. Vide Theorem to queſtion 11th page 75.

EXAMPLES.

1. If 467l. be put out to intereſt for 6 years, what will it amount to in that time at 5 per cent. per annum?

$$\text{Firſt } \frac{467\times5\times6}{100\times1}=\frac{14010 \text{ dividend}}{100 \text{ diviſor}}$$

$$\begin{array}{rrrl} 14010\div100= & 140\text{l.} & 2\text{s} & \text{the intereſt} \\ & 467 & 0 & \text{principal} \\ \hline \text{The amount} & 607 & 2 & =m \end{array}$$

2. What is the amount of 674l. 17s. 6d = p at $4\frac{1}{2}$l. per cent per annum = r, for $5\frac{1}{2}$ years = n?

	l.	s.	d.	qr.	
p =	674	17	6	0	
p×4 is	2699	10	0	0	= a
p÷2 is	337	8	9	0	is b
a+b is	3036	18	9	0	is p×r
			5		
p×r×5 is	15184	13	9	0	is c
p×r÷2 is	1518	9	4	2	is d
c+d is	16703	3	1	2	is p×r×n, dividend
e÷100×t is	167	0	7	$2\frac{30}{100}$	is f, the intereſt
p+f is	841	18	1	$2\frac{30}{100}$	is m, the amount.

What

3. What is the intereſt of 450l. for 9 months, at 4 per cent per annum?

By the rule $450\times9\times4=16200$ the dividend, and the time given in the queſtion being months, we have $12\times100=1200$ the diviſor; then $16200\div1200=13$l. 13s. the intereſt is required.

4. What is the intereſt of 560l. for $3\frac{3}{4}$ years, at $3\frac{1}{2}$ per cent. per annum?

Firſt $3\frac{3}{4}$ years $=15$ quarters, then $560\times3\frac{1}{2}\times15=29400$, the dividend, and the time in the queſtion being quarters, we have $4\times100=400$ the diviſor; then $29400\div400=73$l. 10s. the intereſt required?

5 What will 340l. 10s. 6d. amount to in 3 weeks, at 4 per cent. per annum?

Firſt 340l. 10s. 6d. $=13621$ ſix-pences,

Then $$\frac{13621\times4\times3}{100\times52}=\frac{163452}{5200}=30\frac{1452}{5200}$$ ſix-pences the intereſt, therefore the amount is 341l. 5s. $7\frac{1}{2}$d nearly.

6. What is the intereſt of 7760 d. for 200 days, at $4\frac{1}{2}$ per cent. per annum?

$$\frac{7760\times4\frac{1}{2}\times200}{100\times365}=191\text{ d. }1\frac{135}{365}\text{ qr.}=15\text{s. }11\text{d.}$$

$1\frac{135}{365}$ qr. the intereſt required.

7. What is the amount of 19s. $6\frac{1}{2}$d $=469$ half pence, at 6 per cent per annum, for 3674 days? Anſw. 1l. 11s. 4d. 0. $\frac{18272}{36500}$ qr.

PARTICULAR RULES.

RULE I.

When the rate is at 5 per cent, × the principal in pence by the number of days,dividing the product by 7300, and the quote will be the intereſt in pence.

Note, 7300 is found by multiplying 365 by 100, and dividing by 5.

8. What's the intereſt now due on a bond of 320l. 10s. commencing intereſt July the 10th 1764, this being the 25th of March 1765, at 5 per cent per annum?

From July 10th 1764, to March 25th 1765, are 258 days, and 320l. 10s.=76920d. Therefore 76920×258 ÷7300=2718½d.=11l. 6s. 6½d. the intereſt, ſought.

9. What's the intereſt of 260l. 14s. 7d. for 760 days, at 5 per cent per annum? Anſw. 27l. 2s, 10d. $2\frac{46}{73}$ qr.

10. What will 76l. 13s. 10d. amount to in 3 years 164 days, at 5 per cent. per annum? Anſw. 89l. 18s. 4d. $1\frac{4516}{7300}$ qr.

RULE II.

Twenty years intereſt of any ſum of money at 5 per cent. is equal to the principal. Therefore the intereſt at this rate may be found by taking ſuch proportional part of the principal as the time is of 20 years; and for any other rate, it is but adding or ſubtracting a proportionable part.

11. What's the intereſt of 327l. for 6 years, 7 months, and 12 days, at 5 per cent per annum?

		l.	s.	d.	
p	=	327	0	0	= to 20 years intereſt.
p ÷ 4	is	81	15	0	is a, for 5 years.
a ÷ 5	is	16	7	0	is b, for 1 year.
b ÷ 2	is	8	3	6	is c, for 6 months.
c ÷ 6	is	1	7	3	is d, for 1 month.
d ÷ 5	is	0	5	1⅗	is e, for 6 days.
d ÷ 5	is	0	5	1⅗	is e, for 6 days.
a+b+c+d+2e	=	108 3	7	3⅕	the Anſw.

Now if the intereſt had been required at 4, or 6 per. cent. it would but be ſubtracting or adding a fifth part ;

			l.	s.	d.	
Thus,	x	=	108	3	$7\frac{3}{4}$	= intereſt at 5 per cent.
	x÷5	is	21	12	$8\frac{3}{4}$	is a,
	x—a	is	86	10	11	= intereſt at 4 per cent.
	x+a	is	129	16	$4\frac{1}{2}$	= intereſt at 6 per cent.

Or, having found the intereſt at 5 per cent. ſay, as 5 is the intereſt found :: ſo is any other given rate : to the intereſt required. Thus,

		l.	s.	d.				l.	s.	d.
As 5	:	108	3	$7\frac{3}{4}$	::	4	:	86	10	11
						6	:	129	16	$4\frac{1}{2}\frac{1}{5}$

12. May the 18th, 1762, I put out 624l. 18s. 10d. at intereſt at 5 per cent. per annum ; how much does the intereſt amount to the 23d of July 1765 ?

		y.	m.	d.
From	May 18th 1762 to May 18th 1765 is	3	0	0
	May 18th 1765 to July 18th 1765 is	0	2	0
	July 18th 1765 to July 23d 1765 is	0	0	5

The Anſw. 99l. 7s. 6½d.

R U L E III.

To find the intereſt of any ſum of money for a year, × the principal by the rate of intereſt, and ÷ the product by 100.

13. What is the intereſt of 769l. for a year at 6 per cent. per annum ?

769×6÷100=46l. 2s. 9d. 2 $\frac{40}{100}$ qr. the intereſt.

Note, If the intereſt of one year be multiplyed by any number of years, the product will be the intereſt for that number of years.

14. What is the intereſt of 769l. for 5 years at 6 per cent. per annum?

	l.	s.	d.	qr.	
769×6÷100=	46	2	9	$2\frac{40}{100}$	intereſt for 1 year.
				5	
Anſw.	230	14	0	0	intereſt for 5 years.

Note. The above rules ſerve to calculate any thing that's rated at ſo much per cent: as Factorage, Brokerage, Inſurance, purchaſing of Stocks, &c.

OF FACTORAGE.

Factorage, Commiſſion, or Proviſion, is an allowance made by the Merchants to their Factors or Agents, for buying or ſelling of their goods; and is generally at a certain rate per cent. and calculated, as for one year, by the 3d particular rule.

EXAMPLE I.

What is the commiſſion of 862l. 10s. at 3½l. per cent? Anſw. 30l. 3s. 9d.

EXAMPLE II.

What is the factorage of 568l. 15s. 6d. at 2½l. per cent? Anſw. 14l. 4s. 4d. $2\frac{60}{100}$ qr.

OF BROKERAGE.

Brokerage is the allowance made to Brokers for aſſiſting others in buying and diſpoſing of their goods: and is generally calculated by ÷ing the principal by 100, and taking ſuch a part of the quote as the rate per cent. is of a l.

EXAMPLES.

1. What is the brokerage of 847l. 10s. at 4s per cent?

847l. 10s. ÷ 100=8l. 9s. 6d. ÷ 5=1l. 13s. 10d. $3\frac{1}{5}$qr. the brokerage required. Or by the Rule of Three, thus,

as 100l. : 4s. :: 847l. 10s. : 1l. 13s. 10d. $3\frac{20}{100}$ qr.

2. What is the brokerage of 650l. at 5s. or one 4th per cent? Anſw. 1l. 12s. 6d.

OF INSURANCE.

Inſurance is a ſecurity given that the value of goods, ſhips, houſes, &c. ſhall be reſtored, in caſe of loſs or damage, by ſtorms, fire, &c. For ſuch ſecurity a premium is paid down, at a certain rate per cent. and may be calculated by ſome of the foregoing rules.

EXAMPLES

1. What is the inſurance of 356l. 3s. for 16 months at 7½ per cent. per annum?

By the general rule in page 98.

$$\frac{356l.\ 3s. \times 7\frac{1}{2} \times 16}{100 \times 12} = 35l.\ 14s.\ 3d.\ 2\frac{480}{1200}qr.$$ The Anſw.

2. How much muſt be paid yearly for the inſurance of a houſe, whoſe value is 287l. 10s. per annum, at 3½ per cent? Anſw. 10l. 1s. 3d.

OF STOCKS.

Stocks are the public funds of the nation; the ſhares of which being transferable from one perſon to another occaſion the extenſive buſineſs called, Stock-jobbing. Theſe are commonly beſt calculated by the Rule of Three.

E X A M P L E S.

1. What is the purchaſe of 6490l. 15s. 6d. Bank Stock at 110¾ per cent?

	l.	s.	d.		
As 100l. : 110¾l. ::	6490	15	6	x	
			10		
	64907	15	0	x × 10	a
		10			
	649077	10	0	a × 10	b
	64907	15	0		a
	3245	7	9	x ÷ 2	c
	1622	13	10½	c ÷ 2	d
	7188 | 53	6	7½	a + b + c + d.	

20

10 | 66

12 Anſw. 7188. 10s. 7d $3\frac{98}{100}$ qrs.

7 | 99

4

3 | 98

2. What is the purchaſe of 2400l. South-Sea Stock at 123¾ per cent? Anſw. 2970l.

3. What is the purchaſe of 2474l. 16s. Bank Annuities at 96⅓, or 96l. 6s. 8d. per cent? Anſw. 2384l. 1s. 1d. $3\frac{4}{100}$ qr

What

4. What is the purchaſe of 575l. 10s. South Sea Annuities at 131¾ per cent? Anſw. 758l. 4s. 5d. $\frac{1}{10}$ qr.

5. At 97¼l. per cent. what is the purchaſe of 254l. 17s. Bank Annuities? Anſw. 247l. 16s. 9d. 3 $\frac{96}{100}$ qr.

Of COMPOUND INTEREST.

COmpound Intereſt is that which ariſes from any principal and its intereſt put together, as the intereſt becomes due; that is, when at every payment, or at the time when the payments become due, the amount is converted into a *new principal*. On this account it is ſometimes called intereſt upon intereſt.

For inſtance, were I to put out 100l. for two years at 6l. per cent. per annum, according to the utmoſt improvement, the intereſt as it becomes due not being received but forborn, at the end of the firſt year it would amount to 106l. which 106l. would be my principal for the ſecond year; and at the determination of the ſecond year it would be increaſed to 112l. 7s. 2½d. or 100l. principal, 12l. for two years ſimple intereſt, and 7s. 2½d. the intereſt of 6l. my firſt year's intereſt.

RULE.

Find the amount of the given principal, for the time of the firſt payment, by ſimple intereſt; then make this amount your principal for the ſecond payment, whoſe amount calculate in the ſame manner; and ſo on through all the payments, ſtill accounting the *laſt amount* as the principal of the next payment.

EXAMPLES.

1. What is the compound intereſt of 500l. forborn 3 years, at 5 per cent. per annum?

Firſt 500×5÷100 equal to 25l. the 1ſt year's intereſt.

Then 500 + 25 equal to 525l. the ſecond years principal.

Secondly 525×5÷100 equal to 26l. 5s. the 2d year's intereſt.

Then 525+26l. 5s. equal to 551l. 5s. the 3d year's principal.

Thirdly 551l. 5s.×5÷100 equal to 27l. 11s 3d. the 3d year's intereſt.

	l.	s.	d.	
The 3d year's principal is	551	5	0	Add.
The 3d year's intereſt is	27	11	3	
The total amount is	578	16	3	Sub.
The money firſt lent is	500	0	0	
The total intereſt is	78	16	3	

2. What will 550l. 10s. amount to in 3¼ years, at 6 per cent. per annum, compound intereſt? Anſw. 675l. 6s. 5¼d.

3. What is the amount of 650l. for 5 years at 5½ per cent. per annum compound intereſt? Anſw. 849l. 10s. 5¾d.

Note. If the payments are not yearly, but half yearly, quarterly, monthly, or any other aliquot part of a year, take ſuch aliquot part of the principal and with it work for the intereſt as before. Or work by the general Rule in Simple Intereſt; for the Rule ſtill holds true, provided there be always a complete integer number of times of payments, that is, a certain number of times without a part of another: nor will it be juſt to calculate for a complete time and then to take the ſame part of the reſult as is the part of the time, for the Rule takes no notice of ſuch parts. In this manner ſome Authors have made erroneous calculations. By Logarithms it is as eaſy to perform calculations with parts of times of payments as with whole ones; nor is it impoſſible to perform ſuch calculations without Logarithms, but the trouble in ſome caſes is intolerable.

What

4. What is the amount of 600l. forborn 2 years, at 5 per cent. per annum, ſuppoſing the intereſt payable half yearly? Anſw. 662l. 5s. 9$\frac{1}{8}$d. For 600÷2×5÷100 equal to 15l. the 1ſt half years intereſt. 600+15 equal to 615l the principal for the 2d half year. 615÷2×5÷100 equal to 15l. 7s. 6d. the intereſt for the 2d half year. 615+15l. 7s. 6d. equal to 630l. 7s. 6d. the principal for the 3d half year. 630l. 7s. 6d.÷2×5÷100 equal to 15l. 15s. 2$\frac{1}{4}$d. the intereſt for the 3d half year. 630l. 7s 6d.+15l. 15s. 2$\frac{1}{4}$d. equal to 646l. 2s. 8$\frac{1}{4}$d. the principal for the 4th half year. 646l. 2s. 8$\frac{1}{4}$d.÷2×5÷100 equal to 16l. 3s. 0$\frac{3}{4}$ $\frac{1}{4}$ the 4th half years intereſt. 646l. 2s. 8$\frac{1}{4}$d+16l 3s. 0$\frac{3}{4}$ $\frac{1}{4}$ equal to 662l. 5s. 9$\frac{1}{8}$d. the amount at the end of 2 years, or the 4 half years.

5. What will 600l. amount to in 2 years at 5 per cent. per annum, compound intereſt, ſuppoſing the intereſt payable quarterly?

600×5÷400 equal to 7l. 10s. the intereſt for the 1ſt quarter. 600+ 7l. 10s. equal to 607l 10s. the principal for the d quarter. 607l. 10s. ×5÷400 equal to 7l. 11s. 10$\frac{1}{2}$d the intereſt for the ſecond quarter, and ſo on to the 8th quarter, whoſe intereſt is 8l. 3s. 7$\frac{1}{2}$d. the whole amount being 662l. 13s. 9$\frac{3}{4}$d. the Anſwer required.

6. What will 50l. amount to in 5 years at 5 per cent per annum compound intereſt, ſuppoſing the intereſt payable half yearly? Anſw. 64l. 1d.

7. What is the amount of 217l. forborn 2$\frac{1}{4}$ years, at 5 per cent. per annum, ſuppoſing the intereſt payable quarterly? Anſw 242l. 13s. 4$\frac{1}{2}$d.

See more of Intereſt in Decimals.

REBATE or DISCOUNT.

REBATE, or DISCOUNT, is an allowance for paying money before the time appointed for payment, and is the difference between a ſum of money due a certain time to come, and its preſent worth; which preſent worth, if put to intereſt for the time and rate for which the diſcount is to be made, would amount to the ſum or debt then due.

RULE

Multiply the time by the rate of intereſt; alſo multiply 100 by 1 year, taking the year in the ſame *denomination* as the time propoſed: add the two *products* together, and ſay,

As the ſum of the two products,
Is to the former of the two products;
So is the ſum to be diſcounted, in what denomination ſo ever,
To the diſcount, in the ſame denomination.

The diſcount ſo found ſubtract from the ſum diſcounted; and the remainder is the ready money to be paid down by the diſcounter.

This ready money may alſo be found, without finding the diſcount, thus,

As the total of the two products,
Is to the latter of the two products;
So is the ſum to be diſcounted, in what denomination ſo ever,
To the preſent worth, in the ſame denomination.

EXAMPLES.

1. What is the preſent worth of 463l. 10s. due 9 months hence, diſcount at 4 per cent. per annum?

9 months × 4 = 36 the 1ſt product,
100 × 12 months = 1200 the 2d product.

1236 the ſum of the two products

Then as 1236 : 36 :: 463l. 10s. : 13l. 10s. the diſcount.
Then 463l. 10s.—13l. 10s. equal to 450l. the preſent worth
Or, as 1236 : 1200 :: 463l. 10s. : 450l. the preſent worth.

2. What is the preſent worth of 460 l. due 5 weeks hence, diſcount at 7 per cent. per annum?

5 weeks × 7 = 35 the 1ſt. product.
52 × 100 = 5200 the 2d. product.

5235 the ſum of the two products.

As 5235 : 5200 :: 460 l. : 456 l. 18s. 5d. 3 $\frac{2955}{5235}$ qr. the preſent worth required.

3. How much ready money may be given for a Note of 357 l. 12s. 6d. due 234 days hence, diſcount at 4 per cent. per annum?

234 × 4 = 936 the 1ſt. product.
365 × 100 = 36500 the 2d product.

37436 the ſum of the two products.

357 l. 12s. 6d. = 14305 ſix-pences.
Then as 37436 : 36500 :: 14305 ſix pences : 348l. 13s. 8d $\frac{3104}{37436}$ qr. preſent worth.

Or, as 37436 : 936 :: 14305 : 8l. 18s. 9d. 3$\frac{34332}{37436}$ diſcount.

4. What is the preſent worth of 767l. 17s. 6d. due 3½ years hence, diſcount at 5 per cent. per annum?

7 half years × 5 = 35 the 1ſt product.
2 × 100 = 200 the 2d. product.

235 the total of the two products.

Then as 235 : 200 :: 767l. 17s. 6d : 653l. 10s. 2d. 2$\frac{50}{235}$ the Anſw.

5. A man has a Legacy of 500l. left him, to be paid 20 years hence; but wanting money, he is deſirous to ſell the ſame; how much ready money muſt he receive allowing the purchaſer 5½ per cent? Anſw. 238l. 1s. 10d. $3\frac{9}{21}$ qr.

6. What is the diſcount of 624l. for three months at 6 per cent. per annum? Anſw. 9l. 4s. 5d. +

Note, When the ſign more is annexed to the anſwer, it denotes the anſwer to be a ſmall matter more.

7. What is the rebate of 125l. 10s. payable 10 months hence, at 4½d. per cent. per annum? Anſw. 4l. 10s. 8½d.

8. Sold goods to the amount of 166l. 12s. to be paid 6 months hence, what muſt I have at preſent, diſcount at 8 per cent. per annum? Anſw. 160l. 3s. 10d. +

9. Bought goods to the value of 17l. 14s. to be paid for 8 months hence, what muſt I pay in ready money, diſcount at 7 per cent. per annum? Anſw. 16l. 18s. 2¼d. +

10. If a legacy of 1200l. is left me on the 12th of March 1765, to be paid on the Chriſtmas-day following, what muſt I receive when I allow 5 per cent. per annum diſcount? Anſw. 1154l. 9s. 1d. +

11. At 5 per cent. per annum diſcount, what is the preſent worth of 60l. payable at two 3 months, i. e. half at 3 months, and the reſt at 6 months? Anſw. 58l. 17s. 11d. 2qr. +

12. Sold goods to the value of 420l. which is payable as follows, viz. 100l. at 3 months, 200l. at 6 months, and the reſt at 10 months; what muſt I have in preſent money, allowing diſcount at 6 per cent. per annum? Anſw. 406l. 19s. 7¼d +

Note. When goods are bought or ſold, and diſcount is to be made for preſent payment, at any rate, per cent. without regard to time, the intereſt of the ſum, as calculated for a year, is the diſcount.

Be

13. Bought goods to the value of 160l. 10s. 6d. having diſcount at 5 per cent, how much is the diſcount? Anſw. 8l. 6¼d.+

14. Sold 46 dozen of knives at 10s per dozen, allowing diſcount at 1s. 6d. per L. what is the neat value?

First 46÷2=23l. the groſs value.
l. ſix d. l. ſix d.
Then as 1 : 3 :: 23 : 69 =1l. 14s. 6d. the diſcount.
Hence 23l.—1l. 14s. 6d. =21l. 5s. 6d. the neat value.

Note. The method uſed among Bankers, &c. in diſcounting bills, is to find the intereſt of the ſum drawn for, from the time the bill is diſcounted to the time when it becomes due, (including the days of grace) which intereſt they reckon as the diſcount, thereby making the diſcount more than it really is, the error being in proportion to the time.

EQUATION of PAYMENTS.

EQuation of Payments is, when ſeveral ſums of money are due at different times, to find the time, when the whole may be paid in at once, without loſs to the debtor or creditor.

The common Rule is to multiply every ſum of money by the time it is to continue in the hands of the debtor, and to divide the ſum of the products by the ſum of all the payments, the quote being the equated time.

Note. This rule is not exactly true, though very nearly ſo, and may ſerve very well in common buſineſs; only it makes the mean time a ſmall matter too large.

EXAMPLES.

1. A owes B 110l. whereof 50l. is to be paid at 2 years end, 40l at 3½ years end, and 20l. at 4½ years end; at what time may the whole debt be paid together, without prejudice to either party?

l. years
50 × 2 is equal to 100
40 × $3\frac{1}{2}$ equal to 140
20 × $4\frac{1}{2}$ equal to 90

110 110)330(3 years. Anſwer.
330

0

2. A perſon has three ſeveral ſums of money due at different times, 50l. at the end of 5 months, 84l. at the end of 10 months, and 36l. a year and a half hence, but would receive the whole at once; in what time ſhall he receive the whole ſum? Anſw. $10\frac{38}{170}$ months.

3. A debt of 500l. is to be diſcharged thus, viz. 100l. at preſent, 300l. at 5 months, and the reſt at 15 months, what is the equated time for diſcharging the whole? Anſw. 6 months.

4. A debt is to be diſcharged by paying one half at three months, a third at 5 months, and the reſt at 17 months: What is the equated time for the whole?

Take any number at pleaſure that is diviſible into the propoſed parts without remainder, as 6.

l.
Then half of 6 is 3. 3 × 3 equal to 9
a third of 6 is 2. 2 × 5 equal to 10
And 6—5 is 1. 1 × 17 equal to 17

6)36(6 months Anſw.

5. A debt is to be diſcharged thus, viz one fourth at preſent, and one fourth every three months after, till the

whole be diſcharged: What is the equated time for the whole? Anſw. $4\frac{1}{2}$ months.

The End of the Firſt Chapter.

CHAP. II.

§ I. *Of Vulgar* FRACTIONS.

A FRACTION, ſimply and abſtractedly conſidered, is ſome part or parts either of an unit, or any other number or quantity whatſoever.

A Vulgar Fraction conſiſts of two numbers, called the Numerator and Denominator, placed one above the other with a line of ſeparation between them, as $\frac{3}{4}$ $\frac{\text{Numerator}}{\text{Denominator}}$ which is named three fourths: the Numerator being firſt named, then the Denominator.

The Numerator and Denominator are called Terms of the Fraction.

The Denominator ſhews how many equal parts the whole is divided into, and the Numerator expreſſes how many ſuch parts the Fraction conſiſts of. For inſtance: Imagine the line A B to repreſent one yard, one foot, one pound, one ſhilling, one acre, or any other integer whatſoever, and let it be divided into 4 equal parts; then

One of thoſe parts will be $\frac{1}{4}$, or one fourth.
Two of thoſe parts will be $\frac{2}{4}$, or two fourths.
Three of thoſe parts will be $\frac{3}{4}$, or three fourths.
Four of thoſe parts will be $\frac{4}{4}$, or four fourths, equal

to the whole line. Conſequently $\frac{8}{4}$ is improperly called a fraction.

Now it is eaſy to perceive, that 5 of the above parts, is one fourth part more than the whole line; that 6 is more than the whole by 2 ſuch parts; that 7 exceeds the whole by 3 ſuch parts; and that 8 is equal to 2 whole lines: for if one thing be divided into 4 equal parts, 8 ſuch parts muſt be equal to 2 whole things, and conſequently

$\frac{12}{4}$ equal to three whole things.

$\frac{16}{4}$ equal to four whole things.

$\frac{20}{4}$ equal to five whole things, &c.

S C H O L I U M.

Hence, fractions whoſe Numerators are leſs than, equal to, or greater than their Denominators, are reſpectively leſs than, equal to, or greater than unity, or 1.

C O R R O L L A R Y I.

Whence it follows, that a fraction is but a quotient, ſignifying a part or parts of an Unit, expreſt by a Numerator as a Dividend and a Denominator as a Diviſor.

C O R R O L L A R Y II.

Therefore as the Numerator is to the Denominator, ſo is the Fraction to Unity.

Or as $3 : 4 :: \frac{3}{4} : 1$. i. e. $\frac{3 \times 4}{4 \times 3} = 1$.

L E M M A I.

To multiply a fraction is to multiply the Numerator; but to divide a fraction is to multiply the Denominator, as in the example above.

CORROLLARY III.

Fractions having the ſame Denominators are one to another as their Numerators.

Thus $\frac{3}{8} : \frac{5}{8} :: 3 : 5$.

CORROLLARY IV.

As any fraction is to unity, ſo is unity to the inverſe, or reciprocal of the ſaid fraction.

Note. By the inverſe, or reciprocal of a fraction, is meant the fraction gotten by inverting its terms: thus the reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$, and of 7 or $\frac{7}{1}$ is $\frac{1}{7}$.

Thus $\frac{2}{5} : 1 :: 1 : \frac{5}{2}$.

LEMMA II.

Multiplying or ÷ing both terms of a fraction by one and the ſame number alters not its value, but produces a new fraction equivalent to the given one.

Let both terms of the fraction $\frac{4}{8}$ be ×ed by 3.

$$\text{Then } \frac{4 \times 3}{8 \times 3} = \frac{12}{24} \text{ the new fraction.}$$

Now it is plain that theſe numbers produced are in proportion to each other as the numbers ×ed; becauſe as 4 is the half of 8, ſo 12 is the half of 24, and conſequently the two fractions are of equal value.

Or if we divide a pound into 8 equal parts, 4 of thoſe parts muſt be the half of it; and if a pound be ÷ed into 24 equal parts, 12 of thoſe parts muſt ſtill be the half.

Again, if both terms of the fraction $\frac{12}{24}$ be ÷ed by 3, and afterwards by 2.

$$\text{Thus, } \frac{12 \div 3 = 4 \div 2 = 2}{24 \div 3 = 8 \div 2 = 4} \text{; the new fraction.}$$

is ſtill equivalent to $\frac{12}{24}$.

Let both terms of the fraction $\frac{2}{3}$ be ×ed by 3, 4, and 5.

Then $\begin{array}{l} 2\times3\times4\times5=120 \\ 3\times3\times4\times5=180 \end{array}$ the new fraction, i.e. $\frac{2}{3}=\frac{120}{180}$.

On the contrary both terms of the fraction $\frac{120}{180}$ divided by 60, give $\frac{2}{3}$, $=\frac{120}{180}$.

N O T A T I O N.

There are four ſorts of Vulgar Fractions, diſtinguiſhed by the names of Proper, Improper, Compound and Complex.

1. A proper Fraction (called alſo a pure, ſimple or ſingle Fraction) is that whoſe Numerator is leſs than its Denominator, and repreſents a part or parts leſs than the Denominator, or a whole; as $\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{2}$, &c. which may repreſent $\frac{3}{4}$ of a foot, $\frac{2}{3}$ of an Acre, $\frac{1}{2}$ of ten thouſand pound, &c.

2. An Improper Fraction is that whoſe Numerator is always greater, or equal to, its Denominator, and repreſents a number that is always greater, or equal to, an unit, or one whole thing, as $\frac{5}{3}$ or $\frac{3}{3}$.

3. A Compound Fraction, or Fraction of a Fraction, is that whoſe parts are Vulgar Fractions, and conſiſts of more than one Numerator and Denominator connected with the word of; as $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{7}$; which read thus,—the half of two-thirds of three-fourths of four-ſevenths.

4. A Complex Fraction is that whoſe Numerator, or Denominator, or both, is a Fraction or a mixt number, as $\frac{5\frac{1}{4}}{8}$, or $\frac{5}{8\frac{1}{2}}$ or $\frac{16\frac{1}{4}}{12\frac{1}{2}}$: and if any ſuch Fraction occurs, it only denotes a diviſion of the Numerator by the Denominator. Cor. 1.

If what has been already ſaid, be perfectly underſtood, all operations relating to Fractions admit of very few or no difficulties. I now proceed to Reduction of Vulgar Fractions.

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§ II. *Reduction of Vulgar Fractions.*

REDUCTION is the converting of Fractions out of one form or denomination into another, for the more eaſe in working, or in eſtimating of their value, and is chiefly a preparation of them for Addition or Subtraction.

PROBLEMS.

I. *To expreſs a whole number Fractional wiſe.*

Rule. Place 1 under it for a Denominator.

Example. Suppoſe 3 the whole number, then $\frac{3}{1}$ is the Fraction required.

II. *To reduce a whole number to a Fraction of a given Denominator.*

Rule. Multiply the whole number by the given Denominator, and make the product the Numerator, under which write the given Denominator.

EXAMPLES.

1. Reduce 5 to a Fraction whoſe Denominator may be 7.

Thus $\frac{5\times7}{7} = \frac{35}{7}$ the Fraction required. i. e. $\frac{35}{7} = 5$ by Lemma 2.

2. Reduce 14 to a Fraction whoſe Denominator may be 9.

Anſw. $\frac{126}{9} = 14$

3. Reduce 24 to a Fraction whoſe Denominator may be 30.

Anſw. $\frac{720}{30} = 24.$

III. *To reduce a mixt number into an Improper Fraction of equal value.*

Rule. Multiply the whole number by the Denominator of the Fraction, and to the product add the Numerator, and the ſum is the new Numerator, under which write the Denominator.

EXAMPLES.

1. Reduce $36\frac{3}{4}$ to an Improper Fraction.

Thus $\frac{36\times4,+3}{4} = \frac{147}{4}$ the Fraction required.

For ſuppoſe the whole number to repreſent 36 yards of cloth, and each yard divided into 4 equal parts, or quarters, the 36 yards ſo divided are of courſe equal to 144 quarters; i. e. 36×4=144; to which if the other 3 quarters be +ed, the whole is 147 quarters, or $\frac{147}{4}$ yards.

2. Reduce $32\frac{2}{7}$ to an improper Fraction: Anſw. $\frac{226}{7}$.

3. Reduce $13\frac{3}{10}$ to an improper Fraction, and it is $\frac{133}{10}$.

4. When $7\frac{1}{2}$, $11\frac{3}{4}$, and $14\frac{2}{3}$, are each reduced to an improper Fraction, they become $\frac{15}{2}$, $\frac{47}{4}$, $\frac{44}{3}$.

IV. *To reduce an improper Fraction into its equivalent whole or mixt number.*

Rule. Divide the Numerator by the Denominator, and the quotient is the whole or mixt number.

EXAM-

EXAMPLES.

1. Reduce $\frac{126}{9}$, $\frac{147}{4}$, $\frac{229}{7}$, and $\frac{728}{8}$, each to its whole or mixt number.

$126 \div 9 = 14$ | $229 \div 7 = 32\frac{5}{7}$
$147 \div 4 = 36\frac{3}{4}$ | $728 \div 8 = 91$

2. When $\frac{18}{3}$, $\frac{16}{3}$, $\frac{17}{3}$, $\frac{64}{5}$, $\frac{213}{7}$, and $\frac{403}{9}$, are each reduced to their whole or mixt numbers, they become 6, $5\frac{1}{3}$, $5\frac{2}{3}$, $12\frac{4}{5}$, $30\frac{3}{7}$, and $44\frac{7}{9}$.

V. *To find the greateſt common meaſure (or ÷ſor) for any two numbers, or for the Numerator and Denominator of a Fraction.*

Rule. Divide the greater number by the leſs, and that Diviſor by the Remainder, continuing to make the laſt ÷ſor a Dividend, and the laſt Remainder into a ÷ſor, till nothing remains, and the laſt ÷ſor is the greateſt common meaſure.

Note. If the laſt Diviſor be 1 the numbers are prime to each other.

EXAMPLES

1. Let the numbers propoſed be 168 and 240: their greateſt common meaſure is 24, as found by the Rule

```
thus,        168)240(1
                 168
                 ———
                  72)168(2
                     144
                     ———
The greateſt common meaſure is 24)72(3
                                  72
                                  ——
```

2. What is the greateſt common meaſure for $\frac{312}{1026}$? Anſw. 6.

3. What is the greateſt common meaſure for $\frac{152}{384}$? Anſw. 8.

4. What is the greateſt common meaſure of $\frac{91}{117}$? Anſw. 13.

VI. *To abbreviate or reduce a Fraction to its loweſt terms.*

A GENERAL RULE.

Find the greateſt common meaſure, by which ÷ both terms of the Fraction, and the quotients will be the terms of the Fraction required.

EXAMPLES.

1. Reduce the Fraction $\frac{168}{240}$ to its loweſt terms. The greateſt common meaſure being 24, as found above,

$$\frac{168\div24=7}{240\div24=10},\ \text{i. e. } \frac{7}{10}=\frac{168}{240}$$

2. Reduce $\frac{312}{1026}$ to its loweſt terms. Facit $\frac{52}{171}$.

3. Reduce $\frac{75}{135}$ to its loweſt terms. Facit $\frac{5}{9}$.

4. Reduce $\frac{152}{384}$ to its loweſt terms. Facit $\frac{19}{48}$.

5. Reduce $\frac{192}{576}$ to its loweſt terms. Facit $\frac{1}{3}$

6. Reduce $\frac{5184}{6912}$ to its loweſt terms. Facit $\frac{3}{4}$

PARTICULAR RULES.

1. **Rule.** Any Fraction may be abbreviated by a continual diviſion by 2, if the terms end with an even number or a cypher.

7. Thus $\frac{256}{784} = \frac{128}{392} = \frac{64}{196} = \frac{32}{98} = \frac{16}{49}$ by a continual halving; therefore $\frac{16}{49} = \frac{256}{784}$.

8. $\frac{120}{2368}$ being continually halved is $\frac{15}{296}$.

2. **Rule.** When both terms end with 5, or one with 5 and the other with a cypher, divide, both by 5.

9. Let the Fraction be $\frac{465}{895}$; Then $\frac{465 \div 5 = 93}{895 \div 5 = 179}$

10. $\frac{325}{1300} = \frac{13}{52}$. For $\frac{325 \div 5 = 65 \div 5 = 13}{1300 \div 5 = 260 \div 5 = 52}$.

3. **Rule.** If the ſum of the digits of any number can be ÷ed by 3, the number itſelf may alſo be ÷ed by 3.

As for example, the number 741 is diviſible by 3, becauſe 12, which is the ſum of its digits 7, 4, and 1, is ſo.

EXAMPLES.

1. $\frac{63}{72}, = \frac{7}{8}$, For $\frac{63 \div 3 = 21, \div 3 = 7}{72 \div 3 = 24, \div 3 = 8}$.

2. $\frac{57}{135} = \frac{19}{45}$.

4. **Rule.** When both terms end with cyphers, caſt off as many as are common to both: and reduce the ſignificant figures to lower terms, by the foregoing Rules.

EXAMPLES.

1. $\frac{3500}{56000} = \frac{35}{560} = \frac{7}{112} = \frac{1}{16}$ in its loweſt terms.

2. $\frac{15000}{27000} = \frac{15}{27} = \frac{5}{9}$

5 **Rule.** When any number which is expreſſed by ſeveral others, with the ſign of addition or ſubtraction between them, is to be divided by any number, then muſt all the parts of it be ÷ed by this number.

Thus $\frac{6+9-12}{3} = 2+3-4=1.$

But if the given number is expreſſed by others with the ſign of multiplication between them, only one of them muſt be ÷ed:

So $\frac{4\times9\times12}{3\times8} = \frac{4\times3\times12}{1\times8} = \frac{1\times3\times12}{1\times2} = \frac{1\times3\times6}{1\times1} = 18$

And in this caſe, when the ſame numbers are concerned in both Numerator and Denominator, they may be omitted, or caſt out of both.

EXAMPLES.

$$\frac{9\times5\times7\times3\times2\times8\times7}{5\times7\times2\times5\times3\times9\times6}=\frac{8\times7}{5\times6}=1\frac{26}{30}=1\frac{13}{15}$$ by omitting the common terms, 9, 5, 7, 3, 2.

$$\frac{4\times3\times8\times7}{5\times4\times7\times3}=\frac{8}{5}=1\frac{3}{5}$$ by omitting, 4, 3, 7.

$$\frac{23\times14\times70\times8}{80\times2\times7\times230}=\frac{1\times7\times10\times1}{10\times1\times1\times10}=\frac{1\times7\times1\times1}{10\times1\times1\times1}=\frac{7}{10}$$

VII. *To reduce a Compound Fraction to an equivalent ſingle one.*

RULE. Multiply all the Numerators together for a new Numerator, and all the Denominators together for a new Denominator, of the ſingle Fraction.

Note. If part of the Compound Fraction be a whole or mixt number, reduce it to Fractions by the foregoing rules.

EXAMPLES.

1. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of a pound ſterling to a ſingle Fraction. Fact $\frac{1}{2}$ l.

Thus $\frac{2\times3}{3\times4}=\frac{6}{12}=\frac{1}{2}$ l. the Fraction required.

This is ſo clear that it carries its own evidence along with it: For the value of $\frac{3}{4}$ l. is 15 ſhillings, two thirds of which is 10 ſhillings or $\frac{1}{2}$ l.

2. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of a pound troy or 12 oz. to a ſingle Fraction. Facit $\frac{1}{4}$ lb.

Thus $\frac{1\times2\times3}{2\times3\times4}=\frac{1}{4}$, (by omitting the common terms 2 and 3) or 3 oz.

For $\frac{3}{4}$ of 12 is 9, $\frac{2}{3}$ of 9 is 6, and the $\frac{1}{2}$ of 6 is 3.

Or thus $\frac{1}{2}$ of 12 is 6, $\frac{2}{3}$ of 6 is 4, and $\frac{3}{4}$ of 4 is 3.

Or reckon the Fractions in any other order, the ſolution will be the ſame.

That a compound Fraction, when reduced to a ſingle one by the Rule ſtill retains the ſame value, is eaſily proved by the following example.

3. Reduce $\frac{1}{2}$ of $\frac{1}{2}$ of a yard of cloth to a ſingle Fraction. Facit $\frac{1}{4}$ of a yard.

Now there is nothing more ſelf evident than that the half of half a yard is one quarter, or $\frac{1}{4}$.

4. Reduce $6\frac{1}{2}$d to the Fraction of a pound ſterling.

$6\frac{1}{2}=$d, $\frac{13}{2}$ then $\frac{13}{2}$ of $\frac{1}{12}$ of $\frac{1}{20}$ is $=\frac{13}{480}$ of a pound.

5. Reduce $3\frac{2}{5}$ of $1\frac{1}{2}$ of $\frac{1}{3}$ of 4 to a ſingle Fraction. Facit $\frac{204}{30}=6\frac{4}{5}$

6. Reduce $\frac{3}{8}$ of $\frac{1}{2}$ of $\frac{3}{4}$ to a ſingle Fraction. Facit $\frac{9}{64}$.

7. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of 6l. is $=2\frac{2}{5}=2$l. 8s.

VIII. *To reduce a Complex Fraction to a ſingle or ſimple one.*

1. RULE.

When the fractional part is annext to the Numerator.

1. Multiply the Numerator by the Denominator of the fractional part, and to that product + the Numerator of the fractional part, for a new Numerator.

2 Multiply the Denominator of the fraction, by the Denominator of the fractional part of the Numerator, for a Denominator.

EXAMPLES.

1. Reduce $\frac{3\frac{1}{2}}{6}$ to a ſingle Fraction.

$\frac{3\times2+1}{6\times2}=\frac{7}{12}$ the ſingle Fraction required.

For if we call the Denominator 6 pence, the Numerator muſt be 3 pence half-penny, becauſe both terms of a Fraction are always made up of parts homogeneous, therefore $3\times2+1=7$ half pennys, and $6\times2=12$ half pennys, conſequently $\frac{3\frac{1}{2}}{6}=\frac{7}{12}$ which evidently demonſtrates the truth of the operation.

2. Reduce $\frac{42\frac{3}{4}}{49}$ to a ſimple Fraction. Facit $\frac{171}{196}$

3. Reduce $\frac{34\frac{1}{2}}{46}$ to a ſimple Fraction. Facit $\frac{69}{92}$

4. Reduce $\frac{7\frac{3}{4}}{8}$ to a ſimple Fraction. Facit $\frac{31}{32}$

5. Reduce $\frac{\frac{3}{4}}{6}$ to a ſingle Fraction. Facit $\frac{3}{24} = \frac{1}{8}$

2. RULE

When the Fractional part is annexed to the Denominator.

1. Multiply the Denominator of the Fraction by the Denominator of the fractional part, and to that product add the Numerator of the fractional part for a new Denominator.

2. Multiply the Numerator of the Fraction by the Denominator of the fractional part, for a new Numerator.

EXAMPLES.

1. Reduce $\frac{4}{5\frac{3}{4}}$ to a ſingle Fraction.

Thus $\frac{4 \times 4 = 16}{5 \times 4 + 3 = 23}$ — the ſingle Fraction required.

The truth of this operation may be proved as the firſt Example int he foregoing Rule.

For 4d = 16q. and 5¾d. = 23q. therefore $\frac{4}{5\frac{3}{4}} = \frac{16}{23}$.

2. Reduce $\frac{5\frac{3}{4}}{7\frac{1}{2}}$ to a ſingle Fraction.

Thus $\frac{5 \times 4, + 3, \times 2}{7 \times 2 + 1 \times 4} = \frac{46}{60} = \frac{23}{30}$ the ſingle Fraction required.

Reduce

3. Reduce $\frac{7}{9\frac{1}{2}}$ to a ſingle Fraction. Facit $\frac{14}{19}$

4. Reduce $\frac{50}{60\frac{3}{4}}$ to a ſingle Fraction. Facit $\frac{200}{243}$.

IX. *To reduce Fractions of different Denominators, to thoſe of equal value, having a common Denominator.*

Rule. Multiply each Numerator continually into all the Denominators, except its own, for each new Numerator; then × all the Denominators together for the common Denominator.

Note. When any of the propoſed quantities are whole or mixt numbers, compound, or complex Fractions, reduce them by their proper rules to the form of ſimple Fractions.

EXAMPLES.

1. Reduce $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$ to a common Denominator.

$$\left.\begin{array}{l} 1\times4\times6\times8=192 \\ 3\times6\times8\times2=288 \\ 5\times8\times4\times2=320 \\ 7\times2\times4\times6=334 \end{array}\right\}\text{new Numerator for}\left\{\begin{array}{l} \frac{1}{2} \\ \frac{3}{4} \\ \frac{5}{6} \\ \frac{7}{8} \end{array}\right.$$

$2\times4\times6\times8=383$ the common Denominator: which put under every one of the Numerators laſt found, and you have a new ſet of Fractions,

viz. $\frac{192}{384}$; $\frac{288}{384}$, $\frac{320}{384}$, $\frac{336}{384}$, all of the ſame denomination as appears from the operation itſelf; and all of the ſame value with their reſpective original ones, as will appear by reducing each new Fraction to its loweſt term. Or thus, it evidently appears by the operation, that both terms of each Fraction are ×ed by the ſame numbers, and therefore (by Lemma 2) ſuffers nothing in their value.

4. Reduce $\frac{1}{3\frac{1}{2}}$, $\frac{42\frac{7}{8}}{49}$, $\frac{5\frac{3}{4}}{7\frac{1}{2}}$, $\frac{73}{131\frac{2}{5}}$, to a com. Denom.

Theſe Fractions reduced to ſimple ones, and their loweſt terms are $\frac{2}{7}$, $\frac{7}{8}$, $\frac{23}{30}$, $\frac{5}{9}$; reduced to a common Denominator they become $\frac{4320}{15120}$, $\frac{13230}{15120}$, $\frac{11592}{15120}$, $\frac{8400}{15120}$, the Fractions required.

3. $\frac{4}{7}$, $\frac{5}{8}$, $\frac{6}{9}$, and $\frac{7}{12}$, when reduced, become $\frac{3456}{6048}$, $\frac{3780}{6048}$, $\frac{4032}{6048}$, $\frac{3528}{6048}$.

2. RULE. If the Denominators have a common meaſure ÷ them by it; then × both terms of each given Fraction by the quotes of all the other Denominators, and by this method you will have new Fractions in lower terms than by the 1ſt rule.

EXAMPLES.

1. Reduce $\frac{7}{12}$, $\frac{1}{6}$, and $\frac{8}{15}$, to a common Denominator.

Then

The common meaſure of all theſe Denominators being 3, the quotes are 4, 2, and 5, then $\frac{7}{12}=\frac{7\times2\times5}{12\times2\times5}=\frac{70}{120}$, $\frac{1}{6}=\frac{1\times4\times5}{6\times4\times5}=\frac{20}{20}$ and $\frac{8}{15}=\frac{8\times2\times4}{15\times2\times4}=\frac{64}{120}$.

2. Reduce $\frac{3}{8}$, $\frac{5}{12}$, $\frac{3}{16}$, and $\frac{3}{20}$, to a common Denominator.

3. Reduce $\frac{3}{10}$, $\frac{7}{30}$, $\frac{2}{15}$, $\frac{1}{25}$, and $\frac{3}{20}$, to a commom Denominator.

C O R R O L L A R Y, 1.

Whence in two Fractions, the terms of each muſt be multiplied by the Quote of the other's Denominator.

4. Reduce $\frac{5}{14}$ and $\frac{8}{21}$ to a common Denominator.

Here 7 being the greateſt common ÷ſor, the quotes are 2, 3. Then $\frac{5\times3}{14\times3}=\frac{15}{42}$, and $\frac{8\times2}{21\times2}=\frac{16}{42}$ the Fractions required.

3 Rule. When the greateſt Denominator is diviſible by all the Denominators, × the terms of all the other Fractions by the reſpective quotes.

EXAMPLES.

1. Reduce $\frac{2}{9}$, $\frac{1}{12}$, and $\frac{5}{36}$, to a common Denominator.

Here, the greateſt Denominator being ÷ed by the reſt, the quotes are 4 and 3; then

$\frac{2\times4}{9\times4}=\frac{8}{36}$, $\frac{1\times3}{12\times3}=\frac{3}{36}$, and $\frac{5}{36}$ reſts the ſame; therefore the required Fractions are $\frac{8}{36}$, $\frac{3}{36}$ and $\frac{5}{36}$.

CORROLLARY, 2.

Whence in two Fractions, × the terms of that which hath the leſs Denominator, by the quote ariſing from the diviſion of their Denominators.

2. Reduce $\frac{2}{5}$ and $\frac{4}{15}$ to a common Denominator.

Here the quote is 3 : then $\frac{2}{5}=\frac{2\times3}{5\times3}=\frac{6}{15}$ therefore the required Fractions are $\frac{6}{15}$ and $\frac{4}{15}$

SCHOLIUM.

By this Problem the greater of two or more Fractions may be diſcovered.

X. *Several Fractions being given, to find as many whole numbers in the ſame proportion.*

RULE. Reduce the Fractions to a common Denominator,

nator, and the ſeveral Numerators will be to one another as the given Fractions.

EXAMPLES.

1. Suppoſe the Fractions were $\frac{5}{8}$ and $\frac{3}{4}$.

Theſe reduced to a common Denom. are $\frac{20}{32}$, $\frac{24}{32}$.

Then as 20 : 24 :: $\frac{5}{8}$: $\frac{3}{4}$.

2. The Fractions $\frac{1}{2}$, $\frac{3}{8}$, $\frac{3}{4}$ are as the numbers 32, 24, and 48.

XI. *To reduce Fractions of one denomination to their equivalents of another denomination.*

1. RULE. When the leſs denomination, or integer, *is an aliquot part of the greater*, conſider how many of each leſs denomination are contained in one of the greater; then if the reduction be from a greater denomination to a leſs, × the Numerator, but if from a leſs to the greater, × the Denominator, by all the denominations from the given one to that ſought.

EXAMPLES.

1. Reduce $\frac{3}{8}$l. to the Fraction of a ſhilling.

$\frac{3}{8}$l. $= \frac{3 \times 20}{8} = \frac{60}{8}$s. the Fraction required.

2. Reduce $\frac{60}{8}$s. to the Fraction of a pound.

$\frac{60}{8}$s. $= \frac{60}{8 \times 20} = \frac{60}{160}$l. $= \frac{3}{8}$l. in lower terms.

3. Reduce $\frac{1}{336}$l. to the Fraction of a farthing.

$$\frac{1}{336}\text{l.}=\frac{1\times20\times12\times4}{336}=\frac{960}{336}\text{ q.}$$ the required Fraction.

4. Reduce $\frac{20}{7}$q. to the Fraction of a pound.

$$\frac{20}{7}\text{q.}=\frac{20}{7\times4\times12\times20}=\frac{20}{6720}=\frac{1}{336}\text{l.}$$ the Anſwer.

5. Reduce $\frac{3}{8}$ of a lb. Troy to the Fraction of a grain.

$$\frac{3}{8}\text{ lb.}=\frac{3\times12\times20\times24}{8}=3\times12\times20\times3=2160$$ grains.

6. Reduce $\frac{17280}{8}$ Grains to the Fraction of a lb. Troy.

$$\frac{17280}{8}\text{ grs.}=\frac{17280}{8\times24\times20\times12}\text{lb.}=\frac{17280}{46080}=\frac{3}{8}\text{ lb.}$$

7. Reduce $\frac{3}{8}$lb. Apothecaries weight to the Fraction of a grain.

$$\frac{3}{8}\text{lb}=\frac{3\times12\times8\times3\times20}{8}=\frac{17280}{8}=2160\text{ gr.}$$

8. Reduce $\frac{1}{4}$ of a mile to the Fraction of an inch.

$$\frac{1}{4}\text{m.}=\frac{\times8\times220\times3\times12}{4}\text{ inch.}=\frac{190080}{4}=47520\text{in.}$$

9. Reduce ¾ of a pint of ale to the Fraction of a hhd.

Facit $\frac{3}{1632}$

10. Reduce ¼ of a laſt to the Fraction of a pint.

Facit $\frac{15360}{4}$ pt. = to 3840 pts.

11. Reduce ¾ of an hour to the Fraction of a week.

Facit $\frac{3}{672}$ week.

Note. If a compound whole number be propoſed, reduce it to the loweſt denomination mentioned, and proceed as before.

EXAMPLES.

1. Reduce 6s. 8d. to the Fraction of a pound.

Facit $\frac{80}{240} = \frac{1}{3}$

2. Reduce 5¾d to the Fraction of a ſhilling. Facit $\frac{23}{48}$ s.

3. Reduce 3 oz. 17 dwts. 19 gr. Troy to the Fraction of a lb. Facit $\frac{1867}{5760}$ lb.

2. RULE If the leſs denomination be not an aliquot part of the greater, reduce the Fraction given to ſuch a denomination, whereof a certain number are contained

in the denomination to which the Fraction is to be brought; then reduce this Fraction to the denomination required.

E X A M P L E S.

1. Reduce $\frac{3}{8}$ of a pound to the Fraction of a guinea.

$$\frac{3}{8}\text{l.}=\frac{3\times 20}{8\times 21}=\frac{60}{168}\text{ gui.}=\frac{5}{14}\text{ guinea.}$$

2. Reduce $\frac{3}{4}$ of a Carolus to the Fraction of a Jacobus.

$$\text{Thus }\frac{3\times 25}{4\times 23}\text{ jacobus}=\frac{75}{92}\text{ jacobus.}$$

3. Reduce $\frac{3}{8}$ of a noble to the Fraction of a crown: Facit $\frac{1}{2}$.

4. Reduce $\frac{3}{4}$ of half a crown to the Fraction of a ſhilling.

$$\text{Facit }\frac{15}{8}\text{s.}=1\tfrac{7}{8}\text{s.}$$

5. Reduce $\frac{3}{4}$ of a jacobus to the Fraction of a moidore.

$$\text{Facit }\frac{69}{108}=\frac{23}{36}\text{ moidore.}$$

XII. To find the value of a Fraction, or to reduce it to the known parts of an Integer.

RULE. Multiply the Numerator of the Fraction, by ſuch a number of units of the next leſs denomination, as is requiſite to one of that name which the Fraction is of, dividing the Product by the Denominator, and the quote ſhews the parts ſought.

If there be a remainder, × it by the Units of the next inferior Denomination, and ÷ by the Denominator as before, continuing this Work till it is brought to the loweſt known part of an Integer.

EXAMPLES.

1. What is the value of $\frac{3}{7}$ a pound?

3
20
7)60(8 s.
56
4
12
7)48(6 d.
42
6
4
7)24(3
21
3 rem.
Anſw. 8s. 6d. 3 $\frac{3}{7}$q.

2. What is the velue of $\frac{5}{19}$ of a guinea?

5
21
19)105(5s.
95
10
12
19)120(6 d.
114
6
4
19)24(1 qr.
19
5 rem.
Anſw. 5s. 6d. 1 $\frac{5}{19}$. q.

3. What is the value of $\frac{5}{7}$ of 7l. 11s. 7d. $\frac{3}{4}$?
Anſw. 5l. 8s. 3d. 3 $\frac{2}{7}$ q.

4. What is the value of $\frac{67}{73}$ of a cwt?
Anſw. 3 qr. 18lb. 12 oz. 11 $\frac{29}{73}$ dr.

5. What is the value of $\frac{71}{97}$ of a lb. Troy?
Anſw. 8 oz. 15 dwts. 16 $\frac{4}{97}$ grs.

6. What is the value of $\frac{9}{13}$ of a cwt?
Anſw. 2 qr. 21lb. 8 oz. 9 $\frac{11}{13}$ dr.

7. What is the value of $\frac{3}{4}$ of a lb. Avoirdupoiſe?
Anſw. 12 oz.

8. What is the value of $\frac{7}{9}$ of a ſhilling?
Anſw. 9d. 1 $\frac{1}{3}$ q.

9. What is the proper quantity of $\frac{4}{7}$ of a mile?
Anſw. 4 fur. 125 yds. 2 feet, 1 inch 2$\frac{1}{7}$ b. corns.

10. What is the proper value of $\frac{3}{5}$ of a tun of wine?
Anſw. 2 hhds. 25 gal. 1$\frac{3}{5}$ pt.

11. What is the proper value of $\frac{2}{15}$ of a hhd. of ale?
Anſw. 6 gal. 1$\frac{3}{5}$ qrts.

XIII. *To reduce any given quantity to the Fraction of any greater denomination of the ſame kind.*

Rule. Reduce the quantity to the loweſt denomination mentioned, annexing thereto the Fraction (if any) of that denomination, for a Numerator; then reduce the integral part to the ſame denomination for a Denominator; and laſtly, reduce the Fraction to its loweſt terms.

N. B. Problem XII. and XIII. prove each other.

EXAMPLES.

1. Reduce 8s, 6d. 3$\frac{3}{7}$q. to the Fraction of a pound. Firſt 8s. 6d. 3$\frac{3}{7}$q. = 411$\frac{3}{7}$q. for a Numerator, 1l = 960q. for a Denominator; therefore the required Fraction is $\frac{411\frac{3}{7}}{960}$l. which, reduced to a ſingle Fraction, and then to its loweſt terms, ſtands thus $\frac{411 \times 7 + 3}{960 \times 7} = \frac{2880}{6720} = \frac{3}{7}$l.

2. Reduce 5s. 6d. $1\frac{7}{19}$q. to the Fraction of a guinea.

$$\frac{5s.\ 6d.\ 1\frac{7}{19}q. = 265\frac{7}{19}q.}{1\ \text{guinea} = 1008\ q.} = \frac{6}{19}\ \text{of a guinea.}$$

3. What part of 7l. 11s. 7¾d. is 5l. 8s. 3d. $3\frac{2}{7}$ q?

$$\frac{5l.\ 8s.\ 3d.\ 3\frac{2}{7}q. = 5199\frac{2}{7}q.}{7l.\ 11s.\ 7d.\ 3q. = 7279\ q.} = \frac{36395}{50953} = \frac{5}{7}\ \text{Anſw.}$$

4. What part of a cwt. is 3 qr. 18 lb. 12 oz. 11 $\frac{29}{73}$ drs?

Anſw. $\frac{67}{73}$

5. What part of a pound Troy is 8 oz. 15 dwts. $16\frac{8}{97}$ gr.?

Anſw. $\frac{71}{97}$

6. What part of a cwt. is 2 qr. 21 lb. 8 oz. 9 $\frac{1}{13}$ drs?

Anſw. $\frac{9}{13}$

7. What part of a pound Avoirdupoiſe is 12 oz?

Anſw. $\frac{12}{16}$ = ¾ lb. the Anſw.

8. What part of a ſhilling is 9d. $1\frac{1}{5}$ q? Anſw. $\frac{4}{5}$.

XIV. *To reduce a Fraction to its equivalent that ſhall have any aſſigned Denominator.*

Rule. As the Denominator of the given Fraction is to its Numerator, ſo is the Denominator of the aſſigned Fraction to its Numerator.

EXAM.

EXAMPLES.

1. Reduce $\frac{3}{4}$ to a Fraction of the ſame value, whoſe Denominator ſhall be 12.

Thus as 4 : 3 :: 12 : 9 the new Numerator.

Therefore the new Fraction is $\frac{9}{12}, = \frac{3}{4}$

2. Reduce $\frac{7}{8}$ to a Fraction of the ſame value, whoſe Denominator ſhall be 49. Facit $\frac{42\frac{7}{8}}{49}$

For $7 \times 49 \div 8 = 42\frac{7}{8}$ for the Numerator.

3 Reduce $\frac{3}{4}$ to a Fraction of the ſame value, whoſe Denominator ſhall be 46. Facit $\frac{34\frac{1}{2}}{46}$

4. Reduce $\frac{5}{9}$ to a Fraction whoſe Denominator ſhall be $131\frac{2}{5}$ Thus $131 \times 5 + 2 \div 9 = 73$ the Numerator, hence $\frac{73}{131\frac{2}{5}}$ is the Fraction.

XV. *To reduce a given Fraction to its equivalent, that ſhall have any aſſigned Numerator.*

Rule. As the Numerator of the given Fraction is to its Denominator, ſo is the Numerator of the aſſigned Fraction, to its Denominator.

EXAMPLES.

1. Reduce $\frac{3}{4}$ to a fraction of the ſame value whoſe Numerator ſhall be 9. Facit $\dfrac{9}{12}$

For as 3 : 4 :: 9 : 12 the new Denominator.

2. Reduce $\frac{7}{8}$ to a Fraction of the ſame value whoſe Numerator ſhall be $42\frac{7}{8}$.

Facit $\dfrac{42\frac{7}{8}}{49}$. For $42 \times 8 + 7 \div 7 = 49$ the new Denom.

3. Reduce $\frac{3}{4}$ to a Fraction of the ſame value whoſe Numerator ſhall be $34\frac{2}{4}$.

Facit $\dfrac{34\frac{2}{4}}{46}$.

4. Reduce $\frac{5}{9}$ to a Fraction of the ſame value, whoſe Numerator ſhall be 73.

Facit $\dfrac{73}{131\frac{2}{5}}$

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§ III. *Addition of Vulgar Fractions.*

A GENERAL RULE.

REDUCE mixt numbers to improper Fractions; compound, and complex Fractions, to ſingle ones; and Fractions of different Denominators to a common Denominator. Then add their Numerators into one ſum, under which ſubſcribe the common Denominator, and that Fraction will be the required ſum; then reduce it to its loweſt terms, and, if it be an improper Fraction, to its whole or mixt number.

EXAMPLES.

1. Add $\frac{7}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{1}{8}$ into one ſum.

Here the Denominators being common, add the Numerators together; thus $7+3+5+1=16$ eighths, or $\frac{16}{8}$, $=2$.

2. What is the ſum of $\frac{2}{3}$, $\frac{2}{7}$, and $\frac{4}{9}$? Anſw. $1\frac{25}{63}$.

For when reduced to a common Denominator they are $\frac{126}{189}$, $\frac{54}{189}$, and $\frac{84}{189}$, Then $\frac{126+54+84}{189} = \frac{264}{189}$: the Denominator ſhewing that ſomething is ÷ed into 189 equal parts, and the Numerator denoting 264 ſuch parts, $=1\frac{75}{189} = 1\frac{25}{63}$.

3. Add $1\frac{1}{6}$, $\frac{1}{5}$, $\frac{1\frac{1}{2}}{2}$, and $\frac{2}{3}$ of $\frac{3}{4}$ together.

Firſt $1\frac{1}{6}$, $=\frac{7}{6}$, $\frac{1\frac{1}{2}}{2}$, $=\frac{3}{4}$, and $\frac{2}{3}$ of $\frac{3}{4}=\frac{1}{2}$: then $\frac{7}{6}$, $\frac{1}{5}$, $\frac{3}{4}$, $\frac{1}{2}$, reduced to a common Denominator, and added together, $=2\frac{37}{60}$ the ſum.

4. What is the ſum of $\frac{7}{16}$, $\frac{11}{12}$, and $\frac{4}{9}$? Anſw. $1\frac{115}{144}$

5. What is the ſum of $9\frac{3}{20}$, $\frac{3}{5}$, and $\frac{4}{5}$ of $\frac{1}{3}$?

Anſw. $10\frac{1}{60}$

6. What is the ſum of $\frac{3}{5}$, $\frac{5}{9}$, and $2\frac{1}{6}$? Anſ. $3\frac{87}{270}$.

Note 1. If many Fractions are to be added, the beſt way is, firſt to add two of them together, and to the ſum add a third, and to that ſum a fourth, and ſo on.

EXAMPLES.

1. Add $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$ and $\frac{1}{6}$, together.

Firſt $\frac{1}{2}$ and $\frac{2}{3}$ are reduced to $\frac{3}{6}$ and $\frac{4}{6}$, whoſe ſum is $\frac{7}{6}$.

Then $\frac{7}{6}$ and $\frac{3}{4}$ are reduced to $\frac{28}{24}$ and $\frac{18}{24}$, whoſe ſum is $\frac{46}{24}=\frac{23}{12}$. Then $\frac{23}{12}$ and $\frac{4}{5}$ are reduced to $\frac{115}{60}$ and $\frac{48}{60}$ whoſe ſum is $\frac{163}{60}$. Then $\frac{163}{60}$ and $\frac{1}{6}$ are reduced to $\frac{978}{360}$ and $\frac{60}{360}$; whoſe ſum is $\frac{1038}{360}=2\frac{53}{60}$, the ſum of the five Fractions.

2. What is the ſum of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$?

Anſw. $\frac{183}{60}=3\frac{1}{20}$.

Note 2. When mixt numbers are to be added, reduce their fractional parts to a common Denominator, and add their ſum to the total amount of the whole numbers.

EXAMPLES.

1. Add $2\frac{1}{2}$, $3\frac{2}{5}$, $4\frac{1}{3}$ and $8\frac{1}{4}$ together.

Firſt $\frac{1}{2}$, $\frac{2}{5}$, $\frac{1}{3}$, and $\frac{1}{4}$ are reduced to $\frac{30}{60}$, $\frac{24}{60}$, $\frac{20}{60}$, $\frac{15}{60}$;

Then $2\frac{1}{2} = 2\frac{30}{60}$

$3\frac{2}{5} = 3\frac{24}{60}$

$4\frac{1}{3} = 4\frac{20}{60}$

$8\frac{1}{4} = 8\frac{15}{60}$

$18\frac{29}{60}$ is the ſum required.

Here the ſum of the Fractions is $\frac{89}{60} = 1\frac{29}{60}$, and the ſum of the whole numbers 17;

therefore $17 + 1\frac{29}{60} = 18\frac{29}{60}$.

2. Add $476\frac{1}{3}$, $24\frac{1}{4}$, $74\frac{5}{6}$, $81\frac{7}{8}$, and $152\frac{5}{14}$, together.

Facit $809\frac{109}{168}$.

3. What is the ſum of 4, $3\frac{1}{3}$, 6, $4\frac{1}{2}$, 8, and $10\frac{3}{8}$?

Anſw. $36\frac{5}{24}$.

Note 3. If the Fractions given be of different denominations, reduce them to their proper value by Problem XII in Reduction, and add them as in Addition of different denominations.

EXAM-

EXAMPLES.

1. Add $\frac{1}{3}$ of a pound $\frac{3}{4}$ of a ſhilling and $\frac{3}{4}$d. into one ſum.

	l.	s.	d.	
Thus $\frac{1}{3}$l. =	0	6	8	Reduced ſeverally to their proper values
$\frac{3}{4}$s. =	0	0	9	
$\frac{3}{4}$d. =	0	0	0$\frac{3}{4}$	
The ſum	0	7	5$\frac{3}{4}$	

2. Add $\frac{2}{3}$ of a pound Troy, 1 fifth of an oz. $\frac{2}{3}$ dwt. and 17$\frac{1}{4}$ grains together.

	oz.	dts	grs.
$\frac{2}{3}$ lb. =	8	0	0
$\frac{1}{5}$ oz. =	0	4	0
$\frac{2}{3}$ dwt =	0	0	16
17$\frac{1}{4}$	0	0	17$\frac{1}{4}$
the ſum	8	5	9$\frac{1}{4}$

3. Add $\frac{9}{13}$ cwt. and $\frac{5}{7}$ lb. & 3 fifths of an oz. together?

Facit 2 qr. 22lb. 4 oz. 10 $\frac{138}{455}$ drs.

4. What is the ſum of $\frac{5}{7}$ of 7l. 11s. 7$\frac{3}{4}$d, 5 nineteenths of a guinea, $\frac{3}{7}$ of a pound, and $\frac{3}{4}$ of a ſhilling?

Anſw. 6l. 3s. 2d. 1 $\frac{124}{399}$ qr.

5. What is the ſum $\frac{4}{7}$ of a mile $\frac{3}{5}$ of a furlong, $\frac{2}{3}$ of a yard, $\frac{3}{4}$ of a foot, and $\frac{1}{2}$ of an inch?

Anſw. 5 fur. 38 yds. 1 ft. 11 inch. 0 $\frac{9}{14}$ br. c.

6. Add $\frac{5}{7}$ of a month, $\frac{7}{13}$ of a day, and $\frac{2}{3}$ of an hour, into one ſum.

Facit 2 weeks, 6 days, 13 hrs. 35 m. 23 $\frac{1}{13}$ ſec.

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§ IV. *Subtraction of Vulgar Fractions.*

THE Fraction being prepared here in all reſpects as in Addition, Subtract the Numerators, and under their difference, write the common Denominator.

EXAMPLES.

1. What is the difference between $\frac{3}{4}$ and $\frac{1}{4}$?

$$\frac{3}{4}-\frac{1}{4}=\frac{3-1}{4}=\frac{2}{4}=\frac{1}{2}\text{, the Anſw.}$$

2. What is the difference between $\frac{7}{8}$ and $\frac{3}{8}$? Anſw. $\frac{4}{8}=\frac{1}{2}$.

3. From $\frac{5}{6}$ take $\frac{4}{9}$. Theſe reduced to a common Denominator (by Cor. I. Problem IX. in Reduction) are

$$\frac{15}{18}\text{, and }\frac{8}{18}\text{; then }\frac{15-8}{18}=\frac{7}{18}\text{ the Anſw.}$$

4. What is the difference between $\frac{3}{4}$ of $\frac{2}{5}$ and $\frac{3}{4}$?

$\frac{3}{4}$ of $\frac{2}{5} = \frac{3}{10}$ then $\frac{3}{10}$ and $\frac{3}{4}$, are reduced to $\frac{6}{20}$ and $\frac{15}{20}$;

therefore $\frac{15-6}{20} = \frac{9}{20}$ the Anſw.

5. From $5\frac{3}{8}$ take $\frac{2}{7}$ of $4\frac{1}{6}$.

First $5\frac{3}{8} = \frac{43}{8}$, and $\frac{2}{7}$ of $4\frac{1}{6} = \frac{25}{21}$; then $\frac{43}{8}$ and $\frac{25}{21}$ are reduced to $\frac{903}{168}$ and $\frac{200}{168}$; therefore $\frac{903-200}{168} = \frac{703}{168} =$ $= 4\frac{31}{168}$ the Anſw.

6. What is the difference between $\frac{19}{20}$ and $\frac{1}{7}$ of $\frac{2}{3}$?

Anſw. $\frac{359}{420}$.

7. From $\frac{9}{13}$ take $\frac{2}{3}$ of $\frac{5}{8}$. Facit $\frac{43}{156}$.

8. From $\frac{7}{8}$ of 5 take $\frac{7}{13}$ of $\frac{19}{21}$. Facit $\frac{8491}{2184}$.

Note 1. In ſubtracting one mixt number from another, (or a proper Fraction from a whole number, when the Fraction in the ſubducend is greater than that in the minuend) Subtract the Numerator of the ſubducend from the Denominator, and to the difference add the Numerator of the minuend (if any;) and carry one to the integer (or cypher) in the ſubducend.

EXAMPLES.

1. From $16\frac{1}{4}$ pence.
Take $13\frac{3}{4}$ pence.
Rem. $2\frac{1}{2}$d.

2. From 17
Take $0\ \frac{1}{2}$
Rem. $16\ \frac{1}{2}$

3. From $6\frac{1}{8}$
Take $3\frac{3}{8}$
Rem. $2\frac{6}{8} = 2\frac{3}{4}$

4. From 1
Take $0\frac{3}{4}$
Rem. $0\frac{1}{4}$

Note 2. In mixt numbers, reduce the Fractions to a common Denominator; then take the Fractions from the Fractions, and the whole numbers from the whole numbers.

EXAMPLES.

1. From $16\frac{3}{4} = 16\frac{21}{28}$
Take $9\frac{4}{7} = 9\frac{16}{28}$
Rem. $7\frac{5}{28}$

2. From $86\frac{1}{3} = 86\frac{5}{15}$
Take $57\frac{4}{5} = 57\frac{12}{15}$
Rem. $28\frac{8}{15}$

3. From $34\frac{1}{4}$ take $\frac{2}{5}$ of 34. Facit $20\frac{13}{20}$.

4. From $\frac{1}{3}$ of 64 take $\frac{1}{4}$ of $\frac{1}{2}$ of 42. Facit $9\frac{7}{12}$.

5. From $1\frac{1\frac{1}{3}}{4}$ of $3\frac{2}{5\frac{1}{2}}$ of 6 take $\frac{4}{9}$ of $\frac{3}{4\frac{1}{3}}$.

Facit $26\frac{410}{693}$.

Note 3. In Subtracting Fractions of different denominations, reduce them to their proper value; and ſubtract as before.

EXAMPLES.

	s.	d.	q.
1. From $\frac{3}{8}$ l. =	7	6	0
Take $\frac{4}{13}$ s. =	0	3	$2\frac{10}{13}$
Rem.	7	2	$1\frac{3}{13}$.

	s.	d.	
2. From $\frac{3\frac{1}{3}}{7}$ of a l. =	9	6	$1\frac{5}{35}$
Take $\frac{3}{5}$ of $\frac{2}{3}$ of 1s. =	0	4	$3\frac{7}{35}$
Rem.	9	1	$1\frac{33}{35}$

The Fractions being reduced to a common Denominator.

3. What is the difference between $\frac{3}{5}$ of a guinea, and $\frac{4}{5}$ of a mark? Anſw. 1s. 11⅕d.

4. From ¾ of a lb. Troy, take one ſixth of an oz.
Facit 8 oz. 16 dwts. 16 grs.

5. From ⅘ of a ton take ⅙ of a lb.
Facit 15 c. 3 qr. 27lb. 2 oz. $10\frac{2}{3}$ dwts.

6. From $\frac{1}{6}$ of a yard take $\frac{2}{3}$ of an inch. Facit $5\frac{1}{3}$ in.

7. From $\frac{2}{3}$ of a qr. of corn, take ¾ of a buſhel.
Facit 4 buſhels 2⅓ peck.

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§ V. *Multiplication of Vulgar Fractions.*

R U L E.

REDUCE mixt numbers to Fractions, compound and complex Fractions to ſingle ones, alſo to their loweſt terms; then × all the Numerators together for the Numerator, and all the Denominators together for the Denominator of the required product.

E X A M P L E S.

1. Multiply $\frac{2}{3}$ by ½. Thus $\frac{3\times1}{3\times2}=\frac{2}{6}=\frac{1}{3}$ the product.

The product of any number ×ed by a proper Fraction is always leſs than that number. As is evident by the example above.

S C H O L I U M.

Whence the product of two proper fractions muſt of conſequence, be leſs than either of them.

2. Multiply $\frac{3}{4}$ of a l. by $\frac{8}{9}$ of a l. Facit $\frac{2}{3}$ l.

For $\frac{3}{4}$ l. $\times \frac{8}{9} = \frac{24}{36}$ l. $= \frac{2}{3}$ l. $=$ 13s. 4d. but $\frac{3}{4}$ l. $=$ 15s and $\frac{8}{9}$ l. $=$ 17s. $9\frac{1}{3}$d. therefore their product is leſs than either quantity.

3. Multiply $7\frac{1}{2} = \frac{15}{2}$, by $\frac{2}{3}$. Thus $\frac{15 \times 2}{2 \times 3} = \frac{30}{6} = 5$ the product.

4. Multiply 41 $\frac{4}{5}$ l. by 12 l. Thus $\frac{209}{5}$ l. $\times \frac{51}{4} = \frac{10659}{20}$ l. $= 532\frac{19}{20}$ l. $=$ 532l. 19s. the Anſw.

5. What is the product when $\frac{2}{5}$ of $\frac{3}{4}$ of $\frac{4}{7}$ is multiplied by $\frac{5}{8}$ of $\frac{2}{3}$?

The compounds reduced are $\frac{6}{35}$ and $\frac{5}{12}$, in their loweſt terms.

Then $\frac{6 \times 5 = 30}{35 \times 12 = 420} = \frac{1}{14}$ the Anſw.

Or thus, $\frac{4\times3\times2\times2\times5}{5\times4\times7}\ \frac{}{\times3\times8}=$ (by omitting the common terms 4, 3, & 5) $\frac{2}{2}\times\frac{7}{8}=\frac{4}{56}=\frac{1}{14}$.

This example ſhews, that it is needleſs to reduce compound Fractions before multiplication be performed; becauſe it evidently appears, that multiplication of Fractions, is only reducing a compound Fraction to a ſingle one; for to × $\frac{2}{7}$ by $\frac{1}{4}$, is no more than to take $\frac{1}{4}$ of $\frac{2}{7}$.

CONTRACTIONS.

I. When the Numerator of one and the Denominator of the other, can be ÷ed by any number, take the quotes inſtead of them.

EXAMPLES.

1. Multiply $\frac{5}{12}$ by $\frac{6}{7}$. Divide by 6, and the Fractions become $\frac{5}{2}$ and $\frac{1}{7}$; then $\frac{5\times1}{2\times7}=\frac{5}{14}$ the Product.

2. Multiply $\frac{5}{8}$ by $\frac{4}{15}$. $\frac{5)5}{4)8}\times\frac{4}{15}=\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$ the product.

3 Multiply $\frac{3}{8}$ by $\frac{4}{9}$. $\frac{3)3}{4)8}\times\frac{4}{9}=\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$ the product.

4. Multiply $\frac{3}{8}$ by $\frac{4}{7}$. Facit $\frac{3}{14}$.

II. When a mixt number or Fraction is to be ×ed by a whole number, × the whole number by the whole number, then × the Numerator by the ſaid whole number, and ÷ by the Denominator, and add this qoute to the former product.

E X A M P L E S.

1. Multiply three 5ths by 8. Thus $\frac{3\times 8}{5}=\frac{24}{5}=4\frac{4}{5}$ the product.

2. Multiply $5\frac{2}{3}$ by 14. Thus $14\times 2\div 3=9\frac{1}{3}$, which when added to the product of 5 by 14, thus $14\times 5+9\frac{1}{3}$ $=79\frac{1}{3}$ the product required.

3. What is the product of $4\frac{3}{4}$ by 4? Anſw. 19.

4. What is the product of $3\frac{4}{7}$ by 13? Anſw. $46\frac{3}{7}$.

III. When a Fraction is to be ×ed by a number which happens to be the ſame with the Denominator, take the Numerator for the product.

E X A M P L E S.

1. Multiply *four 7ths* by 7, the product is 4.

2. Multiply 4 by $\frac{3}{4}$, the product is 3.

IV. When ſeveral Fractions are to be ×ed, ſtrike out ſuch ×ers as are found both in the Numerators and Denominators, as before taught in Sect. II. Prob. VI. Rule V.

1. Multiply *four fifths, three eights, two thirds, and five ſixths* together.

Thus $\frac{4\times3\times2\times5}{5\times8\times3\times6}$ = (by omitting the common terms 3 and 5) $\frac{4\times1\times2\times1}{1\times8\times1\times6}$ = (by ÷ing by 4 and by 2) $\frac{1\times1\times1\times1}{1\times2\times1\times3} = \frac{1}{6}$ the product.

2. What is the product of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and 5?
Anſw. $\frac{5}{4}$, or $1\frac{1}{4}$.

Note. A Fraction is beſt multiplied by an integer, by dividing the Denominator by it if poſſible; but if not, multiply the Numerator by it.

EXAMPLES.

1. Multiply $\frac{3}{14}$ by 7. Thus $\frac{3}{14\div7} = \frac{3}{2} = 1\frac{1}{2}$.

2. Multiply $\frac{8}{27}$ by 9. Facit $2\frac{2}{3}$.

3. Multiply $\frac{4}{7}$ by 5. Thus $\frac{4\times5}{7} = \frac{20}{7} = 2\frac{6}{7}$.

4. Multiply *two 3ds* of 5 by 6. Facit 20.

5. What is the product of $\frac{1}{2}$, $\frac{2}{3}$, and 3? Anſw. 1.

6. What is the product of 3l. 19s. 11¾d. by 3l. 19s. 11¾d.?

Firſt as 3l. 19s. 11¾d. = 3839 farthings for a Numerator, and 1l. = 960 farthings for a Denominator, the Fraction becomes $\frac{3839}{960}$.

Then $\frac{3839 \times 3839}{960 \times 960} = \frac{14737921}{921600}$ l. = 15l. 19s. 10 $\frac{1}{3840}$ d. the Anſwer required.

7. What is the product of 6l. 17s. 4¼d. by 2l. 15s. 7¾d.?

Anſw. 19l. 2s. 1d. 3 $\frac{623}{960}$ qrs.

8. What is the product of 4 lb. 9 oz. 4 dwts. 12 grs. by 2 lb. 5 oz. 10 dwts. 20 grs? Anſw. 11 lb. 8 oz. 17 dwts 12⅞ grs.

9. What is the product of 3 yds. 2 feet, 6 inches, 2 bar. corns; by 3 yds. 2 feet, 6 inches, 2 bar. corns?

Anſw. 14 yds. 2 feet, 6 $\frac{10}{81}$ inches.

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§ VI. *Diviſion of Vulgar Fractions.*

FRACTIONS being prepared in all reſpects as for Multiplication, Diviſion is thus performed.

RULE.

When the Numerator and Denominator of the Dividend, can be exactly ÷ed by the Numerator and Denominator of the ÷for, take the quotients for the terms of the required Fraction; but if that cannot be done, multiply both terms of the Dividend by the Diviſor's reverſe Fraction, for the terms of the required quotient.

EXAMPLES.

1. Let $\frac{9}{20}$ be ÷ed by $\frac{3}{4}$. Thus $\frac{9 \div 3}{20 \div 4} = \frac{3}{5}$ the quotient. The truth of this evidently appears, and may be proved by Multiplication, thus $\frac{3 \times 3}{5 \times 4} = \frac{9}{20}$.

2. Divide $\frac{6}{7}$ by $\frac{3}{5}$. The ÷ſor's reſerve Fraction is $\frac{5}{3}$; then thus $\frac{6\times5}{7\times3}=\frac{30}{21}=1\frac{3}{7}$ the quote.

CONTRACTIONS.

I. If a Fraction is to be ÷ed by a whole number, ÷ the Numerator by it, if poſſible; but if not, × the Denominator by it.

II. When the two Numerators, or the two Denominators can be ÷ed by the ſame number, then ÷ them, and take the quotes inſtead thereof.

3. Divide $\frac{9}{11}$ by 3. Thus $\frac{9\div3}{11}=\frac{3}{11}$ quote.

4. Divide $\frac{9}{11}$ by 4. Thus $\frac{9}{11\times4}=\frac{9}{44}$ quote.

5. Divide $\frac{16}{17}$ by $\frac{12}{19}$. First $\frac{16\div4}{17}=\frac{4}{17}$, and $\frac{12\div4}{19}=\frac{3}{19}$. Then $\frac{4}{17}\div\frac{3}{19}=\frac{76}{51}=1\frac{25}{51}$ the quote.

6. Divide $\frac{8}{9}$ by $\frac{2}{45}$. First $\frac{8\div2}{9\div9}=\frac{4}{1}$, and $\frac{2\div2}{45\div9}=\frac{1}{5}$; then $\frac{4}{1}\div\frac{1}{5}=\frac{4\times5}{1\times1}=20$ the quote.

7. Divide $\frac{3}{4}$ l. by $\frac{4}{5}$ s. Firſt $\frac{4}{5\times20}=\frac{4}{100}$ l. $=\frac{1}{25}$ l.

Then $\frac{3}{4}$ l. $\div\frac{1}{25}$ l $=\frac{3}{4\times1}\frac{\times25}{}=\frac{75}{4}=18\frac{3}{4}$ l.

8. What is the quote of $\frac{2}{5}$ by $\frac{2}{3}$? Anſw. $1\frac{2}{5}$.

9. What is the quote of $3\frac{2}{3}$ by $5\frac{1}{4}$? Anſw. $\frac{[illegible]}{63}$

10. What is the quote of $8\frac{1}{3}$ of $2\frac{1}{2}$ by $\frac{3}{4}$ of $\frac{2}{5}$? Anſ $69\frac{4}{9}$

11. What is the quote of 15l. 19s. $10\frac{1}{3840}$ d. by 3l 19s $11\frac{3}{4}$d. Anſw. 3l. 19s. $11\frac{3}{4}$d.

12. What is the quote of 19l. 2s. 1d. $3\frac{623}{960}$q. by 2l. 15s. $7\frac{3}{4}$d? Anſw. 6l. 17s. $4\frac{1}{4}$d.

13. What is the quote of 19l. 2s. 1d. $3\frac{623}{960}$q. by 6 l. 17s. $4\frac{1}{4}$d? Anſw. 2l. 15s $7\frac{3}{4}$d.

14. What is the quote of 11lb. 8 oz. 17 dwts. $12\frac{7}{8}$ grs. by 2lb 5 oz. 10 dwts. 20 grs?
Anſw. 4lb 9 oz. 4 dwts. 12 grs.

15. What is the quote of 11lb. 8 oz. 17 dwts. $12\frac{7}{8}$ grs by 4lb. 9 oz. 4 dwts. 12 grs?
Anſw. 2lb 5 oz. 10 dwts. 20 grs.

16. What is the quote of 14 yds. 2 feet, $6\frac{10}{21}$ inches by 3 yards, 2 feet, 6 inches, 2 bar. corns?
Anſw. 3 yds. 2 ft. 6 in. 2 bar. corns.

§ VII. *Of Decimal Fractions.*

NOTATION.

1. A *Decimal Fraction* is the Numerator of a Vulgar Fraction whoſe Denominator is an Unit with one or more Cyphers; as $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$ &c are Decimal Fractions.

2. In Decimals the Denominator is never expreſſed, but always underſtood to be an Unit with as many Cyphers as there are places in the Numerator, or given Decimal and thereby the Denominator, tho' not expreſſed, may be known.

3. Decimals are ſeparated, and diſtinguiſhed from Integers by a point or comma prefixed, and ſo wrote in one line like whole numbers.

Thus .3 is put for $\frac{3}{10}$, .03 for $\frac{3}{100}$, .003 for $\frac{3}{1000}$, &c.

Obſerve the ſame in mixt numbers, as 36.7 is put for $36\frac{7}{10}$, 3.67 for $3\frac{67}{100}$, 761.012 for $761\frac{12}{1000}$, 81.01261 for $81\frac{1261}{100000}$ &c.

4. The value of a Decimal is diminiſhed by Cyphers on the left, in the ſame tenfold proportion as the value of an Integer is increaſed by having them on the right; Thus $.5=\frac{5}{10}$, $.05=\frac{5}{100}$, $.005=\frac{5}{1000}$, $.0005=\frac{5}{10000}$, &c.

But Cyphers on the right hand of a Decimal alter not its value, for .5 and .50 and .500 &c. are all equal in value, becauſe $\frac{5}{10}, = \frac{50}{100}, = \frac{500}{1000}$ &c. $= \frac{1}{2}$, as is plain from Vulgar Fractions: and therefore Decimals are reduced to a common Denominator by annexing Cyphers. It may here be obſerved, that Cyphers on the right hand of a Decimal number (if nothing follow them) are as inſignificant as Cyphers on the left hand of a whole number; and yet Cyphers are ſometimes placed after Decimals, for the ſake of regularity, or when we want to increaſe the number of decimal places.

5. In Decimals, the Integer or 1, is always ſuppoſed to be ÷ed into 10, 100, 1000, &c. equal parts; that is, 1 is ſuppoſed to be ÷ed into 10 equal parts, and each of theſe parts into 10 equal parts, and each of thoſe parts into 10 parts more, and ſo on, by a continual Subdiviſion. For inſtance, ſuppoſe a foot-rule (or any other meaſure) to be ÷ed into 10 equal parts; then each part is $\frac{1}{10}$; and if every one of thoſe parts be ÷ed into 10 equal parts, the foot (or other meaſure) will be ÷ed into 100 equal parts; thus every part of the firſt diviſion is $\frac{1}{10}$ or $\frac{10}{100}$; and every part of the ſecond diviſion, in reſpect of the whole, will be $\frac{1}{100}$: after the ſame manner we may conceive a foot, a yard, an acre, an hour, a buſhel, a pound, a ſhilling, &c. to be ÷ed into 10, 100, 1000, 10000, &c. equal parts at pleaſure.

6. If, as above, the Integer be ÷ed into 10, 100, &c. equal parts, the figures in the Decimal ſhews how many of ſuch parts are taken as .1 denotes 1 tenth part, .3 three tenth parts, .5 five tenths, or half, .25 denotes 25 parts of 100, or $\frac{1}{4}$; and .75 denotes 75 parts of 100, or $\frac{3}{4}$; alſo .19896 denotes 19896 parts of 100000.

Now becauſe 10 × 10 = 100, and 100 × 10 = 1000, it follows that the 10's are ten times greater than the 100's, the

100's, ten times greater than the 1000's, &c. if the figures were all equal; and in the Decimal .19896, if ever ſo many figures follow the firſt viz. .1, it is greater in value than them all; therefore unity or 1, is greater than .999999 &c. alſo $\frac{1}{10}$ or .1 is greater than .099999 &c. tho' infinitely continued.

7. The following Table will farther illuſtrate the Notation of Decimal Fractions.

The NOTATION TABLE.

Whole Numbers						Decimal parts.							
4	6	7	3	4	5	.6	7	8	9	3	2	4	&c.
100 thouſands	10 thouſands	thouſands	hundreds	tens	units	tenth parts	hundredth parts	thouſandth parts	10 thouſandth parts	100 thouſandth parts	millionth parts	10 millionth parts	

In whole numbers, the 1ſt place contains units, the 2d place to the left, tens; the 3d hundreds, &c. as before ſhewn; but in Decimals, the order of places is retrograde; for the 1ſt place in Decimals is tenths, the 2d place to the right is hundreth parts; the 3d, thouſandth parts, &c. And as all figures on the left hand of the place of units, increaſe in their value, according to their diſtance from it, in a decuple proportion; ſo all figures on the right hand of the place of units, decreaſe in their value in a ſubdecuple proportion: as for inſtance, the number 345.6789 (where 5 ſtands in the place of units) is to be read thus; three hundred and forty five, ſix tenths, ſeven hundredth parts, eight thouſandth parts, nine ten thouſandth parts: or the Decimal parts may be read thus, ſix thouſand ſeven hundred eighty nine ten thouſand

ſand parts, by reducing them all to the Denominator of the greateſt, which is 10000, becauſe the laſt figure, 9 ſtands in the place of ten thouſandth parts. The reaſon of this latter way of reading is plain; for $\frac{6}{10}$ are $\frac{6000}{10000}$, and $\frac{7}{100}$ are $\frac{700}{10000}$, and $\frac{8}{1000}$, are $\frac{80}{10000}$, which being collected together with the remaining $\frac{9}{10000}$ make $\frac{6789}{10000}$. Therefore the numeration, or reading of Decimals, is the very ſame as that of whole numbers, with only adding the name of the parts ſignified by the Decimal.

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§ VIII. *Addition and Subtraction of Decimals.*

RULE.

PLACE all the ſeparating points directly under each other, then tenths will ſtand under tenths, and hundredth parts under hundredths, &c. then add or ſubtract as in the whole numbers: and laſtly, put a point under the other points, which will prick off the number of Decimal places in the ſum, or difference.

EXAMPLES in ADDITION.

	First	Second.	Third	Fourth
	,0200	,873	27,671	2,17
	,0603	,760	17,704	73,76
	,1700	,0731	1,612	,67
	,0120	,0129	0,160	1,61
	,0071	,9302	171,061	,76
Sum	.2694	2.6492	218.208	78.97

5. What is the ſum of 736, 47.6737, 6.7361 360.721, .0126, and 24? Anſw. 1175.1434.

6. What is the ſum of 3.67, 761.2, 22.731, .72, 1.3612, .000276, and .062? Anſw. 789.744476

7. What is the ſum of .76, 7.6, 76, .076, .0076, .00076, and .760? Anſw. 844.44436.

EXAMPLES in SUBTRACTION.

	Firſt.	Second.	Third.
From	.87	.1	1.
Take	.378	.09999	0.9999
Rem.	.492	.00001	.0001
Proof	.870	.10000	1.0000

4. What is the difference between 76.4 and 7.64? Anſw. 68.76

5. What is the difference of 1, and .1? Anſw. .9

6. What is the difference of 26.39 and 26.78? Anſw. .39

7. What number added to 739.712, will make it 801.31? Anſw. 61.598

8. What number +ed to .0679, will make it 20? Anſw. 19.9321

---

§. IX. *Multiplication of Decimals.*

RULE.

MULTIPLICATION in Decimals is alſo performed as in whole numbers, no regard being had to the Decimals as ſuch, till the Product is obtained; but then, ſo many Decimal places muſt be cut off from the right hand of the product, as there are in both factors. If there are not ſo many figures in the product as there ought to be Decimals, ſupply the defect with cyphers on the left.

E X A M P L E S.

	First	Second	Third
Multiply	.3024	32.12	.0006
by	2.23	2.43	1.23
	9072	9636	18
	6048	17848	12
	6048	6424	6
Prod.	.674352	78.0516	.000738

4. What is the product of 741.983 by 3.48 ?
Anſw. 2582.10084

5. Suppoſe there is a ſquare garden, the length of whoſe ſide is 23.53 yards, how many ſquare yards doth it contain ? Anſw. 553.6609

6. In an oblong pavement, whoſe length is 78.451 feet, and breadth 27.8, how many ſquare feet ?
Anſw. 2180.9378

7. A cellar being dug whoſe length is 53.48, breadth 32.5 and depth 10.23 feet, how many ſolid feet of earth did it contain ?

length breadth depth
Anſw. 53.48 × 32.5 × 10.23 = 17780.763 ſolid feet.

8. What is the product of .03217 by .0325 ?
Anſw. .001045525

C O N T R A C T I O N S.

1. When a Decimal, or mixt number, is to be ×ed by an Unit with Cyphers annext, as 10, 100, 1000, &c.

you are only to remove the ſeparating point ſo many places towards the right-hand in the multiplicand as there are cyphers annexed to the unit, ſubjoining cyphers if need be.

E X A M P L E S.

1. The product of 7.374 by 10 is 73.74.

2. The product of 46.2 by 100 is 4620.

3. The product of .8460 by 10000 is 8460.

4. The product of .09642 by 1000 is 96.42.

II. *It often happens in buſineſs, that the factors conſiſt of many Decimal places; and to × them at large wou'd be much trouble to little purpoſe, becauſe a fewer number of places may do the buſineſs as well, if obtained true, which may be done as follows.*

1. Set the unit's place of the leſs factor, or ×er, under ſuch a number of the greater or multiplicand, as you would have kept in the product, and which ſtands immediately before the figures which are to be left out, either in whole numbers or Decimals.

2. Then ſet all the other figures of the ×er in a contrary, or inverſe order, as in the example below.

3. Begin to × each figure in the multiplicand, by that figure in the ×er which ſtands directly under it, rejecting all the figures in the multiplicand which are on the right hand of your ×ing figure.

4. Set the products down ſo, that their right hand figures may fall ſtrait below each other; and carry to ſuch right-hand figures (for the increaſe that would ariſe from the foregoing figures of the multiplicand) 1 from 5 to 15, 2 from 15 to 25, 3 from 25 to 35, &c. and the ſum of theſe ſimple products will exhibit ſo much of the product as was deſigned to be ſecur'd.

E X A M P L E S.

1. Let 58,25435 be ×ed by 21.234, and have in the product (as ſufficient) but two Decimal places.

Now, if they are placed, and the operation wrought, as above directed, they'll ſtand thus,

58,25435
432.12
116509
5825
1165
175
23
1236.97

Or at large thus,

58.25435	
21.234	
23	301740
174	76305
1165	0870
5825	435
116508	70
1236.97	286790

2. Let be required to × 1998612286 Integers by 5342945, and to have five places true in the product.—Alſo .23456 by itſelf and to have ſix Decimals true in the product.

2d.	3d.
1998612286	.234560
5492435	65432.0
99931	46912
5996	7037
799	938
40	117
18	14
106784	.055018

Note 1. When whole numbers are multiplied by this contracted method, the whole multiplier muſt be inverted, as in Example 2.

2. You need not make uſe of any more figures in your ×er than what ſtands even with your multiplicand to the left, as in Example 2,

When

3. When it happens (as it does in Example 3d) that the number of figures produced by this contracted method of ×ing, are not ſo many as are aſſigned for the Decimal places in the product, the defect muſt be ſupplied by prefixing Cyphers, as was obſerved in the general Rule.

This contracted method is of excellent uſe in aſtronomical computations.

4. Multiply 3.141592 by 52.7438, retaining four Decimals in the product. Facit 165.6995.

5. Multiply .248264 by .725234, retaining five Decimals in the product. Facit .18004

6. Multiply 257.356 by 76.48, retaining no Decimals in the product. Facit 19682

7. Multiply 76.84375 by 8.21054, retaining five Decimals in the product. Facit 630.92868

8. Multiply .3570643 by .0210576, retaining nine Decimals in the product. Facit .007518917

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§ X. *Diviſion of Decimals.*

THE manner of operation is exactly the ſame in Diviſion of decimals, as in whole numbers. The only difficulty (if any) is, to determine how many decimal places muſt be pricked off towards the right hand of the quote, which may be done by either of the following general Rules.

1. GENERAL RULE.

Conſider the decimals as whole numbers, and ÷ them accordingly; then cut off from the right hand of the quote, as many decimal places as the dividend hath more than the ÷for: but when the quotient figures are too few, the want muſt be ſupplied with cyphers on the left; for there muſt always be juſt as many decimal figures in the ÷for and quotient together, as there are in the dividend alone; the reaſon whereof is manifeſt from IX Section.

For ſince the ÷for and quote ×ed together are to make the dividend, the ÷for and quote ought to have as many decimal places between them, as there are in the dividend.

Ex. 1. Let 69.9678 be ÷ed by 56.7.

Thus, 56,7)69,9678(1.234 the quote.

Here, by ÷ing as in whole numbers the quote is 1234: but then conſidering that there were four Decimal places in the dividend, and but one in the ÷for, you muſt cut off three places from the right hand of the quote, and ſo make the true quote 1.234

For 1.234×56.7=69,9678 the proof.

Or the value of the quotient figures may be determined by the following Rule.

2. GENERAL RULE.

The firſt figure of the quotient, whether Integers or Decimals, is of the ſame degree, or value, as that figure of the dividend under which the units place of its product ſtands.

Ex. 2. Let .2274202 be÷ed by [illegible]326

```
5.326),2274202(.0427
        21304
        ─────
        .14380
        10652
        ─────
         .37282
          37282
         ─────
         .....
```

Here since 4 (the units place of the product of the ÷for by the quotient figure 4) ſtands under the 2d place of Decimals in the dividend, therefore 4 muſt occupy the ſecond place of Decimals in the quote.

From the above rules are deduced three caſes, by which all operations in diviſion of Decimals may be perfectly underſtood.

CASE 1.

If the number of decimal places in the ÷for and dividend are equal, all the quote will be whole numbers. But note, if in this, or either of the following caſes, there be a *remainder* after all the dividend figures are brought down, you may continue the quote to what number of decimals you pleaſe, by annexing a cypher continually to the laſt remainder, and ÷ing as before.

Ex. 3. Let 227,4202 be ÷ed by .5326 and the quote is 427 whole numbers.

Ex. 4. Divide 172,812 by 1.212.

```
1.212)172,812(142.584158 quote.
      1212
      ----
      5161
      4848
      ----
       3132
       2424
       ----
        7080
        6060
        ----
        10200
         9696
         ----
         5040
         4848
         ----
          1920
          1212
          ----
           7080
           6060
           ----
           10200
            9696
            ----
             504
```

Here, after all the dividend figures are brought down there is a remainder of 708, to which annex a cypher and ÷ as before, and you get one figure more, for a decimal, in the quote: and by annexing a cypher to each ſucceeding remainder, the quote is continued to ſix decimal places: thus by annexing cyphers you may continue the quotient at pleaſure.

Ex. 5. 412,992 ÷ .864=478.

Ex. 6. .0003264 ÷ .0000136=24.

Ex. 7. 69,9678 ÷ .0567=1234.

CASE II.

If the number of decimal places in the dividend exceed thoſe in the ÷ſor, cut off their difference for decimals in the quote.

EXAMPLES.

69,9678 ÷ 56,7=1.234

.699678 ÷ 5.67=,1234

69,9678 ÷ .567=123.4

.00003264 ÷ ,136=.00024

CASE III.

When there are not ſo many decimal places in the dividend as there are in the ÷ſor, annex cyphers to the dividend to make them equal, then will the quote be all whole numbers, as in Caſe I.

EXAMPLES.

1. Divide 6727.5 by .345.
.345)6727.500(195 quote.

2 Divide 128.2 by 5.128
5.128)128.200(25 quote.

3. 11404.8÷3.456=3300 quote.

4. 13475÷.245=55000

5. .75÷.0125=60 quote.

6. 1425÷.6252=2279 quote.

All the variety that can happen in diviſion of decimals are contained in the following Examples, viz.

1. 234)809.64 (3.46

2. 2.34)809.64 (346

3. 2.34)80964.00 (34600

4. .0234)8.0964 (346

5. .0234).80964 (34.6

6. 2.34)8.0964 (3.46

7. 23.4).80964 (.0346

8. 234).80964 (.00346

9. .234)80964.000 (346000

CONTRACTIONS

I. When a decimal or mixt number is to be ÷ed by an integer with cyphers on the right, cut off the cyphers, and remove the ſeperating point in the dividend ſo many places farther to the left as there were cyphers cut off from the right of the ÷ſor: then ÷ as before: prefixing cyphers if need be.

EXAMPLES.

1. Divide 1728.12 by 1200.

Thus 12)17.2812(1.4401 quote.

2. Divide 12.2416 by 40000.

$4).00122416(.00030604$ quote.

3d. $35.8845 \div 470000 = .00007635$ quote.

II. To ÷ by an unit with cyphers, as 10, 100, 1000, &c. it is but removing the ſeperating point in the dividend ſo many places to the left, as there are cyphers annext to the unit.

EXAMPLES.

$2873{,}67 \div 100 = 28.7367$

$2873{,}67 \div 10000 = .287367$

$5.6746 \div 10000 = .00056746$

III. In large diviſions the work may be contracted as follows.

1. Find, by the 2d general Rule, what place of decimals or integers the firſt figure of the quote will poſſeſs.

2. Take as many of the left hand figures of the ÷for, as you judge neceſſary for your firſt ÷for, and juſt ſo many of the left hand figures of the dividend, as will yield you the firſt figure of the quote.

3. Having found the firſt figure of the quote; the following figures may be found thus: take each laſt remainder for a new dividend, and for a new ÷for prick off one figure from the right of each preceeding one; continue the operation till the ÷for is exhauſted.

Note, In multiplying the quotient figure and diviſor, leave out thoſe figures prickt off, only particular regard muſt be had to the increaſe that would ariſe from the laſt figure ſo prickt off, and for ſuch increaſe carry as in the ſecond Contraction of Multiplication.

EXAMPLES.

1. Divide 630,92878056 by 76,8437523

```
76,84375|23)630,92878|056(8.210541
•••••••     61475000
            --------
             1617878
             1536875
             -------
               81003
               76843
               -----
                4160
                3842
                ----
                 318
                 307
                 ---
                  11
                   7
                  --
                   4
```

This contraction is of excellent uſe in large diviſions; eſpecially in aſtronomical computations.

Explanation.

Here 8 is ×ed into 76 84375; then 2 is multiplied into 76,84372 (carrying 1;) then 1 is multiplied into 76,843; the multiplication of 7684 by 0, is omitted; then 768 is multiplied by 5; then 76 by 4, laſtly 7 by 1.

2. Divide 1236.9728679 by 21,234 ſo that the quote may contain three decimals. Facit 58,254

3. Divide 14169.206623951 by 384.672158 ſo that the quote may contain four decimals Anſw. 36,8345

4. Divide 87,076326 by 9,365407 ſo as to have ſix places of decimals true in the quote. Facit 9,297655

---

§ XI. *Reduction of Decimals.*

1. TO reduce a vulgar fraction, to its equivalent decimal one.

RULE.

Add as many cyphers to the numerator as you would have decimal places; then ÷ that ſum by the denominator,

tor, and the quote (if nothing remains) will be a decimal equivalent to the given vulgar fraction. But when there is a remainder, you may by adding more cyphers, proceed ſo as to bring out the quote nearly equal.

EXAMPLES.

1. $\frac{1}{4}=1.00\div4=.25$ decimal, for $4:1::1.00:.25$
2. $\frac{1}{2}=1.0\div2=.5$, for $2:1::1.0:.5$
3. $\frac{3}{4}=3.00\div4=.75$, for $4:3::1.00:.75$

By Problem XIV. Section II.

4. Reduce $\frac{2}{7}$ to a decimal.

Thus $2.000000\div7=.285714=\frac{2}{7}$ nearly, not wanting $\frac{1}{1000000}$ part of an unit.

5. $\frac{5}{8}=.625$ decimal.

6. $\frac{23}{64}=.359375$ decimal.

7. $\frac{3}{32}=.09375$ decimal.

Note 1. If the diviſion ends at a certain number of places (as in the Examples above, except the 4th.) the decimal is called a finite; but that which no where ends, is called an infinite decimal: As in the following examples.

8. Reduce *one third* to a decimal.

Thus 1.0000 &c. $\div3=.3333$ &c.

Note 2. When a figure continually repeats (as .3333 &c.) the decimal is called a ſingle circulate, or recurring decimal. In all circulating numbers daſh the firſt and laſt recurring figure.

9. Change $12\frac{17}{286}$ to decimal. Firſt

$286)17.0000000(.059440559440$ &c.

Then $12\frac{17}{286}=12.059440$ nearly.

Note 3. When a certain number of figures continually repeat in the quote (as in the example above) the decimal is called a compound circulate; for by annexing another cypher to the remainder 170, the number is the ſame as when the diviſion firſt began: Conſequently the ſame figures viz. 59440 will repeat in the quote, ad infinitum.

10. Reduce $\frac{6}{7}$ to a decimal. Facit .857142857142.

11. $\frac{24}{13}$ = 1.8461538 &c.

12. $7\frac{7}{9}$ = 7,77777 &c.

13. $\frac{83}{9768}$ = .0084971334 &c.

14. $\frac{2}{17\frac{3}{4}}$ = .11267605 &c.

II. To reduce decimals or integers to equivalent decimals of a greater denomination.

C A S E 1.

If the decimals or integers propoſed be ſimple, divide continually by all the denominations from the given one to that ſought: As in reductions of integers.

E X A M P L E S.

1. Reduce .75 d. to the decimal of a l.

Thus, .75 ÷ 12 = .0625 s. ÷ 20 = .003125 l.

2. Reduce 1 grain to the decimal of an oz. Troy.

1.0000 ÷ 24 = .0416 dwts. ÷ 20 = .002083 oz.

3. Reduce 11 d. to the decimal of a l.

11.000 ÷ 12 = .9166 &c. ÷ 20 = ,04583 l.

4. Reduce 3.216 pints of wine to the decimal of a hhd.

Facit 3.216 pts. = 0,42 gal. = ,00638 &c. hhd.

5. Reduce 36 minutes to the decimal of a day.

36m. =,6 hr. =.025 day.

6. Reduce 55 yards to the decimal of a mile.

55 yds. =,25 fur. = .03125 mile.

7. Reduce .056 perches to to the decimal of an acre.
Facit .00035.

CASE 2.

If the given part conſiſts of ſeveral denominations, reduce them to that of the loweſt for a numerator, and the integer to the ſame name for a denominator, then annex cyphers and divide as before.

Or thus, begin with the leaſt and place down the ſeveral given denominations, for dividends, orderly under each other on the right hand of a perpendicular line, and on the left of it write againſt each dividend ſuch a number, for a ÷for, as will reduce it to the next greater name; then begin with the upper one, annex the quotient of each diviſion, as decimal parts, on the right hand of the dividend next below it; and let this mixt number be ÷ed by its ÷for, Thus proced thro'. all the denominations, and the laſt quote will be the decimal ſought.

EXAMPLES.

1. Reduce 17s. 6$\frac{3}{4}$ d. to the decimal of a l.

First $\dfrac{17s.\ 9\frac{3}{4}d.}{1l.} = \dfrac{843\ q.}{960\ q.}$ then 843,000000÷960 =.878125 l. the required decimal.

Or thus,

4	3.00 q.
12	6.75 d.
20	17.5625 s.
	0.878125 l. the ſame as above.

2. Reduce 11 oz. 16 dwts. 18 gr. to the decimal of a lb. Troy.

24 { 6 | 18
24 { 4 | (3
20 | 16.75
12 | 11,8375
| .9864583 &c. lb.
| the decimal reqd.

3. Reduce 3 qr. 16 lb. 12 oz. Avoird. to the decimal of a cwt.

16 { 4 | 12
16 { 4 | (3
28 { 4 | 16.75
28 { 7 | 4.1875
4 | 3.59821428571 qr.
| .899553571 4 cwt.

Reduce 2 ſt. 9 in. 2 b. c. to the decimal of a yard.

3 | 2.00 b. c.
12 | 9.6666666 &c. in.
3 | 2.8055555 &c. feet.
| .935185185 &c. yds.

5 Reduce 59 yds. 2 ſt. 9.5 in. to the decimal of a mile.

12 | 9.5 in.
3 | 2.791666 &c. ſt.
220 | 59.930555 &c.
8 | 0.272411616 fur.
| .034051452 m.

6. Reduce 48 m. 37 ſeconds 54 thirds to decimals ; 1 degree being integer.

60 | 54,0 thirds.
60 | 37.9 ſeconds.
60 | 48.6316666 m.
| .810527777 deg.

8. Let 16 ac. 3 ro. 16 pls. be all expreſs'd in acres.

4.0 | 1.6
4 | 3.4
| 16,85 acres.

Hence the following decimal TABLE is made.

MONEY. 1 l. the integer. 1 s. = ,05 1 d. = ,0041666 1 q. = ,0010416	**AVOIRDUPOISE WEIGHT.** 1 cwt. the integer. 1 qr. = ,25 1 lb. = ,00892857 1 oz. = ,00055803
TROY WEIGHT. 1 lb. the integer. 1 oz. = ,0833333 1 dwt. = ,0041666 1 gr. = ,0001736	**LONG MEASURE.** A yard the integer 1 ft. = ,3333333 1 in. = ,0277777 1 b.c. = ,0092592
APOTHECARIES WEIGHT. 1 oz. the integer. 1 dr. = ,125 1 ſcr. = ,0416666 1 gr. = ,0020833	**Square and Solid MEASURE.** 1 in. = ,006945 the decimal of a ſquare foot. 1 in. = ,0005787 the decimal of a cubic foot.
AVOIRDUPOISE WEIGHT. 1 lb. the integer. 1 oz. = ,0625 1 dr. = ,00390625	**TIME.** 1 day the integer. 1 hr. = ,0416666 1 min. = ,0006944 1 ſec. = ,0000115

III. *To reduce a decimal of a ſuperior denomination to its value in the inferior ones.*

R U L E.

Multiply the given decimal by the number of parts contained in the next inferior denomination, cutting off the decimals from the product; (as taught in Multiplication) then × the decimals ſo cut off by the next lower denomination, and you'll have the parts of that denomination; thus proceed to the loweſt name required, or till the decimals pointed off are all cyphers: then the number on the left of the points will expreſs the value of the decimal.

E X A M P L E S.

1. What is the value of ,878125 l?

,878125 l.

20

17,562500 s.

12

6,750000 d.

4

3,000000 q.

Anſw. 17s. 6¼d.

2. What is the value of ,9864583 lb. Troy?

,9864583 lb.

12

11,8374996 oz.

20

16,7499920 dwts.

24

29999680

14999840

17,9998080 grs.

Anſw. 11 oz. 16 dwts 18 grs. nearly.

3. What is the value of ,8995535 cwt.?

4

3,5982140 qrs.

28

16,7499920 lb.

16

11,9998720 oz.

Anſw. 3 qr. 16 lb. 12 oz. nearly.

4. What is the value of ,935185 yd?

,935185 yd.

3

2,805555 ft.

12

9,666660 in.

3

1,999980 b. c.

5. What is the value of ,034051452 mile? Anſw. 59 yds. 2 ft. 9,49 in.

6. What is the value of 16,85 acres? Anſw. 16 acres 3 roods, 16 perches.

7. What is the value of ,6875 yard of cloth?
Anſw. 2 qr. 3 nls.

8. What is the value of ,071428 of a hhd. of wine?
Anſw. 4 gal. 1 qt. 1,999712 pt.

9. What is the value of ,3673 of a year?
Anſw. 134 days 1 hr. 32 m. 52 ſeconds, 48 thirds.

10. What is the value of ,548 degree of a circle?
Anſw. 32 min. 52 ſeconds 48 thirds.

XII. *The Rule of Three Direct, in Vulgar and Decimal Fractions.*

The RULE for *Vulgar Fractions.*

HAVING prepared the numbers by the rules in Reduction, ſo that the firſt and third terms be of one name, multiply continually together the 2d and 3d terms and the reciprocal of the 1ſt, for the anſwer.

The Rule of Three in decimals, is ſo much the ſame with that in whole numbers, that nothing need be ſaid of it, except to illuſtrate it by a few examples.

My chief reaſon for treating the Rule of Three in Vulgar and Decimal Fractions together, is, becauſe I have often found, by experience, that a learner will gain a clearer idea of both rules when wrought together, than when treated ſeparately.

Note. For the ſake of brevity, after the vulgar fractions propoſed in the queſtion, I have placed their equivalent decimals; eſpecially in thoſe that I have ſolved by decimals.

EXAMPLES.

1. If $\frac{3}{5}$ = ,6 of a yard of cloth coſt $\frac{3}{8}$l. = ,375l. what will $\frac{3}{4}$ = ,75 of a yard coſt? Anſw. 9s. 4½d.

Stating. If $\frac{3}{5}$ yd. : $\frac{3}{8}$ l. :: $\frac{3}{4}$ yd:

$$\text{Operation } \frac{5\times3\times3}{3\times8\times4}=\frac{15}{32}\text{ l.}=9\text{s. }4\tfrac{1}{2}\text{d.}$$

By Decimals.

If ,6 yd. : ,375 l. :: ,75 yd. : ,46875 l. = 9s. 4½d. Anſw.

2. A father dying left his ſon a certain portion, of which he ſpent $\frac{1}{4}$th = ,25 part, and $\frac{1}{2}$ = ,5 of what remained, and had then 600l. left: how much was his portion at firſt?

Let $\frac{4}{4}$ = 1, repreſent his portion at firſt; from which take $\frac{1}{4}$, and there remains $\frac{3}{4}$ = ,75 part; $\frac{1}{2}$ of which is $\frac{3}{8}$ = ,375 part, which ſtill remains: then ſay, If $\frac{3}{8}$ part : 600l. :: 1 whole : 600×8÷3=1600 l. Anſw.

By Decimals.

If ,375 part : 600 l. :: 1 whole : 1600 l. his portion at firſt.

3. After paying away *one fourth*, = ,25 and *one fifth*, = ,2 of my money, I found 66 guineas left in my bag: what was in it at firſt? Anſw. 120 guineas.

For $\frac{1}{4}+\frac{1}{5}=\frac{9}{20}$=,45 part paid out: then let $\frac{20}{20}$ = 1 be the quantity at firſt, from which take $\frac{9}{20}$ or ,45 and there remains $\frac{11}{20}$ or ,55 part in = 66 guineas: therefore ſay, as 11 : 66 :: 20 : 120 guineas: the denominators being rejected becauſe common.

By Decimals.

As ,55 : 66 :: 1 : 120 guineas, the Anſwer as above.

4. Out of a caſk of wine, which had leaked away *one third*, 21 gallons were drawn; and then being gauged, it appeared to be half full: how much did it hold?

Anſw. 126 gallons.

For $\frac{1}{3}+\frac{1}{2}=\frac{5}{6}$ part leaked [illegible] and left in, which taken from 1, leaves $\frac{1}{6}$ part drawn o[illegible] = 21 gallons.

Then as $\frac{1}{6}$: 21 :: $\frac{6}{6}$; or, 1 : 21 :: 6 : 126 gallons Anſw.

5. A ſum of money is to be ſhared between two perſons, A and B; ſo that as often as A takes 9l. B is to take 4l. Now it happened, that A received 15l. more than B: their reſpective ſhares are required.

Firſt 4 + 9 = 13; then it is evident A had $\frac{9}{13}$ part and B $\frac{4}{13}$ part, their difference being $\frac{5}{13}$ part, = 15 l.

Then as 5 : 15 l. :: 9, or as 1 : 3 :: 9 : 27 l. A's ſhare,

And as 5 : 15 l. :: 4, or as 1 : 3 :: 4 : 12 l. B's ſhare.

6. It is found by experiment, that heavy bodies near the ſurface of the earth, in their fall deſcend $16\frac{1}{12}$ (equal $\frac{193}{12}$) Engliſh feet in the firſt ſecond of time; how many feet will they deſcend in 4 ſeconds?

Note 1. The ſpaces deſcribed, and the diſtances run thro' by falling weights, will always be as the ſquares of their times ſpent in falling.

2. To ſquare a number, is to multiply it by itſelf. The ſquare of 1 is 1; and the ſquare of 4 is 16.

Then as 1 : $\frac{193}{12}$ :: 16 : 257$\frac{1}{3}$ feet the Anſw.

7. If a body falls 20 feet in 2 ſeconds of time, how far will it fall in 10,5 ſeconds? Anſw. 551 feet.

8. If one bricklayer, A, can build a wall in 40 days, B, in 30, C, in 20, D, in 10, and E, in 5; in how many

days can they finiſh the ſame, if they all work together?

S O L U T I O N.

It is evident that $\frac{1}{40}$ part of the work is to be done by A; $\frac{1}{30}$ by B; $\frac{1}{20}$ by C; [illegible] by D; and [illegible] by E. Now theſe fractions reduced [illegible] common denominator (by the 3d Rule in Prob. IX. [illegible] eduction,) and added together make $\frac{49}{120}$. Then ſay, as 49 : 1 :: 120 : $2\frac{22}{49}$ days the Anſwer. That is, as the ſum of the numerators, is to unity; ſo is the common denominator to the mean time.

9. Some boys A, B, C, D, E, F, G, H, and I, robbed an orchard; A had for his ſhare $\frac{1}{3}$, B $\frac{1}{12}$, C $\frac{1}{8}$, D $\frac{1}{20}$, E [illegible], and F $\frac{1}{4}$ part of the whole; G had 310, H 425, and I 140 apples; how many apples had they in all?

S O L U T I O N.

The above fractions being added together the ſum is $\frac{143}{168}$ (in the loweſt terms) part, taken by A, B, C, D, E, and F (all together;) and 310+425+140=875 apples, the reſidue, taken by G, H, and I, together; now let $\frac{168}{168}$ = 1, repreſent the whole taken by them all together; from which take $\frac{143}{168}$, and there remains $\frac{25}{168}$ part, =875 apples: therefore, by proportion, ſay,
as 25 : 875 ap. :: 168 : 5880 apples the Anſwer.

10. A perſon dying left his wife with child, and making his will, ordered, that if ſhe went with a ſon, *two thirds* of his eſtate ſhould belong to him, and the remainder to his mother; and if ſhe went with a daughter, he appointed the mother *two thirds* and the daughter *one third*; but it ſo happened that ſhe was delivered both of a ſon and a daughter; by which ſhe loſt in equity 2000l. more than if it had only been a girl: what would have been her dowry had ſhe only had a ſon?

S O L U T I O N.

Here if the ſon have $\frac{2}{3}$, and his mother $\frac{1}{3}$, the ſon has twice as much as his mother, and by the ſame way of

reaſoning, the mother will have twi[illegible] as much as the daughter, therefore take three numbers to repreſent each perſons part, that are in proportion to one another, as 2 to 1, ſuch are the numbers 4, 2, 1. Then 4 + 2 + 1 = 7; and as 7 : 4 :: 1 : $\frac{4}{7}$ the ſo[illegible] part,
7 : 2 :: 1 $\frac{2}{7}$ the mo[illegible] part,
7 : 1 :: 1 $\frac{1}{7}$ the daugh[illegible] part.

according to the preſent conditions of the queſtion : but ~~had ſhe been delivered of a girl only, the mother's~~ part would have been $\frac{2}{3}$, which is now but $\frac{2}{7}$ of the eſtate; then $\frac{2}{7}$ taken from $\frac{2}{3}$ leaves $\frac{8}{21}$ their difference.

Then as, $\frac{8}{21}$: 2000 :: 1 : 5250l. the whole eſtate, $\frac{1}{3}$ of which is 1750l. the mother's ſhare if ſhe had only had a ſon.

11. A perſon was poſſeſſed of $\frac{3}{4}$ ſhare of a ſilk mill, and ſold $\frac{3}{4}$ of his intereſt therein for 1710 l. what was the value of the whole mill ? Anſw. 3280l.

12. If $\frac{3}{8}$ of $\frac{4}{5}$ of $\frac{1}{2}$ of a ſhip be worth $\frac{4}{7}$ of $\frac{5}{6}$ of $\frac{11}{45}$ of the cargo, valued at 12000l. : what did both the ſhip and cargo ſtand the owners in ?

Anſw. 15223l. 8s. 10d. 1—q.
9[illegible]

13. A younger brother received 2200l. which was *five twelfths* of his elder brother's fortune, and 3½ times the elder's money was as much as the father was worth : now when the two ſons had received their fortunes, the father gave the reſidue of his money to his ſix daughters, according to their minority as follows, *namely*, as often as A had 6½l. he gave B 5l. C 4l. D 3l. E 2l. and F 1½l. how much was each daughter's fortune ?

Anſw. A's [illegible]665l. B's 2050l. C's 1640l. D's 1230l. E's 820l. and F's 615 l.

14. B. and C. together can build a boat in 18 days; with the aſſiſtance of A, they can do it in 11 days: in what time would A do it [illegible] himſelf? Anſw. 27 $\frac{2}{7}$ days.

15. Sound is ſaid [illegible] move 1142 Engliſh feet in one ſecond of time, how [illegible] will it move in 1,5 minute?
Anſw. 1027[illegible] feet.

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§ XIII. *The Inverſe Rule of Three in Vulgar Fractions.*

R U L E.

MULTIPLY continually together the 1ſt. and 2d. terms and the reciprocal of the 3d. for the anſwer.

EXAMPLES.

1. If the penny white loaf weighs 9 oz. Troy, when wheat is at $4\frac{2}{3}$ s. $=\frac{14}{3}$ s. a buſhel; what ought it to weigh, when wheat is at $6\frac{3}{4}$ s. $=\frac{27}{4}$ s. a buſhel?

$$\text{If } \frac{14}{3}\text{ s.} : \frac{9}{1}\text{ oz.} :: \frac{27}{4}\text{ s.} : \frac{4\times14\times9}{27\times1\times3}=\frac{4\times14}{3\times3}=6\frac{2}{9}\text{ oz.}$$
the Anſw.

2. It is found by experiment that a pendulum muſt be 39,2 inches long to ſwing ſeconds; that is, to make 60 vibrations in a minute; how long then muſt a pendulum be, to make 120 vibrations in a minute?

Note. The length of pendalums are as the ſquares of their vibrations reciprocally. And contrarywiſe.

Therefore 60 × 60 = 3600, and 120 × 120 = 14400:
Then as 3600 : 39.2 :: 14400 : 9,8 inches, the Anſw.

§ XIV. *The Double Rule of Three in Vulgar Fractions.*

R U L E.

HAVING placed down and marked the terms, (according to the rule in page 71) take the continual product of the multiplying terms, and the reciprocals of the dividing terms for the answer.

E X A M P L E S.

1. If the carriage of $3\frac{1}{3}$ cwt. $=\frac{10}{3}$ cwt. $150\frac{1}{4}=\frac{601}{4}$ miles costs $2\frac{1}{10}$ l. $=\frac{21}{10}$ l. what will the carriage of $7\frac{2}{5}$ cwt. $=\frac{37}{5}$ cwt. for $60\frac{3}{5}=\frac{303}{5}$ miles cost at the same rate?

$$\div\frac{10}{3}\text{c.}\qquad\frac{21}{10}\text{l.}\qquad\frac{37}{5}\text{c.}\times$$

$$\div\frac{601}{4}\text{m.}\qquad\qquad\frac{303}{5}\text{m.}\times$$

$$\frac{21\times37\times303\times4\times3}{10\times5\times5\times601\times10}=1\text{l. }17\text{s. }7\tfrac{1}{4}\text{d.}+$$

2. Suppose the weight of a beam, whose length is $40\frac{1}{2}$ feet, breadth $3\frac{1}{5}$ feet, thickness $2\frac{2}{3}$ feet, be 467 ſtones, what will the weight of another beam be whoſe length is 60 feet, breadth 5 feet, and thickneſs $3\frac{1}{2}$ feet?

Anſw. $1418\frac{241}{288}$ ſtones.

3. If 36½ lb. of bread be ſufficient for 3 ſoldiers 3½ days, how many days will 216lb. ſuffice 12 ſoldiers ?

Anſw. $5\frac{13}{73}$ days.

4. If I pay 16s. 4d. for the carriage of 5¼ cwt. 20 miles, what muſt be paid for the carriage of 17¾ cwt. 7½ miles ? Anſw. 1l. 8½d.

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§ XV. *Single Fellowſhip.*

SINGLE FELLOWSHIP (or Partnerſhip) is that which determines how much gain or loſs is due to every Partner concerned; by having the whole gain or loſs, and their particular ſtocks, given.

1. A GENERAL RULE.

Say by the Rule of Three, as the whole ſtock : is to the whole gain or loſs : : ſo is every man's particular ſtock : to his particular part of the gain or loſs.

EXAMPLES.

1. Two Partners A, and B, make a ſtock of 120 l. ; A puts in 40l. ; and B 80l. ; they gain 50l. by Trade. What is the gain of each ?

$$\begin{array}{ll} \text{ſtock} & \text{gain} \\ \text{As } 120\text{ l.} : & 50\text{l.} :: \end{array} \left\{ \begin{array}{l} 40\text{ l.} : 16\text{ l. } 13\text{s. } 4\text{d.} = \text{A's} \\ 80\text{l.} : 33\text{ l. } 6\text{s. } 8\text{d.} = \text{B's} \end{array} \right\} \text{gain.}$$

50l. 0s. 0 d. proof.

The proof is made, by adding together all their ſhares, which muſt be equal to the whole gain or loſs. Or thus, as the whole gain or loſs : is to the whole ſtock : : ſo is each man's ſhare of the gain or loſs, : to his ſhare in the Stock.

2. Three men A, B, C, freighted a ſhip with Wine; A had 284 tuns; B 140, and C 64. By a ſtorm at ſea they were obliged to caſt 100 tuns over board. What loſs doth each ſuſtain?

Firſt 284+140+64=488 the whole ſtock.

ſtock	loſs				
As 488	: 100 :	284	: 58 96/488	=A's	loſs.
		140	: 28 336/488	=B's	
		64	: 13 56/488	=C's	
			100 proof.		

3. Four men bought a barrel of ale for 20s. of which A was to pay one third part, B one fourth, C one fifth, and D one ſixth; what muſt each pay?

Firſt, one third, one fourth, one fifth, and one ſixth of 20s. when added together make juſt 19s. =228d. Then ſay,

As 228d. : 20 : :	80 d.	: 7s. 0d. 0 48/57q.	A's	ſhare.
contracted	60 d	: 5s. 3d. 0 36/57q.	B's	
Or 57 : 5 : :	48 d.	: 4s. 2d. 2 6/57q.	C's	
	40 d.	: 3s. 6d. 0 24/57q.	D's	
		20s. proof.		

2. R U L E.

Where many Partners are concerned; find the ſhare of 1 integer, by dividing the whole gain or loſs by the whole ſtock, and the quotient will be a common multiplier; by that multiply every man's part of the ſtock, and it will give his ſhare of the gain or loſs.

4. Eight Men trade together, and compoſe a ſtock of 2400 l. A put in 96l. 10s. =96,5l B 108,25l. C 162,3 l. D 196.3 l. E 260.4 l F 300 75 l. G 346.25l. H 929.25 l; and they gained 288 l. What is the ſhare of each?

2400

2400)288.00(.12 common ×er, of the gain of 1l.

l.		l.	
96.5	×.12=	11.580	A's ſhare
108.25		12.990	B's
162.3		19.476	C's
196.3		23.556	D's
260.4		31.248	E's
300.75		36.090	F's
346.25		41.550	G's
929.25		111.510	H's

288.00l proof.

5. Eight captains plundered the enemy of 1800l. the firſt had 10 men, the 2d 15, the 3d 20, the 4th 28, the 5th 60, the 6th 76, the 7th 112, the 8th 129. What muſt each captain have in proportion to his number of ſoldiers?

The number of ſoldiers is 450; then 1800÷450=4 a common ×er, or the ſhare of one man. Now each captain's men ×ed by 4 will give his ſhare of the money; ſo the 1ſt captain's ſhare is 40l. the 2d 60 l. the 3d 80l. the 4th 112l. the 5th 240l. the 6th 304l. the 7th 448l. the 8th 516l.

6. Three men compoſed a ſtock thus, A put in 40l. B and C together 170l. they gained 126l. of which B took 42l. for his ſhare. B and C's ſtock, and A and C's gain are required? Anſw. A's gain is 24l. and C's 60l. B's ſtock is 70l. and C's 100l.

7. Two men entered into partnerſhip thus, A put in 170l. and took 4 ſevenths of the gain; what did B put in? Anſw. 127l. 10s.

8. A, B, C, and D, joined their ſtocks in trade and gained a certain ſum of money, of which A, B, and C took 60l. B, C, and D 90l. C, D, and A 80l. D, A and B 70l. what is each perſon's diſtinct gain? Anſw. A's 10l. B's 20l. C's 30l. and D's 40l.

§ XVII.

§ XVI. *Double Fellowſhip.*

THE *Double Rule of Fellowſhip* is that which determines how much gain or loſs is due to every partner concerned; by having the whole gain or loſs, and the particular ſtocks, and their time of continuance given.

1. RULE.

Multiply every man's ſtock by the time it is employed, then by the Rule of Three, ſay, as the ſum of theſe products : is to the whole gain or loſs :: ſo is each of theſe products : to each man's gain or loſs.

EXAMPLES.

1. Three merchants A, B, C, enter into partnerſhip, A put in 60l. for 4 months; B, 50l for 3 months; C 30l. for 2 months. They gain 30l. What is each man's ſhare of the gain?

$$\left.\begin{array}{r}60\times4\\50\times3\\30\times2\end{array}\right\}=\overset{\text{products}}{\left\{\begin{array}{c}240\\150\\60\end{array}\right\}}=\left\{\begin{array}{c}\text{A's}\\\text{B's}\\\text{C's}\end{array}\right\}\text{ſtock ×ed into his time.}$$

$$\underline{}$$

450 the ſum of the products.

$$\begin{array}{l}\text{As }450:30::\\\text{Or }\;\;15:\;\;1::\end{array}\left\}\begin{array}{r}\\240:\\150:\\60:\end{array}\begin{array}{r}\text{l.}\\16\\10\\\underline{4}\end{array}\begin{array}{l}\\=\text{A's}\\=\text{B's}\\=\text{C's}\end{array}\right\}\text{gain.}$$

contracted by ÷ing by 30 — 30 l. proof.

2. Four men A, B, C, D, hold a paſture in common, for which they pay 120l. A had 48 oxen 32 days; B 24 oxen 48 days; C 32 oxen for 24 days; and D had 20 oxen for 30 days. What muſt each pay?

oxen		days		products	
48	×	32	=	1536	the product of each man's oxen ×ed by the days they were feeding.
24	×	48		1152	
32	×	24		768	
20	×	30		600	
				4056	ſum of the products.

l.		l.	s.	d.	q.	
As 4056 : 120 ::	1536 :	45	8	10	$2\frac{6}{169}$	A's ſhare.
Or 169 : 5 ::	1152 :	34	1	7	$3\frac{89}{169}$	B's
contracted by dividing by 24.	768 :	22	14	5	$1\frac{3}{169}$	C's
	600 :	17	15	0	$1\frac{71}{169}$	D's
		120 proof.				

2. RULE.

When many partners are concerned, divide the whole gain or loſs by the firſt term or ſum of the products, and the quotient will be a common multiplier; by which multiplying the ſeveral products, you'll have the ſeveral ſhares.

3. Six merchants trade after this manner.

A puts in 50l. for 6 months, and then puts in 60l. more for 4 months longer.

B puts in 90l. for 8 months.

C puts in 160l. for 5 months, and then takes out 60l. for 4 months more.

D puts in 200l. for 7 months, and then puts in 100l. more for 5 months.

E puts in 300l. for 10 months, and 100l. more for 2 months.

R

F puts in 400l. for 3 months, and 200l. for 4 months more, and then takes out 400l. for 5 months more.

They gained 686,4l. what is the gain of each merchant?

The ſeveral products of the ſtock and time will be as follows.

$$\left.\begin{array}{l}50\times6=300\\60\times4=240\end{array}\right\}\text{add}$$

540 for A.

$90\times8=720$ for B.

$$\left.\begin{array}{l}160\times5=800\\100\times4=400\end{array}\right\}\text{add}$$

1200 for C.

$$\left.\begin{array}{l}200\times7=1400\\100\times5=500\end{array}\right\}\text{add}$$

1900 for D.

$$\left.\begin{array}{l}300\times10=3000\\100\times2=200\end{array}\right\}\text{add}$$

3200 for E.

$$\left.\begin{array}{l}400\times3=1200\\200\times4=800\\200\times5=1000\end{array}\right\}\text{add}$$

3000 for F.

For A. B. C. D. E. F.
$540+720+1200+1900+3200+3000=10560$ the ſum of the products.

10560)686,400(,065 the ſhare of 1l. being a common multiplier.

$$\begin{array}{lc}
 & \text{products}\\
\text{Therefore} & \left.\begin{array}{r}540\\720\\1200\\1900\\3200\\3000\end{array}\right\}\times{,}065=
\end{array}
\begin{array}{c}
\text{l.}\\
\left\{\begin{array}{r}35{,}1\\46{,}8\\78{,}0\\123{,}5\\208{,}0\\195{,}0\end{array}\right\}
\end{array}
=\left\{\begin{array}{l}\text{A's share of}\\\text{B's the gain}\\\text{C's}\\\text{D's}\\\text{E's}\\\text{F's}\end{array}\right.$$

686,4l. proof.

4. Two merchants traded together: A put into ſtock 400l. for 12 months, B put into ſtock a certain ſum which continued in trade 8 months, and they gained 300l. of which B had 160l. what was B's ſtock?

Firſt, 400×12=4800 the product of A's ſtock and time, 300l.—160l =140l.=A's gain: Then

As 140 : 4800 :: 160 : 5485,714=B's ſtock and time, then 8)5485,714(685,71425l =B's ſtock.

5. Three perſons A, B, and C, entered into partnerſhip, their common ſtock being 3822l. A's money was in 3 months, B's 5 months, and C's 7 months; they gained 234l. which was ſo divided, as the half of A's gain was equal to *one third* of B's gain, and equal to ¼ of C's gain: what did each gain and put in?

Suppoſe A's gain was 4l. then B muſt have 6l. and C 8, according to the conditions of the queſtion; then 4+6+8=18, and by the Rule of Three, ſay,

As 18 : 234 :: } 4 : 52 A's gain.
Or 1 : 13 :: } 6 : 78 B's gain.
} 8 : 104 C's gain.

Having the whole ſtock and each man's gain (or loſs) and time given, each man's ſtock may be obtained by the following general rule.

*RULE.

Multiply each man's gain (or loſs) by all the others time, except his own, then ſay, as the ſum of theſe products: is to the whole ſtock :: ſo is each of theſe particular products: to each man's particular ſtock. Now in the preſent queſtion: 52×5×7=1820; likewiſe 78×3×7=1638; and 104×3×5=1560; then 1820+1638+1560=5018, the ſum of the products. Now ſay,

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* I have never as yet met with a true rule, in any Author, for finding each man's ſtock, by having the whole ſtock each man's gain (or loſs) and time given: notwithſtanding it is ſo

	products	l.	s.	d.	
As 5018 : 3822 ::	1820 :	1386	4	4 $\frac{572}{2509}$	A's ſtock.
contracted.	1638 :	1247	11	11 $\frac{43}{2509}$	B's ſtock.
Or 2509 : 1911 ::	1560 :	1188	3	8 $\frac{1924}{2509}$	C's ſtock.
		3822 l.	proof.		

The above queſtion is the ſame with the 6th in HILL's *Arithmetic*, page 284, but there calculated by a falſe rule, by which he finds A's ſtock=468l. B's=1170l. and C's=2184l.

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infallible a method of proving the double rule of Fellowſhip; and for that reaſon, I have here inſerted my method of inveſtigating the rule, which will be eaſily underſtood by any one who hath a ſmall ſmattering of Algebra: and altho' this ſtep is a little out of the way, yet I hope the candid reader will pardon it.

Put $a=52$l. $b=78$l. $c=104$l. $r=3$, $n=5$ $m=7$ months, $s=3822$l. ſtock, and let x be put for A's ſtock: now, as in queſtion 4th, $rx=$ the product of A's ſtock and time; then as $a : rx :: b : \frac{brx}{a}$, the product of B's ſtock and time, which ÷ed by B's time viz. n, gives $\frac{brx}{na}=$ B's ſtock: Again as $a : rx :: c : \frac{rxc}{a}$, the product, of C's ſtock and time, which ÷ed by m, gives $\frac{rxc}{ma}$, = C's ſtock: then $x+\frac{brx}{na}+\frac{rxc}{ma}=s$; which equation:

6. A, B, and C, company: A puts into ſtock 1386l. 4s. $4\frac{572}{2509}$d. for 3 months; B puts in 1247l. 11s $11\frac{13}{2509}$d. for 5 months; C puts in 1188l. 3s. $8\frac{1924}{2509}$d. for 7 months; and they gained 234l. Required each perſon's ſhare thereof? Anſw. A's ſhare is 52l. B's 78l. and C's 104l:

7. A, B, and C, company: A put in his ſhare of the ſtock for 5 months, and took one fifth of the gain; B, put in his for 8 months; C, advanced 400l. for 7 months, and required on the balance two thirds of the gain: what is the ſtock of A, and B? Anſw. A's 168l. and B's 70l.

8. Four merchants trade after this manner. A puts in 100l. for 8 months. B puts in 80l. for 5 months, and then puts in 40l. more for 3 months longer. C puts in 176l. for 4 months, and then takes out 50l. for 4 months more. D puts in 230l. for 6 months, and then takes out the whole. They gained 212l. 10s. what is the gain of each merchant?

	l.	s.	d.
Anſw. A's ſhare is	40	19	8
B's	38	18	8¼
C's	61	17	8½
D's	70	13	11¼.

§ XVII.

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tion reduced gives $x = \frac{sanm}{anm+brm+crn}$, which gives the following analogy,

As $anm+brm+crn : s :: anm$: to A's ſtock: and in like manner, if x be put to repreſent B and C's ſtock ſeverally, by proceeding as above, we ſhall have $x = \frac{sbrm}{anm+brm+crn}$ = B's ſtock, and $x = \frac{scrn}{anm+brm+crn}$ = C's ſtock. That is,

As, $anm+brm+crn : s :: \begin{cases} brm & \text{: to B's ſtock,} \\ crn & \text{: to C's ſtock.} \end{cases}$

Which is the ſame as the rule above given, in words at length.

§ XVII. BARTER.

BARTERING is the exchanging of commodities, ſo as neither party is ſuppoſed to ſuſtain any loſs.

RULE.

1. Find the value of that commodity whoſe quantity is given, then find what quantity of the other commodity, at its propoſed rate, will amount to the ſame money, and ſo much of the latter commodity, muſt be given for the former.

2. When one commodity is advanced above the ready money price, find the advanced price of the other in the ſame proportion, thus: by the Rule of Three, ſay, as the ready money price of the one : is to its advanced price :: ſo is that of the other : to its advanced price.

3. When the commodities to be exchanged are of unequal value, the defect is ſupplied by money, or ſome commodity equivalent in value.

EXAMPLES.

1. How much coffee at 5s. per lb. muſt be given for 367lb. of tea at 8s. per lb? Anſw 587⅕lb.

Firſt as 1lb. : 8s. :: 367lb. : 2936 s. the value of the tea. Now there is no more to do but to find how much coffee at 5s. per lb. may be bought for 2936 s. Thus

As 5s. : 1lb. :: 2936s. : 587⅕lb. the Anſw.

2. How much ſugar at 8d. per lb. may be bought for 12 cwt. of tobacco at 4l. per cwt? Anſw. 1440lb.

Firſt 12×4 = 48 l. the value of the tobacco, = 2880 groats: then if 2 : 1lb. :: 2880 : 1440lb. the Anſw.

3. A and B barter thus: A hath 100 yards of cloth at 5s. 8d. per yard ready money, but in barter he will

have

8s. per yard. B hath hops at 1l. 14s. per cwt. ready money: it is required to find how many cwt. of hops B muſt give A for his cloth, to make his gain in the barter equal to A's.

Firſt 6s. 8d.=20 groats, and 34s.=102 groats,

Then as 20. : 8s. : : 102 : 40,8,s. the advanced price of one cwt. of B's hops. Now proceed as in the two examples above.

Thus 100 × 8=800 s. the advanceed value of the cloth.

Then as 40,8 s. : 1 cwt : : 800 s. : 19 cwt. 3$\frac{3}{51}$qr. the Anſw. required.

4. Two merchants A and B barter with each other thus, A has 460 yards of broad cloth, worth 9 s. 2d. per yard, but in barter he will have 11 s. per yard. B has tobacco worth 2s. per lb. which he charges at 2s. 6d. How much tobacco muſt A receive for his cloth; and what does he gain or loſe by the bargain?

In this queſtion, firſt find what the cloth comes to at the advanced price; then how much tobacco at its advanced price, may be bought for that money, and laſtly the true value of both,

1 yd. : 11s. : : 460 y. : 5060s. the advanced value of the cloth: then as 2½s. : 1lb. : : 5060s. : 2024lb. of tobacco received. Next find the true value of both at their prime coſt. Thus,

1 y. : 9⅙s. : : 460 y. : 4216⅔ s. the value of the cloth.
and 1lb. : 2s. : : 2024lb : 4048 s. the value of the tobacco.

Difference = 168⅔ s. which A loſt by the bargain.

5. A and B barter: A has 140 l. 12 oz. of plate, at 6s. 4d. per oz. which in truck he rates at 7s. 2d. an ounce and allows a diſcount on his part, to have *one ſeventh* of that in ready money. B has tea worth 9s. 6d. the pound, which he rates at 11s. 2d. when they ſtruck the ba-

lance, A received but 7 c. 2 qr. 18 lb. of tea: what diſcount did A allow B; which of them had the advantage, and how much, in an article of trade thus circumſtanced?

Firſt 140 lb. 11 oz.=1691 oz. 7s. 2d.=86 d.

then 1 oz. : 86d. :: 1691 oz. : 145426 d. = 605 l. 18s. 10 d. the value of the plate at the advanced price; *one ſeventh* of which is 86 l. 11 s. 3$\frac{1}{7}$ d. the ready money to be paid, which taken from 605 . 18 s. 10 d. leaves 519 l. 7 s. 6$\frac{6}{7}$ d.=124650$\frac{6}{7}$ d. to be laid out in tea at 11 s. 2d. =134 d. per lb.

Then as 134 d. : 1 lb. :: 124650$\frac{6}{7}$ d. : 8 c. 1 qr. 6$\frac{108}{469}$ lb.

from which take the quantity received, viz. 7 c. 2 qr. 18 lb. and there remains 2 qr. 6$\frac{108}{469}$ lb. which at 11 s. 2d. per lb. amounts to 40 l. 6 s. 6$\frac{6}{7}$ d. the diſcount.

	l.	s.	d.
B's gain at 20 d. per lb. upon the tea he delivered, viz. 7 c. 2 qr. 18 lb. amount to	71	10	0
A's gain at 10 d. per oz. upon his plate delivered, viz. 140 lb. 11 oz. amounts to	70	9	2
The difference of their gain in barter is	1	0	10
But A allows B diſcount to the amount of	40	6	6$\frac{6}{7}$
Therefore B has the advantage by	41	7	4$\frac{6}{7}$

6. How much tea at 9s. per lb. can I have in barter for 4$\frac{1}{2}$ cwt. of chocolate, at 4s. per lb? Anſw. 2 cwt.

7. How many knives at 5s. 2d. per doz. muſt be delivered in barter for 3 cwt. 2 qr. 16 lb. of ſteel, at 37 s. 4d. per cwt.

Anſw. 26$\frac{10}{31}$ doz.

8. A and B barter thus: A has 53$\frac{1}{2}$ quarters of corn, at 1 l. 10 s. per quarter; for which B would give 13$\frac{1}{7}$ cwt. of

ſugar at 4l. 12s. per cwt. and the balance in raiſins, at 6$\frac{1}{2}$d. per lb. how many pounds of raiſins muſt be given?

Anſw. 737$\frac{67}{91}$ lb.

9. A and B barter: A gives to B 90 doz. of penknives at 7s. 8d. per doz. for which B gives to A 10$\frac{1}{4}$l. in money and 500lb. of tobacco; what was it valued at per lb?
Anſw. 11d. 2,08 qr.

10. A and B barter: A hath 20 cwt. of cheeſe at 21,5s. per cwt. B hath 8 doz. of handkerchiefs at 3$\frac{7}{10}$l. per doz. I deſire to know who muſt receive the difference, and how much? Anſw. B muſt receive of A 8 l. 2 s.

11. A and B barter: B hath cheeſe at 30s. per cwt. which is vendable but for 26s. he would have *one third* ready money, and gain 10 per cent: A hath handkerchiefs at 4s. per piece ready money: how muſt the handkerchiefs be valued per piece, to equal the barter?

Anſw. 5$\frac{4}{13}$ s.

12. A has tea at 6s. per lb. which he barters with B at 7 s. 6 d. per lb. and is willing to loſe 10 l. per cent. to have *one third* ready money: what is the juſt price of a yard of velvet delivered at 22s. to equal the barter?

Anſw. 18,7s.

13. If 56 yards of cloth be given for 6$\frac{1}{2}$ cwt. of raiſins at 2,4l. per cwt. what is the cloth valued at per yard?

Anſw. 5s. 6d. 3$\frac{2}{7}$q.

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§ XVIII. LOSS and GAIN.

BY this rule we are taught how to raiſe or ſettle the price of any commodity, ſo as to gain or loſe ſo much per cent. or by the whole, and is of excellent uſe to traders. A great variety of queſtions may be ſtarted in this rule; which may be eaſily ſolved, if you conſider that the gains and loſſes are always in proportion as the given quantities, and the contrary. Therefore this rule is only direct proportion.

E X A M P L E S.

1. Bought 5 cwt. 3 qr. 21 lb. of ſugar, at 2 l. 16 s. per cwt. and ſold it again for 8 d. per *pound*: how much was gained or loſt by the whole, and how much per cent?

Firſt 5 cwt. 3 qr. 21 lb. = 665 lb. and 2 l. 16 s. = 2,8 l. then 112 lb. : 2,8 l. :: 665 lb. : 16,625 l. = 16 l. 12 s. 6 d. = 3990 d. the prime coſt:

Again, 1 lb. : 8 d. :: 665 lb. : 5320 d. = 22 l. 3 s. 4 d. ſold for; therefore the gain upon the whole is 5 l. 16 s. 10 d. Next to find the gain per cent. ſay,

As 3990 d. : 5320 d. :: 100 l. : 133 l. 6 s. 8 d. therefore the gain per 100 l. is 33 l. 6 s. 8 d.

2. A Mancheſter chapman going to a fair, ſold fuſtians for 11 s. 6 d. the end, wherein was gained 15 l. per cent. and ſeeing no other chapman had ſo good, raiſed them at the latter end of the fair to 12 s. I demand what he gained per cent. by this latter ſale? Anſw. 20 l.

For 15 l. + 100 = 115 l. what 100 l. is advanced to by the firſt ſale; then as 11,5 s. : 115 l. :: 12 s. : 120 l. the amount of 100 l. by the latter ſale; then 120 l. minus 100 l. = 20 l. the Anſwer.

N. B. *This queſtion is the ſame with the eighth queſtion in loſs and gain in* HILL's Arithmetic *page* 289, *but there calculated by a falſe method; for the anſwer there brought out is* 15,652 *l. which is* 4,348 *l. too little; and in like manner ſeveral Authors have run into the ſame error in queſtions of this ſort, by making the gain or loſs of* 100 *l the ſecond term in the ſtating, inſtead of its amount (in caſe of gain) or depreſſion (in caſe of loſs).*

3. A grocer bought coffee at 7 s. per lb. which, being damaged, he is willing to ſell out ſo as to loſe 8 l. per

cent. at what rate per pound muſt it be ſold ? Anſw. 6 s. 5 d. 1,12 q.

For 100—8=92 the depreſſion of 100l. then ſay, As 100l : 92 l. :: 7 s. : 6,44 s. the Anſw.

4. If 5 cwt. of chocolate coſt 252 l. at what rate per lb. muſt it be ſold, that the gain upon the whole may be 18 l. 13 s. 4 d ? Anſw. at 9 s. 8 d. per lb.

5. If tea be bought for 7 d. per ounce, and ſold for $8\frac{3}{4}$ d. what is the gain per cent ? Anſw. 25 l.

6. If needles be bought for 2 s. 6 d. a groſs, how many may be ſold for 1 d. to gain 20 l. per cent ? Anſw. 4.

7. A had 15 pipes of wine which he parted with to B, at 4 l. 6 s. 8 d. per cent. profit, who ſold them to C, for 38 l. 11 s. 6 d. advantage; C made them over to D for 500 l. 16 s. 8 d. and cleared thereby, $6\frac{1}{2}$ l. per cent. what did this wine coſt A per gallon ? Anſw. 4 s. $4\frac{3}{4}$ d.

8. If when I ſell cloth at $8\frac{1}{2}$ s. per yard, I loſe 5 *per cent*. what do I gain or loſe per cent. when I ſell it for 9 s. per yard ? Anſw. 11 s. $9\frac{2}{17}$ d. gain.

9. Paid 69 l. for 1 ton of ſteel, which is retailed at 6 d. per lb. what is the profit or loſs by the ſale of 14 tons ? Anſw. 182 l. loſs.

10. Having ſold 12 lb. of tea for 5 l. 14 s. and thereby gained 8 l. per cent. what was the prime coſt of a lb ? Anſw. 8 s. 9 d. $2\frac{2}{9}$ q.

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§ XIX. ALLIGATION.

ALLIGATION teaches how to mix ſeveral ſorts of ſimples or ingredients together, ſo that the compoſition may be of a middle quality. Alligation is commonly diſtingiſhed into two principal caſes, called *alligation medial* and *alligation alternate*.

Case

CASE I. ALLIGATION MEDIAL.

Alligation Medial teaches how to find the mean rate of a mixture or compound, when the particular quantities are mixt, and their ſeveral rates are given.

Note. That by the rates are meant the numbers which determine, or define the proportion of the qualities of the ſimples, and compound; ſuch as the given prices of their integers, or numbers expreſſing their degrees of fineneſs, or any other numbers proportional to them. And if any one of the ſimples be of little or no value with reſpect to the reſt, its rate is ſuppoſed to be 0; as water mixt with wine, or alloy with gold or ſilver.

R U L E.

Multiply each quantity by its rate; then divide the ſum of the products, by the ſum of the quantities, (or the whole compoſition) and the quote will be the rate of the mixture (or compound) required.

E X A M P L E S.

1. If 20 buſhels of wheat at 5s. 6d. per buſhel, be mix'd with 30 buſhels of rye at 3s. 6d. a buſhel. At what price per buſhel muſt the mixture be ſold?

buſh. d. d.

20 × 66 = 1320 the value of the wheat.

30 × 42 = 1260 the value of the rye.

50 50)2580(51,6d. = 4s. 3d. 2,4 qr. the mean rate, or value per buſhel of the miſlegin.

The proof is made, by finding the value of the whole mixture at the mean price; which muſt be equal to the total value of the ſeveral ingredients.

Thus, 51,6 d. × 50 = 2580 d. as above.

2. A vintner would mix 40 gallons of malaga, at 8 s. the gallon; with 25 gallons of ſherry at 5s. 6d. 16 gallons of canary at 7s. and 14½ gallons of white wine at 4s. 6d. how muſt the mixture be ſold?

Anſw. 6s. 7d. $3\frac{7}{191}$ q.

3. A maltſter mixed 24 quarters of high dried malt, at 25s. a quarter, with 30 quarters of brown malt, at 28s. and 46 quarters of pale malt at 30s. What is the value of 1 quarter of this mixture? Anſw. 28,2s.

4. A mixture being made of 20 gallons of ale at 10d. a gallon, 18 at 8d. 14 at 6$\frac{1}{2}$d. 10 at 5d. and 7$\frac{1}{2}$ at 4d. what will it be worth per gallon? Anſw. 7d. 1$\frac{22}{139}$ q.

5. Having melted together 10 oz. of gold of 22 carracts fine, 30$\frac{1}{2}$ oz. of 20 carracts fine, and 16 oz. of 18 carracts fine: I demand the fineneſs of the compoſition? Anſw. 19$\frac{82}{113}$ carracts fine.

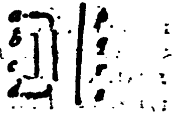
6. Suppoſe I mix 4lb. of ſilver of 10 oz. fine, with 6lb. 8 oz. of 8 oz. fine, and 2lb. 6 oz. of alloy: of what fineneſs is that compoſition? Anſw 7$\frac{7}{9}$ oz. fine.

CASE II. ALLIGATION ALTERNATE.

Alligation alternate ſhews how to find the particular quantities concerned in any mixture, when the particular rates of each ſort, and alſo the mean rate, are given. *Alligation alternate* is the *reverſe* of *alligation medial*, and may be proved by it.

PREPARATION.

Set down the ſeveral rates in order from the greateſt to the leaſt, as the letters *a*, *b*, *c*, *d*; and the mean rate (*m*) to the left in its due order.



Connect or link every two rates together by an arched line or brace, ſo as one rate may be greater and another leſs than the mean, till all be coupled.

Where *note*, that one rate may be coupled with ſeveral others one by one, as oft as you pleaſe.

Take the difference between each rate and the mean rate, and place it down *alternately*, that is, againſt all its yoke fellows. Do thus with all the rates ; and the differences will ſtand as p. q. r. s. When ſeveral differences happen to ſtand againſt one rate, add them all together. Then,

1. RULE.

When no quantity is given of any of theſe ſorts, the numbers (or differences) ſtanding againſt the ſeveral rates, are the quantities required.

EXAMPLES.

1. A vintner would mix canary at 2s. 9d a quart, with ſherry at 1s. 6d. per quart, to ſell the whole at 2s. 3d. per quart. How much of each muſt he take ?

$$\begin{array}{rl} & \text{d.} \\ \text{m. } 27 & \left\{\begin{array}{r} 33 \\ \\ 18 \end{array}\right\} \begin{array}{l} 9 \text{ quarts of canary} \\ \\ 6 \qquad\qquad \text{ſherry} \end{array} \left.\begin{array}{l} \\ \\ \\ \end{array}\right\} \text{the Anſwer.} \end{array}$$

Which may be proved by alligation medial : Thus,

9 qrts. canary at 33d. amounts to 297d.
6 ſherry at 18d. amounts to 108d.

15 total quantity. Total amount. 405 d.

Then $405 \div 15 = 27$d. the mean rate.

Note. The queſtions of this caſe are called indeterminate, becauſe each admit of various anſwers : from an Algebraic proceſs it appears that they will have infinite varieties of anſwers ; nay, if the expreſſion may be allowed, that they will admit of infinite varieties of infinite varieties of anſwers : In the queſtion above, altho' 9 and 6 do truly anſwer the conditions, yet any two numbers that bear the ſame proportion to each other as 9 to 6, will as truly anſwer it. Thus,

$$9 : 6 :: \left\{\begin{array}{rcr} 3 & : & 2 \\ 6 & : & 4 \\ 12 & : & 8 \\ 15 & : & 10 \end{array}\right\} \text{\&c. ad infinitum.}$$

Here 3 and 2, or 12 and 8, or 15 and 10, are numbers that will as truly anſwer the conditions of the queſtion (as thoſe found by the rule) 9 and 6.

2. How much ſherry, at 2s 3d. canary at 2s. 9d. and white port at 1s. 2d. per quart, muſt a vintner mix together to be ſold for 18d. per quart?

18d.	33	4	4 qrts. canary	Anſwer.
	27	4	4 ſherry	
	14	9.15	24 w. port	

Here the difference between 18 and 14 is 4, which I ſet againſt 33, and alſo againſt 27. The difference between 18 and 27 is 9, which I place againſt 14.

The difference between 18 and 33 is 15, which I alſo ſet againſt 14. Then 9 added to 15 is 24. So the differences, to work by, will be 4, 4, 24.

3. How much malaga, at 2s. 11d. per quart, canary at 2s. 6d. ſherry at 1s. 8d. and white port, at 1s. 2d. will compoſe a mixture to be ſold for 2s. 8d. per quart?

	d.			
32d.	35	2 . 12 . 18	32 qrts malaga	Anſwer.
	30	3	3 canary	
	20	3	3 ſherry	
	14	3	3 w. port	

4. How much malaga, at 2s. 11d. per quart, canary at 2s. 6d. ſherry at 1s. 11d. and white port at 1s. 2d muſt be mixed, to be ſold at the mean rate of 20d. per quart?

Anſw. malaga, canary, ſherry, 6 qrts. each, and 28 qrts. of w. port.

5. Suppoſe it be required to make a mixture of ſix ſorts of wine, viz. malaga at 2s. 11d canary at 2s. 6d. gallicia at 2s. ſherry at 1s. 8d. rheniſh at 1s. 6d. and white port at 1s. 2d. per quart, how much of each muſt be uſed, that the compoſition may be ſold for 1s. 11d. per quart?

Anſw. malaga 9, canary 9, gallicia 3, ſherry 3, rheniſh 9, white port 12 quarts. And by various coupling of the quantities, various anſwers may be obtained; or by the rule of proportion.

RULE.

ALLIGATION PARTIAL.

In alligation partial, where one of the quantities (to be mixed) is given; ſay, by the Rule of Three,

As the difference ſtanding againſt the price of the given quantity:
Is to the given quantity ::
So are the ſeveral other differences:
To the reſpective quantities required.

EXAMPLES.

1. A farmer would mix 15 buſhels of *wheat* at 5s. with *rye* at 3s. *barley* at 2s. 9d. and *oats* at 1s. 10d. to be ſold at 2s. 10d. per buſhel. How much rye, barley, and oats muſt he take?

			B	
M. 34	wheat	60	12	The differences found by the firſt rule.
	rye	36	1	
	barley	33	2	
	oats	22	26	

Note. Theſe differences are not the very quantities that the limits of the queſtion require, but correſponding proportionals to them; from which the quantities themſelves may be found (according to the above rule) as follows.

As 12 : 15 :: { 1 : 1¼ buſhels of rye } to be mixed
Contr 4 : 5 :: { 2 : 2½ barley } with 15 buſhels
{ 26 : 32½ oats } of wheat.

This may be proved by alligation medial.

2. A tobacconiſt would mix three ſorts of tobacco, at 20d. at 14d. and at 10d. per lb. with 8lb. at 24d. per lb.

How much of each ſort muſt be taken, ſo as the whole compoſition may be ſold for 18d. per lb?

1ſt.	24	8 lb.	2d	24	4lb.
m.	20	4	m.	20	8
18	14	2	Or thus 18	14	6
	10	6		10	2

Now the differences gained by the firſt method, viz. 4lb. at 20d. 2lb. at 14d. and 6lb. at 10d. per lb. are ſuch as will ſufficiently anſwer the propoſition, becauſe 8lb (found by alligation, is the ſame as the quantity given in the queſtion; but the differences gain'd by the ſecond method will require the rule of proportion to bring out the true quantities.

As 4 : 8 :: {8 : 16lb. / 6 : 12 / 2 : 4} at {20 / 14 / 10} amount to {320 / 168 / 40}
i. e. 1 : 2 ::

And the given quantity (8lb) at 24d. 192

The total quantity 40)720(18 mean rate.

3. How much ſilver bullion of 10 dwts. and another ſort of 12 dwts. fine, muſt be melted down with 20lb. that is 6 dwts. fine, ſo that the whole mixture may bear 9 dwts fine? Anſw. 15lb of each.

4. How much gold of 22, and 24 carracts fine, alſo how much alloy, muſt be mixed with 20 ounces of 20 carracts fine, that the whole maſs may be 18 carracts fine? Anſw. 20 oz. of 22 carracts fine, alſo 20 oz. of 24 carracts fine, and 13⅓ oz. of alloy.

3. RULE.

ALLIGATION TOTAL.

In alligation total, where the total ſum of the quantities (to be mixt) is given; add up all the differences together, then ſay by the Rule of Three,

As the ſum of the differences :
Is to the total quantity given ::
So is every particular difference :
To its reſpective quantity.

EXAMPLES.

1. A goldſmith would mix gold of 24 carracts, with ſome of 23 carracts, ſome of 22 carracts, and ſome other of 21 carracts fine, and with a due quantity of alloy; ſo that 200 oz. might bear 20 carracts fine: how much of each ſort muſt he take?

Here the mean rate muſt be ſubtracted from the ſeveral given rates, and the ſeveral differences placed againſt 0: and 0 from the mean rate, to be placed againſt the ſeveral given rates; as below.

$$\begin{array}{ll} \text{m.} & \\ 20 & \end{array} \left\{ \begin{array}{l|l} 24 & 20 \\ 23 & 20 \\ 22 & 20 \\ 21 & 20 \\ 0 & 4 \cdot 3 \cdot 2 \cdot 1 \end{array} \right| \begin{array}{r} 20 \\ 20 \\ 20 \\ 20 \\ 10 \\ \hline \end{array}$$

Here alloy is reckoned 0 carracts.

The ſum of the difference 90

Then $90 : 200 :: 20 : 44\frac{4}{9}$ oz. of the 4 ſorts of gold.
or $9 : 20 :: 10 : 22\frac{2}{9}$ oz. of alloy.

2. A refiner has ſix ingots of ſilver of different fineneſs, viz. of 4, 5, 6, 8, 11 and 12 ounces fine, of which he would mix 500lb. weight, ſo as to make it 9 oz. fine: how much of each ſort muſt he take?

In the ſolution of this queſtion, I ſhall ſet down four ways of obtaining the differences between the mean rate and the ſeveral ſimple rates, but ſhall leave the linking of the ſeveral ſimple rates for the learner's practice.

First way.

m. 9	4	2	2
	5	2	2
	6	3	3
	8	3	3
	11	4 . 5	9
	12	1 . 3	4

Second way.

m. 9	4	3	9
	5	3	3
	6	2 . 3	5
	8	2 . 3	5
	11	1 . 3	4
	12	3 . 4 . 5 . 1	13

Third way.

m. 9	4	2	2
	5	2 . 3	5
	6	2 . 3	5
	8	3	3
	11	3 . 4 . 5	12
	12	1 . 3 . 4	8

Fourth way.

m. 9	4	3	3
	5	2 . 3	5
	6	2 . 3	5
	8	2 . 3	5
	11	1 . 3 . 4	8
	12	1 . 3 . 4 . 5	13
			39

The operation by the laſt way, is thus

39 : 500 :: 3 : $38\frac{18}{39}$ oz. of 4 oz. fine.
5 : $64\frac{4}{39}$ of 5, 6, and 8 oz. fine.
8 : $102\frac{22}{39}$ of 11 oz.
13 : $166\frac{26}{39}$ of 12 oz.

SCHOLIUM.

Although the ſeveral ways of combining and coupling the rates, as before directed, afford ſo many different ſolutions to the queſtion; yet they do not give all the anſwers the queſtion is capable of. To remedy which, and to make the method more general, you may repeat any two alternate (or correſponding) differences as often as you will; and the like for any other two, &c. This will give a great variety of ſolutions, from which the eaſieſt and moſt ſuitable may be ſelected. Or rather proceed by the following rule.

4. RULE, Univerſally.

Having coupled the rates as before directed, inſtead of any couple of the differences, take any equimultiples thereof; that is, multiply them both by any number you will; do the like for any other couple, &c. By this means you'll have a new ſet of differences to work with.

3. A grocer would mix three ſorts of ſugar, at 6d. 9d. and 12d. per lb. to have a quantity of 50 lb. to be ſold at 8d. per lb. How much of each ſort muſt he take?

Common way

m. 8 {12 | 2 | 2 ; 9 | 2 | 2 ; 6 | 4 1 | 5} 11

m. 8 {12 | 2 × 3 | 6 ; 9 | 2 × 4 | 8 ; 6 | 4 × 3 . 1 × 4 | 16}

30

Here the couple of differences againſt 12 and 6 being 2 and 4, × them both by 3, and they become 6 and 12. Again the couple of differences againſt 9 and 6 being 2 and 1, × them both by 4 and they become 8 and 4. Then you'll have 6, 8, and 16 for a new ſet of differences, whoſe ſum is 30.

30 : 50 :: {6 : 10 lb. at 12d. ; 8 : 13⅓ at 9d. ; 16 : 26⅔ at 6d.}

The proof 50

4. A wholeſale brewer having four ſorts of ale, at 12d. 10d. 5d. and 4d. per gallon, deſires to have a mixture of 100 gallons made out of them, ſo as that it may be ſold for 7d. per gallon: how much of each ſort muſt he take?

d.

m. 7d. {12 | 2 × 2 = 4 ; 10 | 3 × 3 = 9 ; 4 | 3 × 3 = 9 ; 5 | 5 × 2 = 10} A new ſet of differences.

Sum of the new differences 32

gal.

As 32 : 100 :: {4 : 12½ at 12d. ; 9 : 28⅛ at 10d. and alſo at 4d. ; 10 : 31¼ at 5d.}

Or 8 : 25 ::

5. A Goldſmith would make a maſs containing 80 oz. and bearing 18 carracts fine, of the following ſorts of gold, viz. that of 23, 22, and 21 carracts fine, and of alloy, of each a ſufficient quantity: how much of each ſort muſt he make uſe of?
Anſwer 21$\frac{9}{11}$ oz. of each ſort of gold, and 14$\frac{6}{11}$ oz. of alloy.

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§ XX. EXCHANGE.

EXCHANGE properly conſiſts in giving monies in one *place* to receive the value at a certain price agreed on in another, and is much like the bartering of commodities.

There are five things to be defined before we come to the practice of Exchange; and theſe are, 1ſt. The real money of every *country*. 2d. The imaginary money of it. 3d. The *par* of *exchange*. 4th. The *courſe* or current rate of exchange. and 5th. The *ages*.

1ſt. The real money of any *country*, is a quantity of ſome metal coined by the authority of that ſtate and current at a certain rate; as a *guinea*, a *crown*, a *ſhilling*, &c. in *England*. 2d. The imaginary money is a denomination which has no real *ſpecies* or *coin* to anſwer it; as a *pound*, a *mark*, an *angel*, a *penny*, &c. in *England*.

3d. The par of exchange is, when any two *coins* of different countries do contain the ſame quantity of pure gold or ſilver, although of a different weight. Hence it has been found that the ecu or French crown is, to the Engliſh crown, as 9 to 10, and therefore the *par* of the French crown is 4s. 6d. in ſterling.

4th. The courſe of exchange does not always run at *par*. but often riſes above and ſettles below it, according to the plenty or ſcarcity of money, or bills, and the preſent rate at any time, is called the *courſe* of *exchange*.

5th. The ageo is, the difference of the value of current money, and *bank money*: for the bank money in foreign places is finer than the current money in them.

Note. It is by comparing the bank money with ſterling that the par is aſcertained. Alſo the exchange is always ſuppoſed to be made in bank money; and if there is a neceſſity for taking currency in caſe of a defect of the bank to anſwer the bills, the more current money muſt be received, and that in proportion to the ageo.

It will not be conſiſtent with my intended brevity, to treat of the exchanges of every country, for that would make a volume of itſelf, neither is it any wiſe neceſſary: I ſhall therefore confine myſelf to the exchanges made between England and the chief countries of commerce in Europe. The courſe of exchange of any country being given, the queſtions appertaining thereto, may be ſolved by the Rule *of* Three, *or* Practice; *and ſince theſe rules have already been ſufficiently treated of, I think it unneceſſary to expreſs any operations at large; but only to ſhew the proportion by the ſtating of the queſtion in each caſe, and ſubjoin the anſwer.*

1. *With* FRANCE.

In France accompts are kept in livres, ſols, and deniers: but they exchange with other kingdoms, &c. upon the crown (or ecu) of 60 ſols, or 3 livres tournois.

Note 1. The livres are imaginary money, but the crowns, ſols, deniers are real: and the term tourneis is only a note of diſtinction of the French money, as ſterling is of the Engliſh.

2. In the French exchange you muſt diſtinguiſh between exchange money and book money. The exchange money is called money d'or, and is expreſſed in crowns, ſols, and deniers d'or, and theſe are ſumed up like l. s. d.; for 12 deniers make a ſol of the crown, and 20 ſols of the crown, make a crown.

The book money is called tournois, and is expreſſed in livres, ſols, and deniers tournois; and theſe are ſumed up like l. s. d.; for 12 deniers make a ſol of the livre, and 20 ſols of the livre make a livre. So that crowns, ſols, and deniers d'or (or exchange money) multiplied by 3 through out, give livres, ſols and deniers, tournois, or book money. On the contray, book money divided by three throughout, gives exchange money.

E X A M P L E S.

1. How many livres &c. will 345l. 6s 7d. ſterling, amount to, exchange at 31¾ d. per ecu?
As 31¾ : 1 ecu :: 345 l. 6s. 7d. : 2610 ecus 7 ſols 3 den. d'or, near. Which ×d by 3, gives 7831 liv. 1s. 9 den. tournois.

2. To how much ſterling will 7831 livres, 1 ſol 9 den. amount to at 31¾d. per ecu?
As 3 liv. : 31¾d. :: 7831 livres 1 ſol 9 den : 345l. 6s. 7d. ſterling. the Anſwer.

3. How much ſterling muſt be paid in London, to receive in Paris 14875 livres tournois, at 23 livres per pound ſterling?

As 23 liv. : 1l. :: 14875 liv. : 646l. 14s. 9 $\frac{2}{23}$d. ſterling. the Anſwer.

4. London remits to Paris 646 l. 14s. 9 $\frac{2}{23}$d. what is the value in French livres, at 23 livres per pound ſterling?

As 1l. : 23 liv. :: 646l. 14s. 9 $\frac{2}{23}$d. : 14875 liv. the Anſw.

5. Remitted to Paris 168l. 18s. ſterling at 23⅞d per crown; how many crowns, ſols, and deniers d'or does it come to?

Anſwer 1697 cr. 16 ſols. 10 $\frac{58}{191}$ den.

6. How much ſterling for 2064 crowns d'or, exchange at 56d. each?

Anſwer 467 l. 14s.

7. How many ecus, for 345l. 6s. 7d. ſterling, at 32d. per ecu? Anſw. 2589 ecus. 19 ſols 4½ den.

8. Marſeilles remits to London 47329 livres, 10 ſols. 6d. exchange at 27 liv. 10s. per pound ſterling; what ſterling money will the ſaid remittance amount to?
Anſw. 1721l. 1s. 5¾d.

2. *With* S P A I N.

In Spain they keep their accompts in piaſtres, rials, and maravedies; reckoning 34 maravedies to a rial, and 8 rials to a piaſtre, *peſo*, or piece of *eight*, old money, by which they exchange, whoſe par is 4s. 6d. The piaſtre is imaginary, but the rials and maravedies are real. London exchanges with Spain in peſos (or piece of eight) old money, of 8 rials, or 272 maravedies; and in new money, of 10 rials.

E X A M P L E S.

1. If London draws or remits on Spain 555l. 5s. 5d. at 43¾d. ſterling per peſo, how much muſt be paid, or received there?

d. peſo l. s. d. peſos marav.
As 43¾ : 1 :: 555 5 5 : 3046 16 the Anſw.

2. Cadiz remits to London 7075 peſos, 3 rials, 10 marav. at 52¼d. per peſo; what will this remittance amount to in London?

peſo d. peſos r. m. l. s. d.
As 1 : 52¼ :: 7075 3 10 : 1540 7 6¼ the Anſw.

3. How many piaſtres, &c. ſhall I receive at Bilboa for 612l. 10s. 4d. ſterling exchange at 52⅞d. ſterling per piaſtre? Anſw. 2780 piaſtres, 1 rial, $25\frac{67}{423}$ marav.

4. A merchant at Cadiz remits to London 2547 peſos, at 56d. ſterling each; how much ſterling is the ſum?
Anſw. 594l. 6s.

5. How many pieces of eight, &c. will 2500 l. amount to at $57\frac{1}{2}$d. ſterl. per piece of eight? Anſw. 10434 peſos, 6 rials, $8\frac{20}{23}$ marav.

3. *With* ITALY.

At Leghorn and Genoa they keep their accompts in livres, ſols, and deniers, as in France, but they exchange by the piaſtre or dollar, as in Spain, which at Genoa is accounted 5 livres, and at Leghorn 6. Alſo at *Venice*, accounts are kept by ſome in the ſame manner, and by others in ducats and groſs. Alſo at Florence by ducatoons. A ducatoon is equal to 4s. 4d. at *par*.

Note, 6 ſoldi = 1 groſs, and 24 groſs = 1 ducat.

EXAMPLES.

1. Suppoſe there be owing to me by a correſpondent at Genoa 3675 livres, 15 ſols, 10 deniers; how much ſterling does it amount to, exchange at 50d. ſterling per piaſtre *dollar* or piece of eight?

First 3675 liv. 15 ſols. 10 den, ÷ 5 = 735 dollars, 3 ſols, 2 den.

	dol.		d.		dol.	ſol.	den.		l.	s.	d.	
As	1	:	50	::	735	3	2	:	153	3	$1\frac{11}{12}$	the Anſw.

2. A merchant remitted 153l. 3s. $1\frac{11}{12}$d. ſterling to Genoa; how many dollars muſt he receive there, the exchange being 50d ſterling per dollar?

First 153l. 3s. $1\frac{11}{12}$d. = $36757\frac{11}{12}$d. and $36757 \times 12 + 11 = 441095$ twelfths of pence;

Alſo $50 \times 12 = 600$ twelfths of pence; then

			dol.			dol.	ſols.	den.	
As	600	:	1	::	441095	: 735	3	2	the Anſw.

3. Genoa draws upon London for 8504 liv. 17 ſols 1 den: how much ſterling will ſatisfy the draught, exchange at $53\frac{3}{8}$d. ſterl. per dollar?

Anſw. 378l. 5s. 9d. $1\frac{4}{15}$q.

5. How

4. How many livres, &c. muſt be received at Leghorn for 705l. 16s. 4d. ſterling, exchange at 51½d. ſterling per piaſtre?

Anſw. 19735 liv. 9 ſols $1\frac{53}{103}$ den.

5 A merchant ſold goods to Florence, for 250 ducatoons, at 54d. ſterl. each; what is the value in pounds ſterling? Anſw. 56 l. 5s.

At Venice ſome keep their accompts in ducats of the bank, and others in ducats current, but the Republic keeps them in ducats groſs. The ducat of the bank (which is imaginary) is worth 24 groſs; and 100 ducats banco are worth 120 ducats current; ſo that 5 ducats banco are equal to 6 ducats current; and the ageo conſtantly 20 per cent.

To reduce bank money into current.

R U L E.

Divide the given ſum by 5, the quote is the ageo which added to the bank money gives the current.

E X A M P L E.

Reduce 13650 ducats banco into current.

13650÷5=2730 ageo, and 13650+2730=16380 ducats current.

To reduce current money into bank money.

R U L E.

Divide the current money by 6 and the quote will be the ageo, which ſubtract from the current money, and the remainder will be the bank money.

E X A M P L E.

Reduce 16380 ducats current into ducats banco.

16380÷6=2730 ageo; and 16380−2730=13650 ducats banco.

EXAMPLES.

1. London remits to Venice 764l. ſterl. the exchange 52d. ſterl. per ducat banco, how many ducats banco will this remittance come to? Firſt 764l.=183360d.

d: duc. d. ducats groſs
As 53 : 1 :: 183360 : 3526 $3\frac{2}{13}$d. banco, and 4231 duc. $9\frac{1}{13}$ groſs current.

2. Suppoſe my correſpondent at Venice owes me 3526 duc. $3\frac{2}{13}$ groſs banco, the exchange at 52d. ſterl. per ducat, what ſterling money will this remittance produce at London?

duc. d. ducats groſs l.
As 1 : 52 :: 3526 $3\frac{2}{13}$: 764 ſterling.

3. How many Venice ducats at 4s. 5d. ſterl. each, for 60l. 14s. 7d. ſterling? Anſw. 275.

4. How many pounds ſterling, for 275 Venice ducats, at 4s. 5d. ſterl. per ducat? Anſw. 60l. 14s. 7d.

4. *With* PORTUGAL.

The Portugeſe keep their books and accompts in milrees, which are real, ſeparating the thouſands from the inferior ſums called rees: therefore 1000 rees = 1 milree, whoſe *par* is about 6s. 8½d. or 6s. 9d. ſterling.

EXAMPLES.

1. To how many milrees will 199l. 15s. amount, exchange at 5s. 8d. per milree?

As 5s. 8d. : 1 mil. :: 199l. 15s. : 705 milrees.

2. To how much ſterling will 705 milrees amount, exchange at 5s. 8d. per milree?

As 1 mil. : 5s. 8d. :: 705 mil. : 199l. 15s. ſterling.

3. Liſbon remits to London 4617 milr. 216 rees, exchange at $65\frac{3}{4}$d. ſterling per milree: what muſt be received at London?

	rees		d.		milr.	rees		l.	s.	d.	
As	1000	:	$65\frac{3}{4}$	::	4617	,216	:	1264	18	5	$3\frac{101}{125}$q.

ſterling.

4. To how many milrees at $65\frac{3}{4}$d. each, will 1264l. 18s. 6d. amount? Anſw. 4617 milr. $216\frac{192}{263}$ rees.

5. *With HOLLAND, FLANDERS, and GERMANY.*

I take theſe countries together, becauſe they keep their accompts, and make exchange with London in the ſame manner.

In *Amſterdam*, *Antwerp*, Bruſſels, *Rotterdam*, and *Hamburgh*, ſome keep their accompts in *pounds ſhillings* and pence, (or groats) as in *England*; others in guilders, ſtivers and pennings;

16 pennings = 1 ſtiver, or 6 groats
20 ſtivers = 1 guilder, or florin.
and 50 ſtivers = 1 rixdollar.

The money of *Holland* and *Flanders* is diſtinguiſhed by the name *flemiſh*, and they exchange with London by the pound ſterl. = 33s. 4d. flemiſh at *par*.

1 penny *flemiſh*, is ½ ſtiver, or 8 pennings.
1 ſhilling *flemiſh*, is 12 pence flemiſh, or 6 ſtivers.
1 pound *flemiſh*, is 20 ſhillings flemiſh or 6 guilders.

Holland. The bank of *Amſterdam* was eſtabliſhed by an ordinance of the States in 1609, in which it was decreed, that all bills of exchange ſhould be paid for in bank money, and for merchandize in current or groſs money, unleſs the ſum ſhould be under 360 florins.

Note. The agio, or difference between the bank money and current money, is 3, 4, 5 or 6 per cent; ſometimes more, and ſometimes leſs.

To reduce current money into bank money.

RULE.

As 100 current money with the ageo added to it, is to 100 bank money, ſo is the current money given, to the bank money ſought.

EXAMPLE.

Change 8162 guilders 12 ſtivers current money into bank money, the ageo being $3\frac{1}{4}$ per cent?

guil. cur. guil. b. guil. ſtiv. guil. ſtiv. penn.
As $103\frac{1}{4}$: 100 :: 8162 12 : 7905 13 5 + banco.

Anſw. 7905 guil. 13 ſtiv. 5 pennings banco, with a ſmall remainder.

To change bank money into current money.

RULE.

As 100 guilders bank, is to 100 with the ageo added; ſo is the bank given, to the current required.

EXAMPLE.

Suppoſe the ageo $5\frac{1}{8}$ per cent. how much current money will 4954 guil. 9 ſt. 8 penn. bank amount to?

bank current g. bank ſt. p. guil. ſt. p.
As 100 : $105\frac{1}{8}$ :: 4954 9 8 : 5208 7 $13\frac{32}{100}$ current.

To change ſterling money into flemiſh.

RULE.

As 1l. ſterl. is to the given rate; ſo is the ſterl. given to the flemiſh ſought.

To change flemiſh into ſterling.

RULE.

As the given rate, is to 1l. ſterling, ſo is the flemiſh ſum given, to the ſterling required.

EXAMPLES.

1. A merchant at *London* remits to Rotterdam 3 bills of exchange, 200l. each, one at 33s. 6d. another at 33s. 9d. and the other at 33s. $5\frac{1}{2}$d. flemiſh per pound ſterling.

How many guilders, &c. bank money does each of theſe bills come to? And how much current money in them all, ageo at $4\frac{7}{8}$ per cent?

	l.	s. flem.	l.	l. ſt.	s.	d.	guild.	ſt.	p.
As	1 :	$33\frac{1}{2}$ ::	200 :	335	0	0 =	2010	0	0
	1 :	$33\frac{5}{12}$ ::	200 :	334	3	4 =	2005	0	0
	1 :	$33\frac{11}{24}$ ::	200 :	334	11	8 =	2007	10	0
						Banco	6022	10	0
at $4\frac{7}{8}$ per cent. the ageo of the whole is							293	11	15
						current money	6316	1	15

2. Suppoſe there be due to me at Rotterdam 6316 guil. 1 ſt. 15 pennings, current money, and the ageo be $4\frac{7}{8}$ per cent. how much ſterling money does it come to, exchange at 33s. $5\frac{1}{2}$d. flemiſh per pound ſterling?

Firſt reduce the current money into banco, thus

As $104\frac{7}{8}$ cur. : 100 bank :: 6316 cur. 1ſt. 15p. : 6022 guil. 10 ſtiv. banco.

Next reduce the banco money into flemiſh pounds, thus 6022 guil. 10 ſt. ÷ 6 = 1003l. 15s. flemiſh; which reduce into ſterling, (exchange being 33s. $5\frac{1}{2}$ per l. ſterl.) thus as 33s. $5\frac{1}{2}$d. : 1l. :: 1003l. 15s. : 600l. ſterling, the Anſw.

3. To how much flemiſh will 754l. 10s. ſterling amount, exchange at 33s. 6d. flem. per l. ſterling? Anſw. 1263l. 15s. 9d. flem.

4. To how much ſterling will 1263l. 15s. 9d. flemiſh amount, exchange at 33s. 6d. per l. ſterling? Anſw. 754l. 10s.

5. How many guilders, &c. may I have for 173l. 14s. 2d. ſterling, exchange at 35s. $3\frac{1}{2}$d. flem. per l. ſterling? Anſw. 1839 guil. 2 ſtiv. $11\frac{1}{2}$ pennings.

6. How much flemiſh currency will 290l. 11s. 10d. ſterling amount to, exchange at 33s. 10d. flem. per l. ſterling, and the ageo 4½ per cent? Anſw. 513l. 14s. 1d. $1\frac{917}{3000}$q.

7. How much ſterling will 8000 guilders amount to, at 11 guil. 3 ſtiv. 10 pen. per l. ſterling? Anſw. 715l. 9s. $8\frac{76}{1789}$d.

8. To how much ſterling will 6712 guil. 10 ſtiv. current money amount, at 34s. 10d. per l. ſterl. and ageo at 4¼ per cent? Anſw. 596l. 2s. 0d. 3q. +

6. *With* IRELAND, AMERICA, *and the* WEST-INDIES.

Accompts are kept in Ireland, America, and the Weſt-Indies, in pounds, ſhillings, and pence, as in England: and exchange per cent. ſterl. 100l. ſterling = 108⅓ l. Iriſh at par, or 1l. ſterl. = 1l. 1s. 8d. Iriſh: alſo 5l. ſterl. is accounted worth 7l. of the currency of the Weſt-Indies, becauſe of the great plenty of the foreign coins there; but on the continent of America there is very little coin of any ſort circulating.

EXAMPLES.

1. How much in Dublin for 123l. 4s. 5d. in London, exchange at 107 per cent?

ſterl. Iriſh ſterl.
As 100 : 107 l. :: 123l. 4s. 5d. : 131l. 16s. $11\frac{11}{100}$d. the Anſw.

2. Dublin remits to London 345l. at 108 per cent. how much muſt be received at London?

Iriſh l. En. Iriſh l. s. d.
As 108 : 100 :: 345 : 319 8 10 Eng. Anſw.

3. London remits to Dublin 123l. 4s. 5d. at 107⅛ per cent. what muſt be received there? Anſw. 132l. Iriſh.

4. London remits to Barbadoes 789l. 10s. 11d. what currency muſt be received for it, exchange at 107½ per cent?

Anſw. 848l. 15s. 2d. $3\frac{3}{10}$q. Barbado. cur.

5. How much Engliſh for 436l. 8s. 10¼d. Barbadoes currency, at 131 per cent?

Anſw. 333l. 3s. 2d. $3\frac{115}{131}$q.

6. London remits to Jamaica for 678l. 9s. 10d. ſterling: what muſt be received for it, exchange at 135l. per cent?

Anſw. 915l. 19s. 3d. 1⅕q.

7. Jamaica remits to London for 135l. 7s. 9¼d. currency: what muſt be received for it, exchange at 110½ per cent?

Anſw. 122l. 10s. 5d. $2\frac{138}{221}$q. Eng.

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§ XXI. CONJOINED PROPORTION.

THIS chiefly conſiſts in comparing ſeveral different ſorts of things together, as to their value; and the following rule teaches to find, how many of one ſort is equal to a given number of another ſort.

RULE.

Place the terms in two perpendicular columns, ſo that there may not be found in either column, two terms of one kind. Then the numbers in the leſs column muſt be xed for a diviſor; and the numbers in the greater column, where the odd term is, for a dividend. The quotient is the Anſwer.

Note: To abridge the work, throw out any numbers that you can find in both columns.

EXAM.

EXAMPLES.

1. If 100lb. at London be equal to 92lb. at Paris, and 46lb. at Paris be equal to 44lb. at Rouen, and 88lb. at Rouen be equal to 91 at Rochelle, how many pounds at Rochelle, for 50lb. at London?

100lb. Lond. = 92lb. Paris
46lb. Paris = 44lb. Rouen
88lb. Rouen = 91lb. Rochelle
How many lbs Rochelle = 50lb. London

thus.

$$\frac{92\times44\times91\times50}{100\times46\times88}=\frac{18418400}{404800}=45\tfrac{1}{2}\text{ lb. the Anſw.}$$

2. If 3 pair of gloves be worth 2 yards of lace, 3 yards of lace worth 7 dozen of buttons, 6 dozen of buttons equal to 2 penknives, and 21 penknives to 18 pair of buckles; how many pair of gloves is equal to 28 pair of buckles?

3 gloves = 2 lace,
3 lace = 7 buttons,
6 buttons = 2 knives,
21 knives = 18 buckles,
28 buckles

31752 504

504)31752(63.

Or thus,

$$\frac{3\times3\times6\times21\times28}{2\times7\times2\times18}=\frac{3\times21\times28}{2\times7\times2}=3\times21=63.$$

3. Exchange between Dublin and London being 107l. per cent. between London and Amſterdam 36 ſch. 1 den. gros per l. ſterl.; what is the amount at Amſterdam of 234 Iriſh remitted from Dublin, by the way of London, allowing London $\frac{1}{2}$ per cent. or a 200th part, for commiſſion and charges?

Note 1. When commiſſion is allowed in one or ſeveral places, it is beſt to make calculations for theſe commiſſions, on the dividend, before the laſt diviſion is made. If the affair under conſideration be a circulation or chain of remittances, ſubtract the commiſſion found: again on the remainder calculate the ſecond commiſſion, which ſubtract from the ſaid remainder; and ſo continue to calculate for as many commiſſions as are to be allowed.

But in a circulation of draughts, the commiſſion found muſt be added, and the new commiſſions calculated on the totals laſt found.

2. The rate of commiſſion is different in different places, in ſome it is ¼ per cent, that is to ſay, the 400th part, in ſome one third per cent or the 300th part; in others ½ per cent, or the 200th part.

If 107l. Iriſh = 100 l. Eng.
and 1l. Eng. = 433 den gros.
how much at Amſt. = 234 l. Iriſh ?

Diviſor 107. From 10132200 dividend = x.
deduct a 200th part of x. = 50661 Lond. commiſſion.

107)10081539(94220 den. gros. near.

4. Exchange between London and Antwerp being 36 ſch. 8d. gros per l. ſterl.; betwixt Antwerp and Paris 57 den gros per ecu of 3 liv. tourn; betwixt Paris and Oporto 450 rees per ecu tourn; how much in Oporto for 456 l. ſterl. remitted from London thro' Antwerp and Paris, allowing Antwerp and Paris each ½ per cent. for commiſſion and charge ?

1l. ſterl. = 440 den. gros.
57 den. gros = 1 ecu tourn.
1 ecu tourn = 450 rees.
How many rees = 456l. ſterl.

$$\frac{440\times450\times456}{57} = \frac{440\times450\times8}{1} =$$

1584000 rees.
a 200th part is 7920 for Antw. commiſſion.

1576080
a 200th part is 7880 for Paris commiſſion.

1568,200 amount in Oporto.
mil. rees.

5. A remittance circulating entirely; that is to ſay, ending where it began.

London remits 500l. to Cadiz, at 40d. ſterling per piece of 8, to be remitted thence to Leghorn, at 125 pieces of 8 for 100 Leghorn dollars; thence to Venice at 90 Leghorn dollars for 100 ducats banco; thence to Lyons at 66 duc. banco for 100 ecus tourn; thence to London at 33d. ſterl. per ecu tourn; how much will this remittance produce in London, allowing to each of the other four places, ½ per cent. for commiſſion?

40d. ſterl = 1 pce of 8.
125 pieces of 8 = 100 Leghorn d.
90 Leg. dol. = 100 duc. bo.
66 duc. bo = 100 ecus tourn,
1 ecu tourn = 33d ſterl.
how much is = 500l. ſterling?

From theſe two columns, the diviſor will be found to be 9, and the dividend 5000, when each are properly contracted.

From the dividend 5000 = x
Subtr. the 200th part of x = 25 Cadiz Comm.

From 4975 = a
Subtr. the 200th part of a = 24,875 Leg. Comm.

From 4950,125 = b
Subtr. the 200th part of b = 24,750 Ven. Comm.

From 4925,375 = c.
Subtr. the 200th part of c. = 24,627 Lyons Comm.

4900,748 = d.
d÷9=544.527 l. in London; or 544 l. 10s. 6½d. ſterl. the neat produce.

§XXII. INVOLUTION.

INVOLUTION is the raiſing of powers from any propoſed number, called the root or firſt power, and is nothing but a continual multiplication of the given quantity or root into itſelf a certain number of times.

Thus: 4 is the root, or firſt power of 4.
4×4 = 16 is the ſquare, or ſecond power of 4.
4×4×4 = 64 is the cube, or third power of 4.
4×4×4×4 = 256 is the biquadrate, or 4th power of 4.

SCHOLIUM.

Some of theſe powers have borrowed their denomination from local extenſion. For a line having but one dimenſion, viz. length, ×ed into itſelf, produces a ſquare-plane. And that a ſquare having two dimenſions, viz. length and breadth, ×ed into itſelf, produces a cubed-ſolid. This cube has three dimenſions, viz. length, breadth, and thickneſs; but the nature and property of ſpace admits of no other extenſion.

Whence it follows, that the root or firſt power being taken as a ſide, the ſecond power will be a ſquare, the third a cube.

A TABLE *of the firſt three powers of the nine digits.*

Roots	1	2	3	4	5	6	7	8	9
Squares	1	4	9	16	25	36	49	64	8
Cubes	1	8	27	64	125	216	343	512	729

Note 1. The number which exceeds the multiplications by one is called the index, or exponent of the power: ſo the index of the firſt power is 1, that of the ſecond power is 2, that of the third is 3, &c.

2. Powers are commonly denoted by writing their indices above the firſt power: ſo the ſecond power of 4 may be denoted thus 4^2, the third power thus 4^3, the fourth power thus 4^4, &c. and if the given number

conſiſts of ſeveral figures, a crooked line is ſometimes drawn between them and the index: thus $\overline{408}\rceil^5$, denotes the fifth power of 408.

E X A M P L E S.

1. What is the 5th power of 8? Anſw. 32768.

For 8 × 8 = 64 the 2d power, and 64 × 8 = 512 the 3d power, and 512 × 8 = 4096 the 4th power, and 4096 × 8 = 32768 the 5th power.

2. What is the biquadrate of 27? Anſw. 531441.

3. What is the ſixth power of 503?
Anſw. 16196005304479729.

Note, 1. To involve a fraction, raiſe each of its terms to the power required.—And if a mixt number be propoſed, either reduce it to an improper fraction, or reduce the vulgar fraction to a decimal, and proceed as above.

2. The ſum of any two or more indices, or exponents of powers, will be the index of a power reſulting from the multiplication of the ſeveral powers into each other.

As $5^2 \times 5^3 = 5^5 = 3125$. Alſo $3^2 \times 3^2 \times 3^4 \times 3^5 = 3^{12} = 531441$. and $3^{12} \times 3^{12} = 3^{24}$: by this method the raiſing of powers may be a little ſhortened, for if any power be multiplied into itſelf, a power is produced whoſe index is double to that which was multiplied: for the 2d power multiplied into itſelf gives the fourth power, and the 4th multiplied into itſelf gives the 8th power, and this into itſelf, will give the 16th power, &c.

E X A M P L E S.

1. What is the third power of $\frac{7}{8}$? Anſw. $\frac{343}{512}$

For $\frac{7 \times 7 \times 7}{8 \times 8 \times 8} = \frac{343}{512}$ the Anſw.

2. What is the 5th power of $\frac{5}{7}$? Anſw. $\frac{3125}{16807}$

3. What is the 2d power of $4\frac{4}{25}$? Anſw. 17,3056.

4. What is the ſixth power of ,09?

Firſt ,09 × ,09 × ,09 = ,000729, the third power, or cube, and ,000729 × ,000729 = ,000000531441 the Anſw.

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§ XXIII. EVOLUTION.

EVOLUTION, or the extraction of roots, being directly contrary to Involution, or raiſing of powers, is performed by converſe operations, in order to find the roots from the given powers, either accurately or in decimals till the error be inſignificant.

NOTATION of ROOTS.

Roots are moſtly repreſented by writing $\surd$ before the power with the index of the root againſt it; ſo the third root of 60 is $\sqrt[3]{60}$, and the ſecond root of it is $\sqrt{60}$, the index 2, being omitted; which index is always underſtood when a root is named or wrote without one. But if the power is expreſſed by ſeveral number with the ſign +, or −, &c. between them, then a line is drawn from the top of the radical ſign, over all the parts of it: ſo the third root of 60 + 14 is $\sqrt[3]{60+14}$, or of 60 − 14 is $\sqrt{60-14}$. And ſometimes roots are expreſſed like powers by a crooked line drawn over the number, with the reciprocal of the index of the root above it: ſo the ſquare root of 60 is $\overline{60}\rceil\frac{1}{2}$, the cube or third root of it is $\overline{60}\rceil\frac{1}{3}$: Alſo the third root of 60 + 14 − 6 is $\overline{60+14-6}\rceil\frac{1}{3}$; the fourth root of it is $\overline{60\times14-6}\rceil\frac{1}{4}$, and the *m* root of $\overline{60\times14-6\times2}\rceil^{3}$ is $\overline{60\times14-6\times2}\rceil\frac{3}{m}$.

RULE *for extracting the ſquare root of* integers or decimals, or both mixt together.

1. Divide the given number into periods of two figures each: Thus begin at the units place, and put a point

over it, alſo put a point over the top of every other figure, to the left hand in whole numbers, and to the right in decimals: ſo will the given number be pointed into ſo many periods, as there will be figures in the required root.

2. Find the greateſt ſquare that is contained in the firſt period, towards the left hand. Set the root in the quotient and ſubtract the ſquare from the figures of that period.

3. To the remainder bring down the two figures under the next point, for a reſolvend. This is always to be repeated.

4. Double the quotient for a diviſor, and ſee how oft it is contained in the reſolvend (except the laſt figure); and ſet the anſwer in the quotient, and alſo after the diviſor. This muſt always be repeated; for a new diviſor muſt be found for every figure.

5. Then multiply the whole diviſor by that quotient figure, and ſubtract the product from the whole reſolvend; but if that product be greater, a leſs gure muſt be placed in the quotient. Thus go on till all the figures or periods are brought down, but if there be a remainder bring down cyphers two at a time, and ſo carry the root into decimals at pleaſure.

Note, Inſtead of doubling the quotient every time for a diviſor, you may always add the laſt quotient figure to the laſt diviſor, for a new diviſor, and proceed as before.

EXAMPLES.

1. Extract the ſquare root of 32176552,863844 and the operation will be as follows.

root

```
32176552,863844(5672,438
25
--------
     106 | 717
      +6 | 636
     ---------------
    1127 | 8165
      +7 | 7889
     ---------------
   11342 | 27652
      +2 | 22684
     ----------------
  113444 | 496886
      +4 | 453776
     ------------------
 1134483 | 4311038
      +3 | 3403449
     -------------------
11344868 | 90758944
         | 90758944
     -------------------
```

The proof is, to multiply the root by itſelf, and add the remainder; which muſt be equal to the number to be extracted, if the work be right.

Thus 5672,438 × 5672,438 = 32176552,863844 the given number.

Note, In large operations the work may be much abbreviated thus, when one more than half the figures of the root are found, by the common method; all the reſt may be found as truly, by dividing the laſt dividend by the laſt diviſor, annexing a cypher to every dividual, as in diviſion of decimals; or rather, without annexing cyphers, by omitting continually the right hand figure of the diviſor, after the ſame manner of the third contraction in diviſion of decimals, in page 170.

Ex. 2. Extract the ſquare root of 0,002468.

0.002468(0.049678969394 the root near.
16
89) 868
+9 801
986) 6700
+6 5916
9927) 78400
+7 69489
99348) 891100
+8 794784
993569) 9631600
8942121
689479
596141
93338
89420
3918
2981
937
894
43
40
3

Here having by the rule found the firſt ſix figures, I find the five laſt by plain diviſion : only it may here be obſerved, that the laſt figure found by diviſion is moſtly a ſmall matter too large ; as 4, the laſt figure in this Example by the rule wou'd but come out 3.

3. What is the ſquare root of 3272869681 ?
Anſw. 57209.

4. What is the ſquare root of 787833.76?
Anſw.8 7,6

5. What is the ſquare root of ,001234?
Anſw .0351283361 nearly.

6. Extract the ſquare root of 3, to thirteen places?
Facit. 1.7320508075688

To extract the ſquare root of a Vulgar Fraction.

RULE.

1. Reduce the Fraction to its loweſt terms.

2d. When the terms of the given Fraction are both of them perfect ſquare numbers, extract the root out of the Numerator and Denominator, for the reſpective terms of the root required.

3. When either of the terms of a propoſed Fraction hath not a perfect ſquare root, then reduce it to decimals, and proceed as before.

4. Mixt numbers may be either reduced to improper Fractions, and extracted as above, or you may reduce the Vulgar Fraction to a Decimal, and annex it to the whole number, then extract the root of the whole.

EXAMPLES.

1. What is the ſquare root of $\frac{36}{49}$? Anſw, $\frac{6}{7}$.

2. What is the ſquare root of $\frac{384}{864}$? Anſw. $\frac{2}{3}$

For $\frac{384}{864}$ in its loweſt terms is $\frac{4}{9}$ whoſe ſquare root is $\frac{2}{3}$

3. What is the ſquare root of 6225 $\frac{21}{100}$? Anſw. 78.9

4. What is the ſquare root of 85$\frac{1}{1}\frac{4}{1}$? Anſw. 9.26 &c.

Ap.

Application of the Square root.

To find a mean proportional between any two given numbers.

Rule. Multiply the two numbers together, and extract the ſquare-root of the product.

Ex. 1. What is the mean proportional between 4 and 9?
$\sqrt{9 \times 4} = 6$ the Anſw.

2. What is the mean proportional between 49 and 64? Anſw. 56

3. What is the diameter of a circle equal in area to an ellipſis whoſe longeſt diameter is 768, and ſhorteſt 48? Anſw. 192.

To find a ſide of the ſquare equal in area to any given ſuperficies.

Rule. Extract the ſquare-root of the given area, for the ſide of the ſquare ſought.

Ex. 1. If the area of a circle be 1296, what is the ſide of an equal ſquare. Anſw. 36.

2. An army of 32400 men is to be formed into a ſquare battle; how many men muſt be in rank, and how many in file? Anſw. 180.

3. Suppoſe 12544 ſoldiers be diſpoſed into the form of an oblong, ſo that the number of men in rank may be 4 times as many as thoſe in file; how many muſt be placed in rank, and how many in file?

Firſt $12544 \div 4 = 3136$; whoſe ſquare root is 56 in file; and $12544 \div 56 = 224$ in rank.

4. A certain ſociety collect among themſelves a ſum amounting to 15l. 5s. 0¼d. every one contributing as many farthings as there were members in the whole Society: I demand the number of members. Anſw. 121 members.

Two

Two ſides of a right angled triangle being given to find the other ſide.

Case. 1. The two perpendicular ſides, or legs being given to find the other ſide, or hypothenuſe,

Rule. Square each ſide, add the two ſquares together, and the ſquare root of this ſum gives the hypothenuſe required.

Ex. Suppoſe a garriſon's wall the height of which is 18 feet, with a moat ſurrounding it, whoſe breadth is 24 feet, what muſt be the length of a ſcaling ladder, to reach from the outſide of the moat, to the top of the wall?

Note. The moat and wall are the two perpendicular legs of a right angled triangle, and the ladder the hypothenuſe therefore the ſquare of 18 added to the ſquare of 24 equal 900, whoſe ſquare root is 30 feet, the ladder's length.

Case 2. The hypothenuſe and either leg being given to find the other leg.

Rule. From the ſquare of the hypothenuſe ſubtract the ſquare of the given leg, and the ſquare root of the remainder will be the leg required.

Ex. A ladder whoſe length is 30 feet, being fixed on the brink of a moat, will reach the top of a wall on the oppoſite ſide whoſe height is 18 feet: from hence is required the breath of the moat.

From the ſquare of 30 take the ſquare of 18, and the ſquare root of the remainder is 24, the breadth of the moat.

If the length of the ladder and breadth of the moat be given to find the height of the wall, then from the ſquare of 30 take the ſquare of 24, and the ſquare root of the remained is 18.

A general rule for extracting the roots of all powers.

1. Prepare the given number for extraction, by pointing it into periods of two, three, four, five or ſix, &c. fi-

gures each, according as the index of the power directs; beginning at the units place and from thence proceeding to the left hand in whole numbers, and to the right in Decimals. By this means the number will be divided into ſo many periods as there are figures in the required root.

2. Enquire which is the greateſt cube, biquadrate, or 5th power, &c. in the firſt period, and the root of that power will give the firſt figure of the required root. Subtract the greateſt cube, biquadrate, or 5th power, &c from the firſt period, and to the remainder annex the firſt figure of your ſecond period, which will give your dividend.

3. Involve the root, or quotient figure, already found to a power leſs by unity than that whoſe root is ſought, × it by the index of the given power for a diviſor, by which divide the dividend, and the quotient will be the ſecond figure of the root required.

4. Involve the part of the root already found, to the power whoſe root is required for a ſubducend, and if that ſubducend be found equal to, or leſs than the two firſt periods of the given number, the ſecond figure of the root is right. But if it be found greater, you muſt diminiſh the ſecond figure of the root till the ſubducend be found equal to, or leſs, than thoſe periods of the given number.

5. Take the ſubducend from the figures of the two firſt periods, and to the remainder annex the firſt figure of the next period for a new dividend.

6. Find a new diviſor, and proceed thro' the whole given number in all reſpects as before, finding the third figure by means of the two firſt, as you found the ſecond by the firſt, and afterwards find the fourth figure (if there be a 4th period) after the ſame manner from the three firſt.

Note 1. After you have gone thro' the number propoſed, if there is a remainder, you may continue the operation by adding periods of cyphers to that remainder, and carry the root into decimals, to any degree of exactneſs.

2. In extracting the cube root, after you have got two or three figures in the quotient, (or root) you may bring down a

whole period, a figure at a time, and ſo get three places of figures more by diviſion, as in Example 2d: The ſame may be obſerved in other roots.

3. The extraction of roots is greatly expedited by obſerving what integer numbers multiplied together produce the index of the required root, and making ſuch extractions as are denominated by theſe numbers.

Thus, inſtead of the biquadrate root, extract the ſquare root twice, i. e. extract the ſquare root, and then the ſquare root of that root; inſtead of the ſixth, extract firſt the ſquare root, and then the cube root of that; inſtead of the eight, extract thrice the ſquare root; and inſtead of the ninth, extract twice the cube root.

EXAMPLES.

1. What is the cube root of 146363183?

146363183(527 the root of
125 = cube of 5.

$5^2\times3=75$) 213 = dividend

146363 = two firſt periods.
Subducend 140608 = cube of 52.

$\overline{52}|^2\times3=8112$)57551 = new dividend.

146363183 the three periods.
Subducend 146363183 = cube of 527.

0 remains.

EXPLANATION.

Here the greateſt root in the firſt period is 5, whoſe cube is 125, which taken from the 1ſt period leaves 21, to which annex 3 the firſt figure of the ſecond period and you have 213 for the dividend, and 3 times the ſquare of 5, =75 for the diviſor, which goes twice in 213, ſo ſet 2 after 5 in the root and you have 52, whoſe cube is 140608; which taken from the two firſt periods leaves 5755, to which bring down 1, the firſt figure of the third period, and you have 57551 for a new dividend, and 3 times the ſquare of 52=8112 for a diviſor, which goes 7 times in 57551, ſo ſet 7 after 2 in the root and it makes it 527, whoſe cube is 146363183 equal the given number: therefore 527 is the true cube root ſought.

2. What is the cube root of ,067507824239?

$,06\dot{7}50\dot{7}82\dot{4}23\dot{9}(,407178$ root.
,064 = cube of 4.

$4^2\times3=48)\ 35$ dividend

,067507 the two firſt periods
,064000 = cube of ,40

$\overline{40}|^2\times3=4800)35078$ dividend.
33600

,067507824 = 3 firſt periods
,067419143 = cube of ,407

$\overline{407}|^2\times3=496947)886812$ dividend
496947

3898653
3478629

4200249
3975576

224673

Here becauſe the cube of ,407 is ſo nearly equal to the three firſt periods, I take in three of the laſt figures by diviſion, but if this method in any caſe ſeems not to hold true, you may proceed according to the rule, viz. by cubing the root for every new figure annexed, &c.

3. What is the cube root of 2? Anſw. 1.2599, &c.

2.000000000000000 (1.2599
1 = cube of 1

$1\times3=3)10$ dividend
2 000 = two firſt periods

ſubduct 1,728 = cube of 12.

$\overline{12}\rceil^2\times3=432)2720$ dividend

2.000000 = the three firſt periods,
ſubducend = 1.953125 = the cube of 1.25

$\overline{125}\rceil^2\times3=46875)468750$

2.000000000 = the 4 firſt periods.
ſubducend = 1.995616979 = the cube of 1,259

$\overline{1259}\rceil^2\times3=4755243)43830210$ dividend.

4. What is the ſurſolid, or 5th root of 12309502009375

12309502009375(415 root.
$1024=4\rceil^5$

$\overline{4}\rceil^4\times5=1280)2069$ = the dividend

1230950020 = the two firſt periods:
ſubducend = 1158562001 = the 5th power of 41

$\overline{41}\rceil^4\times5=14128805)72388190$ = the dividend.

12309502009375 = the 3 periods.
ſubducend = 12309502009375 = $415\rceil^5$

5. What is the 7th root of 3829.86553955078125 ?

3829,86553955078125(3,25

$\overline{3}\rceil^{7}=2187$

$\overline{3}\rceil^{6}\times 7=5103)16428$ dividend

38298655395=two firſt periods.

$\overline{32}\rceil^{7}=34359738368$=the ſubducend.

$\overline{32}\rceil^{6}\times 7=7516192768)39389170275$ dividend

$\overline{325}\rceil^{7}=38298655395507812 5$ Subducend.

0 remains

6. What is the 9th root of

40452954761505126953125 ?

40452954761505126953125(325 root.

$\overline{3}\rceil^{9}=19683$

$\overline{3}\rceil^{8}\times 9)207699$ dividend

40452954761505=two firſt periods.

$\overline{32}\rceil^{9}=35184372088832$=ſubducend.

$\overline{32}\rceil^{8}\times 9)\ 52685826726731$ dividend.

$\overline{325}\rceil^{9}=40452954761505126953125$ Subducend.

0 remains

In dividing the dividend by its reſpective diviſor, the quotient figure will ſometimes come out too much, but in ſuch caſe, the ſubducend will be greater than the number of periods from which it is to be taken: therefore you may any time know how to diminiſh the quotient figure if it happens to be too large. In example 3d, the 1ſt diviſor 3, may be had 3 times in the dividend, 10, but then the ſubducend ariſing therefrom, viz. the cube of 13, would be greater than 2,000 the two firſt periods. And in example 5th the diviſor 5103 wou'd go three times in the dividend 16428; but then the 7th power of 33, would be greater than the two firſt periods.

7. What is the biqnadrate root of
1035330554199747·518002456336?

Firſt extract the ſquare root and it makes 32176552,863844. Again extract the ſquare root of 32176552,863844 and you'll have 5672,438 for the root required. See Note 3. page 234.

8. What is the biqnadrate root of
1680362,6204252162565? Anſw. 36,004.

9 What is the cube root of 46671553.728064? Anſw. 360,04

10. What is the cube root of 182519501173,877531672? Anſw. 5672,438.

11. What is the fifth root of 2? Anſw. 1,148699 &c.

12. What is the fifth root of 8349416423424? Anſw. 384,3 &c.

13. What is the ſixth root of 21035896,12735? Anſw. 16,61474 &c.

14. What is the ſeventh root of 12311715481324093441? Anſw. 384,42 &c.

To extract any root of a vulgar fraction.

R U L E.

1. If the given fraction be a complete power, i. e. have a finite root of the kind required, extract the root out of the *numerator* and *denominator*, for the terms of the root required.

2. But if the fraction is not a complete power, reduce it into a decimal, and then extract the required root.

3. Mixt numbers may either be reduced to improper fractions or decimals, and then extracted.

EXAMPLES.

1. What is the cube root of $\frac{27}{343}$? Anſw. $\frac{3}{7}$.

2. What is the cube root of $8\frac{4}{5}$? Anſw. 2,08

3. What is the cube root of $\frac{512}{729}$? Anſw. $\frac{8}{9}$.

4. What is the cube root of $\frac{6}{7}$? Anſw. .949 &c.

5. What is the cube root of $42\frac{7}{8}$? Anſw. $3\frac{1}{2}$.

The APPLICATION of the Cube Root.

PROBLEMS.

I. *To find the ſide of a cube that ſhall be equal in ſolidity to any given ſolid, as a cone, priſm, globe, cylinder, &c.*

Rule. Extract the cube root of the ſolidity for the ſide of the required *cube.*

Ex. If the ſolidity of a *cone priſm, globe,* or *cylinder* be 15625 inches, what is the ſide of a cube of equal ſolidity? The cube root of 15625 = 25 inches the Anſw.

¶ All ſimilar ſolids are in proportion to one another as the cube of their ſides, and all ſpheres as the cube of their diameters.

II. *The ſide of one cube being given, to find the ſide of another cube, that ſhall be 2, 3, 4 times, &c. greater or leſs.*

Rule 1. To make a cube any number of times greater, × the cube of the ſide by that number, and extract the cube root of the product.

2. To make a cube any number of times leſs, ÷ the cube of the ſide by that number, and the cube root of the quotient will be the ſide required.

Ex. 1. If the ſide of a cube be 12 inches, and its ſolidity 1728: required the ſides of ſix other cubes, whoſe ſolidity of the firſt three ſhall be 2, 3, and 6 times greater; of the latter three 2, 3, and 6 times leſs than the given cubes.

$\sqrt[3]{1728\times2}$ = 15,1 Alſo $\sqrt[3]{1728\times3}$ = 17,3 and $\sqrt[3]{1728\times6}$ = 21,8 the ſides of the greater cubes.

$\sqrt[3]{1728\div2}$ = 9 5 Alſo $\sqrt[3]{1728\div3}$ = 8,3 and $\sqrt[3]{1728\div6}$ = 6,6 inches, the ſides of the three leſs cubes.

Ex. 2. There is a ſphere, or globe whoſe diameter is 6 inches and ſolidity 113,097; what is the diameter of another ſphere whoſe ſolidity is 8 times greater than the former?

Multiply 113,097 by 8, extract the cube root of the product, and you'll have 9 6 the diameter ſought.

III. *To find two mean proportionals between two given numbers.*

Rule. Divide the greater number by the leſs, take the cube root of the quotient, which, ×ed by the leſs number, gives the leſs mean, × the ſaid cube root by the leſs mean, and the product will be the greater mean proportional.

Ex. 1. What are the two mean proportionals between 9 and 576?

Firſt 576÷9=64, whoſe cube root is 4. And 9×4=36 the leſs mean. And 36×4=144 the greater mean.
Proof 9 : 36 :: 144 : 576.

2. What are the two mean proportionals between 8 and 512? Anſw. 32 and 128.

¶ Like ſolids are in a triple proportion to their homologous, or like, ſides, diameters, lines, &c. Therefore the weight of globes are as the cube of their diameters, and the burthen of ſhips, &c. as the cube of their like dimenſions.

QUESTIONS.

1. If a bullet, whoſe diameter is 4 inches, weigh 9lb. what will a bullet of the ſame metal weigh, whoſe diameter is 8 inches?

Firſt 4×4×4=64, and 8×8×8=512.
Then as 64 : 9lb. :: 512 : 72lb. Anſw.

2. If a bullet whoſe diameter is 4 inches weighs 9lb. what is the diameter of another bullet of the ſame metal, whoſe weight is 72lb.? Here 4×4×4=64, the cube of the diameter. Then

As 9lb. : 64 :: 72lb. : 512 whoſe cube root is 8, the Anſw.

3. If the diameter of a globe be 1 inch, and the ſolidity thereof ,5236 of an inch; what is the ſolidity of another globe, whoſe diameter is 10 inches?

	cube D.		solid.		cube D.		ſolid.
As	1	:	,5236	::	1000	:	523,6 inches Anſw.

4. If the length of a ſhip's keel piece be 44 feet, depth of the hold 9 feet, and midſhip beam 20 feet, what muſt theſe dimenſions be in another ſhip of the ſame mould to carry a double burthen?

The cube of 44 ×ed by 2 = 170368, whoſe cube root is 55,44 feet for the keel's length ſought.

$$\text{Then as } 44 : 55{,}44 :: \begin{cases} 20 : 25{,}22 \text{ midſhip beam.} \\ 9 : 11{,}34 \text{ depth of the hold.} \end{cases}$$

Thus you ſee, having found any one of the dimenſions, the reſt may be had by the Rule of Three.

§ XXIII.

§ XXIV. *Arithmetical Progreſſion.*

ARithmetical Progreſſion, or Proportion, is when numbers do proceed by equal differences, either increaſing or decreaſing. Such a rank of numbers is ſometimes termed a ſeries. If the ſucceeding terms of a progreſſion exceed each other, it is called an aſcending progreſſion or ſeries; if the contrary, a deſcending ſeries.

As, 1. 2. 3. 4. 5. 6. 7. 8. &c. is an aſcending ſeries, increaſing by the continual addition of (the common difference) 1. And 8. 7. 6. 5. 4. 3. 2. 1. &c. is a deſcending ſeries, decreaſing by the continual ſubtraction of 1.

Note. The firſt and laſt terms of a progreſſion are called the extreams; and the other terms the means.

THEOREMS.

In any number of continued arithmetical proportionals:

1 *If the number of terms be even*; the ſum of the extreams, and that of every two terms equally diſtant from them, are equal.

2. *If the number of terms be odd*; then thoſe ſums are equal to the double of the middle term.

As ſuppoſe 2. 4. 6. 8. 10. 12 be an even number of terms, then $2+12=4+10=6+8$; and if 2. 4. 6. 8. 10. be an odd number, then $2+10=6\times2$.

In Arithmetical Progreſſion, theſe five things are to be obſerved, viz.

$$\text{The}\left\{\begin{array}{l}\text{Leaſt Term}\\ \text{Greateſt Term}\\ \text{Common Difference}\\ \text{Number of Terms}\\ \text{Sum of the Series}\end{array}\right\}\text{which put}\left\{\begin{array}{c}\\ \\ =\\ \\ \\ \end{array}\right\}\left\{\begin{array}{l}l.\\ g.\\ d.\\ n.\\ s.\end{array}\right\}$$

Any three of theſe terms being given, the other two are eaſily found. But not having room here to treat the whole at large, I ſhall therefore be obliged to expreſs the rules (or theorems) algebraically, only ſome of the moſt material in words.

The rules algebraically expreſſed (which in my opinion is far the beſt way of expreſſing them) may be eaſily underſtood, and applied, by a common Arithmetician; and turned into words if required.

PROBLEMS.

I. *Given one of the extreams, the common difference (d), and the number of terms (n) of an arithmetical ſeries; to find 1. The other extream. 2. The ſum of the ſeries (s).*

1. RULE. $\begin{cases} g=l+\overline{d\times n-1} \\ l=g-d\times\overline{n-1} \end{cases}$ That is, to or from the given term, according as it is the leaſt or greateſt, add or ſubtract the product of the common difference ×ed into 1 leſs than the number of terms, and the ſum or difference will be the term required.

2. RULE. $s=g+l\times\frac{1}{2}n$: That is, multiply the ſum of the extreams by half the number of terms, and the product will be the ſum of the ſeries.

EXAMPLES.

1. What is the greateſt term and ſum of an arithmetical ſeries, whoſe leaſt term (l) is 5, common difference (d) 3, and number of terms (n) 52?

1. RULE. $g=5+\overline{3\times51}=158$, the greateſt term.

2. RULE. $s=158+5\times\frac{1}{2}51=4238$ the ſum of the ſeries

2. Given the greateſt term (g) 128, the common difference (d) 4, and the number of terms (n) 31: to find (l) the leaſt term, and (s) the ſum of the ſeries.

1 RULE. $l=128-\overline{30\times4}=8$ the leaſt term.

2. RULE. $s=\overline{128}+8\times31\div2=2108$ the ſum of the ſeries.

3. If a debt can be diſcharged in 2 years, by paying 2s. the firſt week 5s. the ſecond, and ſo on, encreaſing every

week's payment by 3s. what is the laſt payment, and how much the debt? Anſw. The laſt payment will be 15l. 11s and debt 813l. 16s.

4. What is the greateſt term, and ſum of the ſeries $0+1+2+3+4$ &c. continued to 1000 places? Anſw. the greateſt is 999, and ſum of the ſeries 24975.

II. *Given the extreams* (l *and* g) *and number of terms* (n,) *to find*, 1. *the common difference* (d), 2. *the ſum of the ſeries* (s).

1. Rule. $d = g - l \div n - 1$: That is, divide the difference of the extreams by 1 leſs than the number of terms, the quote will be the common difference.

2. Rule. $s = l + g \times \frac{1}{2} n$: As the 2d. Rule Prob. I.

EXAMPLES.

1. Given the extreams 5 and 158, and the number of terms 52; what is the common difference, and ſum of the ſeries?

1. Rule. $d = g - l \div n - 1 = 158 - 5 \div 52 - 1 = 3$, the common difference.

2. Rule. $s = g + l \times \frac{1}{2} n = 4238$ the ſum of the ſeries.

2. Find three arithmetical means between 2 and 12.

Here the number of terms is 5; therefore $12 - 2 \div 4 = 2\frac{1}{2}$ the common difference.

Then $2 + 2\frac{1}{2} = 4\frac{1}{2}$, the 2d term; $4\frac{1}{2} + 2\frac{1}{2} = 7$, the 3d term; and $7 + 2\frac{1}{2} = 9\frac{1}{2}$, the 4th term: Therefore the required means are $4\frac{1}{2}$, 7, $9\frac{1}{2}$. And the ſeries, 2, $4\frac{1}{2}$, 7, $9\frac{1}{2}$, 12.

Note, Any arithmetical ſeries may be conſtructed, from any given term, and with any given difference, thus, for an aſcending ſeries, add the difference to the firſt term, the ſum will be the ſecond; add the difference to the ſecond, the ſum will be the third; and ſo on, continually adding the difference to the term laſt found for the next ſucceeding term. And for a deſcending ſeries, ſubtract continually the common difference for the ſeveral ſucceeding terms.

3. If the extreams be 8 and 128, and the number of terms 31; what is the common difference, and ſum of the ſeries? Anſw. The difference is 4 and ſum 2108.

4. What debt can be diſcharged in 2 years by weekly payments in Arithmetical Progreſſion, whereof the firſt term or payment is 2s and laſt 15l. 11s. and what is the common difference of the ſeries of payments?

Anſw. The difference is 4s. and the debt 831l. 16s.

III. *Given* l, g *(the extreams,) and* d *(the common difference,) to find* n *and* s.

1. Rule. $n=\overline{g-l+d}\div d$: That is to the difference of the extreams add the common difference; divide the the ſum by the common difference and the quote will be the number of terms.

2. Rule. $s=\overline{g+l}\times\overline{g-l+d}\div d$: i.e. $s=\overline{g+l}\times\frac{1}{2}n$.

EXAMPLES.

1. Given the leaſt extream (l) 5, the greateſt (g) 158, and common difference (d) 3; required (n) the number of terms, and (s) the ſum of the ſeries.

1. Rule. $n=\overline{g-l+d}\div d=\overline{158-5+3}\div 3=52$, the number of terms.

2. Rule. $s=158+5\times\frac{1}{2}52=4238$, the ſum of the ſeries.

2. If the extreams be 8 and 128, and the common difference 4. what is the number of terms and ſum of the ſeries? Anſw. The number of terms is 31, and the ſum is 2108.

3. What debt can be diſcharged, and in what time, ſuppoſing the firſt weeks payment is 2s. and the payments every week following to increaſe by 3s. till the laſt payment be 15l. 11s? Anſw. The debt is 831l. 16s. and will be diſcharged in two years.

IV. *Given* l, g; *and* s; *to find* n, *and* d.

1. **Rule.** $n = 2s \div g + l$: That is, divide twice the ſum, by the ſum of the extreams, the quote will be the number of terms.

2. **Rule.** $d = \overline{gg - ll} \div \overline{2s - g - l}$: Or having found n, then d, may be found as in Problem II.

E X A M P L E S.

1. If the extreams be 5 and 158, and the ſum of the ſeries 4238, what is the number of terms and common difference?

1. **Rule.** $n = 4238 \times 2 \div 158 + 5 = 52$ the number of terms.

2 **Rule.** $d = 158 - 5 \div 52 - 1 = 3$, the common difference.

2. If the extreams be 8 and 128, and the ſum of the ſeries 2108: what is the number of terms, and the common difference? Anſw. The number of terms is 31, and the common difference 4.

3. In what time will a debt of 813l. 16s. be diſcharged by weekly payments in Arithmetical Progreſſion, the firſt payment being 2s. and the laſt payment 15l. 11s. and what will the common difference of the ſeries be?

Anſw. The common difference will be 3s. and the debt will be diſcharged in two years.

V. *Given one extream, the ſum of the ſeries, and number of terms; to find the other extream, and common difference.*

1. **Rule.** $g = \overline{2s \div n} - l$, or $l = \overline{2s \div n} - g$.

2. $\begin{cases} d = 2s - 2nl \div nn - n, \text{ the leſs extream being given.} \\ d = 2ng - 2s \div nn - n, \text{ the greater ext. being given.} \end{cases}$

Or having both the extreams and d, the com. diff. may be found, by prob. 2. or 4.

EXAMPLES.

1. If the leaſt term be 5 (l), the number of terms 52 (n), and ſum of the ſeries 4238 (s); what is the greateſt term (g), and com. diff. (d)?

1. Rule $g=\overline{4238\times2\div52}-5=158$, the greateſt term.

2. Rule. $d=158-5\div51=3$ the common difference.

2. If the greateſt extream be 128 (g), the number of terms 31, and ſum of the ſeries 2108; what is the leaſt term, and common difference?

1. Rule. $l=\overline{2108\times2\div31}-128=8$ the leaſt extream, and $d=4$ the com. diff.

3. A debt of 813l. 16s. can be diſcharged in 2 years by weekly payments in Arithmetical Progreſſion, the leaſt term or payment being 2s. what will be the greateſt payment, and the com. diff. of the payments? Anſw. The com. diff. will be 3s. and the laſt payment 15l. 11s.

VI. *Given* d, n, *and* s; *to find* l, *and* g.

Rule. $l=\frac{s}{n}-\frac{n-1}{2}\times d$, and $g=\frac{s}{n}+\frac{n-1}{2}\times d$. That is, divide the ſum of the ſeries by the number of terms; then to and from the quotient, add and ſubtract half the product of the common difference into 1 leſs than the number of terms, and the ſum and difference will be the two extreams.

EXAMPLES.

1. If the common difference be 3, the number of terms 52, and the ſum of the ſeries 4238; what are the extreams?

Firſt $l=\frac{4238}{52}-\frac{52-1}{2}\times3=8$ the leſs extream.

Secondly $g=\frac{4238}{52}+\frac{52-1}{2}\times3=158$ the greateſt extr

2. If the common difference be 4, the number of terms 31, and the ſum of the ſeries 2108; what are the extreams? Anſw. 8, and 128.

3. If a debt of 813l. 16s. can be diſcharged in two years, by weekly payments in Arithmetical Progreſſion, whoſe common difference is 3s. what are the firſt and laſt payments? Anſw. 2s. and 15l. 11s.

VII. *Given one of the extreams, the common difference* (d) *and the ſum of the ſeries* (s) *to find the other extream and number of terms* (n.)

1. Rule.

$$\left\{\begin{array}{l} g = \sqrt{\overline{l \backsim \frac{1}{2}d}^{\,2} + 2ds} - \frac{1}{2}d \\ l = \frac{1}{2}d \pm \sqrt{\overline{g + \frac{1}{2}d}^{\,2} - 2ds} \end{array}\right\}$$

2. Rule.

$$\left\{\begin{array}{l} n = \dfrac{\sqrt{2sd + ll + \frac{1}{4}dd - ld} \pm \frac{1}{2}d \backsim l}{d} \\ n = \dfrac{\frac{1}{2}d + g \mp \sqrt{\overline{g + \frac{1}{2}d}^{\,2} - 2ds}}{d} \end{array}\right.$$

Or, having now both extreams and the common difference, the number of terms may be found by the firſt rule of Problem III.

Note, That $\backsim$ denotes the difference or exceſs of any two numbers or quantities.

EXAMPLES.

1. If the leaſt term be 5 (l,) the common difference 3 (d), and the ſum of the ſeries 4238 (d); what is the greateſt term, and the number of terms.

1. Rule. $g = \sqrt{\overline{5 - \frac{1}{2}3} \times \overline{5 - \frac{1}{2}3} + 4238 \times 2 \times 3} - \frac{1}{2}3 = 158$ the greateſt term.

2. Rule. $n = \dfrac{\sqrt{4238 \times 2 \times 3 + 5 \times 5 + \frac{1}{4} \times 3 \times 3 - 5 \times 3}}{3} - \dfrac{5 - \frac{1}{2}3}{3} = 52$ the number of terms.

Or, $n = \overline{158 - 5 + 3} \div 3 = 52$. See Rule 1. Prob. 3.

2. If the greateſt term be 128 (g), the com. diff. 4 (d), and the ſum of the ſeries 2108 (s); what is the leaſt term, and the number of terms?

1ſt. $l = \frac{1}{2}4 + \sqrt{\overline{128+\frac{1}{2}4} \times 128+\frac{1}{2}4 - 2108 \times 2 \times 4} = 8$ the leaſt term.

Secondly $n = \overline{128-8+4} \div 4 = 31$ the number of terms.

3. In what time will a debt of 813l. 16s. be diſcharged by weekly payments in Arithmetical Progreſſion, the firſt term or payment being 2s. and the common difference 3s. and what will the laſt payment be? Anſw. The laſt payment will be 15l. 11s. and the debt will be diſcharged in 2 years.

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§ XXV. *Geometrical Progreſſion.*

Geometrical Progreſſion, (or proportion continued), is when a ſeries of numbers do proceed by equal ratio's, that is, by one common ×er or ÷ſor, either increaſing or decreaſing; that is, either aſcending or deſcending.

As { 1, 2, 4, 8, 16, 32, &c. is an aſcending Geom. ſeries.
32, 16, 8, 4 2, 1, is a deſcending Geom. ſeries.

THEOREMS.

In any number of continued geometrical proportionals,

1. If the number of terms be even;
The product of the extreams, and that of every two terms, equally diſtant from them are equal.

2. If the number of terms be odd;
Then thoſe products are each equal to the ſquare of the middle term.

As 1, 3, 9, 27, 81, 243, 729, 2187, even.

Thus $1 \times 2187 = 3 \times 729 = 9 \times 243 = 27 \times 81$.
Or as 1, 3, 9, 27, 81, 243, 729, odd.
Thus, $27 \times 27 = 9 \times 81 = 3 \times 243 = 729$.

In Geometrical Progreſſion, theſe five things are to be obſerved, viz.

The	Firſt or leaſt term.	which put	=	a.
	Laſt or greateſt term,			z.
	Ratio (or com. ×er or ÷for)			r.
	Number of terms			n.
	Sum of the ſeries			s.

Any three of theſe terms being given, the other two are eaſily found.

PROBLEMS.

I. *Given one of the extreams (a or z), the ratio (r), and the number of terms (n) to find the other extream (z or a), and the ſum of the ſeries (s).*

1. Rule $\begin{cases} z = a \times r^{n-1}, \text{ when a is given.} \\ a = z \div r^{n-1}, \text{ when z is given.} \end{cases}$ That is, multiply or divide the given extream by ſuch power of the ratio whoſe index is one leſs than the number of terms, and the product or quotient will be the required term, according as it is the greater or leſs extream.

2. Rule. $\begin{cases} s = \overline{rz - a} \div \overline{r - 1} \\ s = \overline{z - a} \div \overline{r - 1} + z \end{cases}$ That is, × the greateſt term by the ratio, from the product ſubtract the leaſt term, then divide the difference by the ratio leſs 1, and the quotient will be the ſum of the ſeries. Or, divide the difference of the extreams by the ratio leſs 1; to the quotient add the greater extream, and it will give the ſum of the ſeries.

EXAMPLES.

1. Given the leaſt term 3 (a), the ratio 2 (r), and the number of terms 12 (n); required the greateſt term (z) and the ſum of the ſeries (s).

Here $r=2$ which being involved to the 11th power is 2048: viz. to a power leſs by 1 than the number of terms: then by the

1. Rule. $z=3\times2048=6144$ the greateſt term. And by the

2. Rule. $s=\overline{6144\times2-3}\div2-1=12285$ the ſum of the ſeries.

2. If the greateſt term be 1310720 (z), the ratio 4 (r), and the number of terms 10 (n); what is the leaſt term, and the ſum of the ſeries?

Here, $r=4$, whoſe 9th power is 262144, then by the

1. Rule. $a=1310720\div262144=5$ the leaſt term.

2. Rule. $s=1310720-5\div4-1+1310720=1747625$ the ſum of the ſeries.

3 What debt will be diſcharged in a year or 12 months, by paying 3s. the firſt month, 9s. the ſecond, 27s. the third, and ſo on, each ſucceeding payment being trebble the laſt; and what will the laſt payment be? Anſw. the debt is 39858l. and the laſt payment 26572l. 1s.

II. *Given* a, r, *and* z, *to find* s, *and* n.

1. Rule. $s=zr-a\div r-1$: the ſame as 2d Rule, in prob. I.

2. Rule. $n=\dfrac{\text{log.}z-\text{log.}a.}{\text{Log.}\,r}+1$: That is, divide the difference of the Logarithms of the extreams by the logarith of the ratio; add 1 to the quotient, and the ſum will be the number of terms. Or divide the greateſt term by the leaſt; find what power of the ratio is equal to the quotient; then add 1 to the index of that power, and the ſum will be the number of terms.

EXAMPLES.

I. *Given the extreams* 3 (a), *and* 6144 (z), *and the ratio* 2 (r); *required the ſum of the ſeries* (s), *and the number of terms* (n).

1. Rule $s = \overline{6144 \times 2 - 3} \div \overline{2 - 1} = 12285$ the ſum as before.

1. Rule.
Log. of 6144 is 3. 7884512
Log. of 3 is 0. 4771213
Log. of 2 is 0. 301300)3. 3113299(11
+1
The number of terms n. = 12

Or thus; 6144 ÷ 3 = 2072, which being continually divided by the ratio (2), the number of thoſe diviſions will be (11) the number of terms wanting one.

2. If the extreams be 5 and 1310720, and the ratio 4; what is the ſum of the ſeries, and the number of terms?
Anſw. The ſum is 1747625, and the number of terms 10.

3. What debt will be diſcharged by monthly payments in Geometrical Progreſſion, whereof the firſt is 3s. and the laſt 26572l. 1s. the ratio being 3; and in what time will it be diſcharged?

Anſw. The debt is 39858l. and will be diſcharged in a year.

III. *Given* r, s, *and one of the extreams; to find the other extream, and the number of terms* (n).

1 Rule.
$z = \overline{rs + a - s} \div r$; when the extream a is given.
$a = zr + s - rs$; when the extream z is given.

2 Rule.
$n = \log. z - \log. \overline{s + rz - sr} \div \log. r$: by having (z) the greater extream.
$n = \log. \overline{sr + a - s} - \log. a \div \log. r$: by having (a) the leſs extream.

Or, having found both the extreams, the number of terms may be found as in problem 2.

EXAMPLES.

1. If the leaſt term be 3 (a), the ſum of the ſeries 12285 (s), and the ratio 2 (r;) what is the greateſt term (z), and the number of terms (n)?

1. Rule. $z = 12285 \times 2, + 3 - 12285 \div 2 = 6144$ the greateſt term.

And the number of terms (found by prob. 2.) 12.

2. Given the greateſt term 1310720, the ſum of the ſeries 1747625, and the ratio 4; what is the leaſt term, and the number of terms?

1. Rule $a = 1310720 \times 4 + 1747625 - 1747625 \times 4 = 5$ the leaſt term. And the number of terms 10.

3. In what time will a debt of 132860 l. be diſcharged by monthly payments in Geometrical Progreſſion, whereof the firſt term is ½ l. and the ratio 3; and what will the laſt payment be? Anſw. The laſt payment will be 88573½ l. and the debt will be diſcharged in a year.

IV. *Given* a, n, *and* z; *to find* r, *and* s.

1. Rule. $r = \overline{z \div a}\rceil^{\frac{1}{n-1}}$: That is, ÷ the greater extream by the leſs, and extract ſuch root of the quotient whoſe index is equal to the number of terms leſs .

2. Rule. $s = \overline{\overline{z - a}, \div \overline{z \div a}\rceil^{\frac{1}{n-1}} - 1}, + z$: Or having found the ratio, the ſum of the ſeries may be found, as in problem 1.

EXAMPLES.

1. Given the extreams 3 (*a*), and 6144 (*z*), and the number of terms 12 (*n*); to find the ratio (*r*), and the ſum of the ſeries (*s*.)

1. Rule. $r = \overline{6144 \div 3}\rceil^{\frac{1}{11}} = 2$ the ratio: That is, $6144 \div 3 = 2048$, whoſe 11th root is 2.

2. Rule. $s = \overline{6144 - 3 \div 2}, + 6144 = 12285$ the ſum of the ſeries.

2. If the extreams be 5, and 32805, and the number of terms 9; what is the ratio, and the ſum of the ſeries?

Anſw. The ratio is 3; and the ſum 49205.

3. What debt can be diſcharged in a year by monthly payments in a Geometrical Progreſſion, whereof the firſt payment is 10s. and the laſt 88573.5l. and what will the ratio of the ſeries be?

Anſw. The ratio will be 3, and the debt 132860l.

V. *Given* a, z, *and* s; *to find* r, *and* n.

1. Rule. $r = s - a \div s - z$.

2. Rule. $n = \dfrac{\log.\ z - \log.\ a}{\log.\ \overline{s-a} - \log.\ \overline{s-z}} + 1$. Or, having found r, n may be found as in *Problem* 2.

E X A M P L E S.

1. Given the extreams 3 (a), and 6144 (z), and the ſum of the ſeries 12285 (s); to find the ratio, and number of terms.

1. Rule. $r = 12285 - 3 \div 12285 - 6144 = 2$ the ratio.
And 2dly $n = 12$ found by Problem 2.

2. If the extreams be 5, and 32805, and the ſum of the ſeries 49205; what is the ratio, and number of terms? Anſw. The ratio is 3, and the number of terms 9.

3. In what time will a debt of 132860l. be diſcharged by monthly payments in Geometrical Progreſſion, the firſt payment being 10s. and the laſt 88573l. 10s. and what is the ratio of the ſeries? Anſw. The ratio is 3, and the debt will be diſcharged in a year.

VI. *Given* r, n, *and* s, *to find* a, and z.

1. Rule. $a = rs - s \div r^{n} - 1$.

2. Rule $z = \overline{r-1} \div \overline{r^{n}-1} \times sr^{n-1}$. Or, having found the leaſt term, the greateſt may be found by Problem 1.

EXAMPLES.

1. If the ratio be 2 [r], the number of terms 12 [n], and the ſum of the ſeries 12285 [s]; what are the extreams?

Here r^n = 12th power of 2, viz. 4096; alſo r^{n-1} = 2 involved to the 11th power, viz. 2048; then $\frac{r-1}{r^n-1}=\frac{2-1}{4095}=\frac{1}{4095}$: Then per

1. Rule. $a=\frac{1}{4095}\times 12285=3$ the leaſt term. And by the

2. Rule. $z=\frac{1}{4095}\times 12285\times 2048=6144$ the greateſt term.

2. Given the ratio 3, the number of terms 9. and the ſum of the ſeries 49205; what are the extreams?

Anſw. 5 and 32805.

Of decreaſing Geometrical Progreſſion.

IN *finite decreaſing progreſſion* the ſame rules will ſerve the like Problems, if the ſeries be inverted, ſo that the leaſt term be the firſt, and the greateſt the laſt. But in an infinite decreaſing progreſſion or ſeries, that is, when the number of terms are infinite, then ſhall (z) the laſt term, be equal to (o) nothing. For becauſe n, and conſequently r^{n-1} is infinite, $z=a\div r^{n-1}=0$, for it is leſs than any aſſignable number whatever. The ſum of ſuch a ſeries $s=a\times r\div r-1$; which is a finite ſum, though the number of terms be infinite.

PROBLEMS.

I. *Given the firſt term* [a], *and ratio* [r], *of an infinite decreaſing progreſſion, or ſeries, to find the ſum of the ſeries* [s].

Rule

I. Rule. $s = a \times r \div \overline{r - 1}$. That is, ÷ the product of the firſt term and ratio, by the ratio leſs 1.

EXAMPLES.

1. What is the ſum of the ſeries $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ &c. infinitely continued?

Here $s = a \times r \div r - 1 = 1 \times 2 \div 2 - 1 = 2$ the ſum of the ſeries required.

2. What is the ſum of the infinite ſeries ,666 &c. whoſe ratio is 10, and firſt term ,6?

Anſw. $s = ,6 \times 10 \div 9 = \frac{2}{3}$ the ſum required.

3. Suppoſe a body to move eternally in this manner, viz. the firſt hour 20 miles, the 2d 19 miles, the 3d $18\frac{1}{20}$, and ſo on in the ſame Geometrical Progreſſion whoſe ratio is $\frac{20}{19}$, how far will it move in an eternity? Anſw. 400 miles.

That is, a moveable body continuing its motion in that ratio eternally, would only run 400 miles, or more than any thing that is leſs than 400 miles.

II. *Given the ratio* [r], *and the ſum* [s] *of an infinitely deſcending ſeries; to find the firſt term* [a].

Rule. $a = \overline{rs - s} \div r$. That is, from the product of the ratio and ſum, ſubtract the ſum, and divide the remainder by the ratio.

EXAMPLES.

1. Given the ratio 5 [r] and the ſum $\frac{1}{4} = ,25$ [s] of an infinite ſeries; to find the firſt term (a).

Rule. $a = \overline{5 \times ,25 - .25} \div 5 = ,2$ *or one fifth* the firſt term.

2. What is the firſt term of the infinite ſeries whoſe ratio is 10, and ſum $\frac{2}{3}$? Anſw. $\frac{3}{5}$.

3. Suppoſe, that a body, if continued moving eternally, would move over 400 miles; and in ſuch manner,

that the ſpaces paſſed over at the end of each hour, are in a deſcending progreſſion, whoſe ratio is $\frac{20}{19}$; how much does it move the firſt hour? Anſw. 20 miles.

III. *Given the ſum* [s], *and firſt term* [a], *of an infinite deſcending ſeries, to find the ratio* [r].

Rule $r=s\div s-a$. That is, divide the ſum by the difference between the ſum and firſt term, for the ratio.

EXAMPLES.

1. Given the ſum $\frac{1}{4}$=,25 [s], and firſt term *one fifth*=.2 [a], of an infinite deſcending ſeries; to find the ratio [r].

Rule. $r=s\div s-a=.25\div .25-.2=5$ the ratio ſought.

2. What is the ratio of an infinite ſeries, whoſe firſt term is $\frac{3}{5}$, and ſum $\frac{2}{3}$? Anſw. 10

3. If a body, in an eternity, move over 400 miles, in ſuch ſort, that at the end of any equal intervals of time, the ſpaces moved over be in a geometrical progreſſion; what will the ratio of the ſeries be? Anſw. $\frac{20}{19}$.

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§ XXVI. *Of* PERMUTATION.

PERMUTATION is the changing or varying the order of things.

Rule. Multiply all the given terms continually one into another, and the laſt product will be the Anſwer.

Exam. 1. How often might a family of 7 perſons dine together, and be placed every day in a different poſition?

$1\times2\times3\times4\times5\times6\times7=5040$ days the Anſw. required.

Exam. 2. How many changes may be rung on 12 bells, and how long would they be ringing but once over, allowing 3 ſeconds to every round?

Anſw. 479001600 changes. The time 1437004800 ſeconds = 45 years, 27 weeks, 6 days, 18 hours.

§ XXVII. *Intereſt and Annuities.*

TO write all the *rules* in words at length, relating to the buſineſs of Intereſt and Annuities, would ſwell this Treatiſe beyond its intended limits: therefore I ſhall expreſs the rules (or Theorems) algebraically.

I. *Of a ſingle ſum of money paid either before, or after it is due.*

1. SIMPLE INTEREST.

For the definition, ſee page 98.

Let p, n, m, be put to repreſent the ſame things here as in page 98, and r = the ratio of the rate, or intereſt of 1l. for a certain time, as a year, &c. Then by proportion.

As { 1 l. : r :: p : pr, the intereſt of the principal (p) for a year
1 yr. : pr :: n : prn, the intereſt of p for the time n.

Then p + prn, = the amount or arrear at the end of the time n.

Hence we have theſe four Theorems.

1. $m = p + prn$, when p, r, n, are given.
By this Theorem Table I. was conſtructed.

2. $p = m \div \overline{rn+1}$, when m, r, n, are given.

3. $n = \overline{m-p} \div pr$, when m, p, r, are given.

4. $r = \overline{m-p} \div pn$, when m, p, n, are given.

Note. The ratio (r) or intereſt of 1l. for a year is found thus: as 100 l. is to its rate of intereſt ſo is 1l. to its rate of intereſt for a year.

Thus as 100 l. : 3 l. :: 1l. : ,03 = r, at 3 per cent.
100 l. : 3,5 :: 1l. : ,035 = r, at 3½ per cent.
100 l. : 4 :: 1l. : ,04 = r, at 4 per cent. &c.

2. COM-

2. COMPOUND INTEREST.

Let p, n, r, m repreſent the ſame as before, and put $x = 1 + r$ the amount of 1l. for any given time, as a year &c. at the rate r. Then x may be found thus, viz.

$$\text{As}\left\{\begin{array}{l} 100 : 104{,}5 :: 1 : 1{,}045 = x, \text{ at } 4\tfrac{1}{2} \text{ per cent.}\\ 100 : 105 :: 1 : 1{,}05 = x, \text{ at } 5. \text{ per cent.}\end{array}\right\}$$

and ſo on for any other rate of intereſt.

$$\begin{array}{l}\text{In geometrical}\\ \text{prop. continued.}\end{array}\left\{\begin{array}{l} 1\text{l} : x :: p : px = 1\text{ſt.}\\ 1\text{l.} : x :: px : pxx = 2\text{d.}\\ 1\text{l.} : x :: pxx : pxxx = 3\text{d}\end{array}\right\}\begin{array}{l}\text{year's}\\ \text{amount}\\ = m.\end{array}$$

And as 1l. : x^n :: p : px^n = nth, years amount (= m). That is any principal (p) or ſum of money ×ed by the amount of 1l. involved to a power whoſe index is denominated by the number of years, gives its amount. Whence we have theſe four theorems.

1. $m = p \times x^n$, or $m = \log.\ p + \overline{\log.}\ x \times n$, when p, n, x are given.

2. $p = m \div x^n$, or $p = \log.\ m - \overline{\log.\ x} \times n$, when m, n, x are given.

3. $n = x^n, \div\div\ x$, or $\log.\ n = \overline{\log.\ m - \log.\ p} \div x$, when p, n, x are given.

Note. $\div\div$ ſignifies a continual diviſion by x, (ſee Prob. 1ſt, in Geom. Prog.) and $x^n = m \div p$; the amount of 1l. at the given rate in the required time.

4. $x = \overline{m \div p}\,{}^{\frac{1}{n}}$, or $x = \overline{\log.\ m - \log.\ p} \div n$, when p, m, n, are given.

Note. Table II. is conſtructed by ſucceſſively involving to the 30th power theſe numbers, 1.03, 1.035, 1.04, 1.045, and 1.05, being the amount of 1l. for a year at theſe ſeveral rates of intereſt viz. 3, 3-half, 4, 4-half, and 5 per cent.

The ſeveral powers (or tabular numbers) are the amounts of 1l. for ſuch a number of years as is expreſſed by the index of the power to which the firſt years amount of 1l. is raiſed. Any ſum of money multiplyed by the amount of 1l. for any number of years, (found in the table) will give the amount of that ſum of money for the ſaid number of years The ſame as expreſſed by Theorem 1ſt. from which the other three are deduced.

QUESTIONS *to excerciſe the firſt Theorem, both of ſimple and compound Intereſt.*

1. If 575 l. (p), be put out to intereſt, what will it amount to in 7 years (n), at 4 per cent. *ſimple* or *compound Intereſt?*

Here $r=.04$ and $x=r+1=1.04$; then $m=p+prn$ $=575+575\times.04\times7=736$l. the amount required at ſimple intereſt. Again, $m=p\times x^{n}=575\times1.04)^{7}=756,66071$. Or 756l. 13s. 2½d. the amount required at compound intereſt.

By the Tables. Look into Table I. under 4 per C. and againſt 7 years you'll find 1.28 the amount of 1l. for 7 years at 4 per cent.

Then $m=575\times1.28=736$l. as before, And in Table II. you'll find the amount of 1l. for 7 years at 4 per cent. com. intereſt to be 1.31593; then 575×1.31593 $=756.65975$ nearly as before.

2. If 125l. be put out to intereſt, what will it amount to in 15 years, at 5 per cent. ſimple or compound intereſt? Anſw. At ſimple intereſt 218l. 15s. at compound 259l. 17s. 3d. 3.6q.

3. If 50l. 10s. 6d. be put out at intereſt, what will it amount to in 30 years, at 5 per cent. ſimple or compound intereſt? Anſw. At ſimple intereſt 126l. 6s. 3d. and at compound 218l. 7s. 3d. 3.37q.

QUESTIONS *to exerciſe the ſecond Theorem, both of ſimple and compound intereſt.* By the 2d Theorem (both ſimple and compound) diſcount is calculated.

1. If 736 l. (m) was to be paid 7 years hence (n) what is the preſent value (p), diſcounting at 4 per cent. ſimple intereſt?

By Tab. I. the amount of 1l. for the given time and rate is $1.28 = \overline{n \times r + 1}$; then $p = m \div \overline{n \times r + 1} = 736 \div 1{,}28 = 575$ l. the preſent value ſought.

2. If 756,65975 l (m), was to be paid 7 years hence (n). what is its preſent worth, diſcounting at 4 per cent. compound intereſt?

By Table II. the amount of 1 l. is 1.31593 (=1.04 raiſed to the 7th power) for the given time and rate; then $p = m \div x^n = 756.65975 \div 1.31593 = 575$l. the preſent worth ſought.

3. What principal put out for 15 years will amount to 218l. 15s. at 5 per cent. ſimple intereſt, and 259l. 17s. 3 d. 3. 6 q. at compound? Anſw. 125 l.

4. What principal put out for 30 years at 5 per cent. will amount to 126 l. 6 s. 3 d. ſimple intereſt and to 218l. 7s. 3d. 3. 37 qr. at compound? Anſw. 50l. 10s. 6d.

QUESTIONS *to exerciſe the third Theorem both ſimple and compound intereſt.*

1. In what time will 575l. (p), amount to 736l. (m), at 4 per cent. ſimple intereſt, and to 756.65975l. (m), at 4 per cent. compound intereſt?

Here $n = m - p \div pr = 736 - 575 \div 575 \times .04 = 7$ years at ſimple intereſt.

Again, is $x^n = m \div p = 756.65975 \div 575 = 1.31593$ the amount of 1l. which being continually divided by 1,04 (x) till nothing remains, the number of thoſe diviſions will be 7. Which $x^n \div x = 7$ years at compound. Or ſeek 1,31593 in Table II. col. 4. per C. and you'll find it againſt 7 years, the time ſought.

2. In what time will 125l. amount to 218l. 15s at 5 per cent. ſimple intereſt, and to 259l. 17s. 3d. 3.6 qr. at 5 per cent. compound intereſt? Anſw. In 15 years.

3. In what time will 50l. 10s. 6d. amount to 126l. 6s 3d. at ſimple intereſt, and to 218l 7s. 3d. 3.37 q at compound intereſt, the rate being 5 per cent each?
Anſw. 30 years.

QUESTIONS *to exerciſe the 4th Theorem, both of ſimple and compound intereſt.*

1. At what rate will 575l. (p), amount to 736l. (m), at ſimple intereſt, and to 756.659751 (m), at compound intereſt, in 7 years?

Firſt $i=m-p\div p\times n=736-575\div575\times7=.04$ the ſimple intereſt of 1l. and as 1l. : .04 :: 100l. : 4 the rate per cent.

2dly. x^n or $x^7=m\div p=756.65975\div575=1.31593$ the amount of 1l. for 7 years at compound intereſt; then

$x=\overline{m\div p}|^{\frac{1}{n}}=\overline{1.31593}|^{\frac{1}{7}}=1.04$ the amount of 1l. for a year at 4 per cent. Or thus, ſeek the number 1.31593 (m÷p) over againſt 7 years in Tab. II. and you'll find it under 4 per cent. the rate ſought.

2. At what rate will 125l. amount to 218l. 15s. at ſimple intereſt, and to 259l 17s 3d. 3.6 q. at compound intereſt, in 15 years? Anſw. At 5 per cent.

3 At what rate will 50l. 10s. 6d. amount to 126l. 6s. 3d. at ſimple intereſt, and to 218l. 7s. 3d. 3.37qr at compound intereſt in 30 years? Anſw. At 5 per cent.

II. *Of Annuities, Rents, or Penſions, &c. in Arrear.*

ANNUITIES, RENTS, or PENSIONS, &c. are ſeveral equal payments, due at ſeveral equal times, as yearly, half yearly, or quarterly: and when they remain unpaid for any number of payments, they are then ſaid to be in arrears.

1. *Allowing ſimple Intereſt.*

Put $a =$ *annuity, rent,* or *penſion*; $n =$ years, or number of payments it is forborn; $r =$ intereſt of 1l. for a year, &c. $m =$ whole arrear. 'Tis evident, $1 : r :: a : ar$, that is, the intereſt of a. at the rate r, per 1l. per annum is ar.

But in arithmetical progreſſion.
$$\left.\begin{cases} a \\ a+1\,ar \\ a+2\,ar \\ a+\overline{n-1}\times ar \end{cases}\right\}\text{is the}\left\{\begin{matrix} 1\text{ſt} \\ 2\text{d} \\ 3\text{d} \\ n\text{th} \end{matrix}\right\}\text{years amount }(m)$$

Therefore by the 2d Rule. Prob. 1. in Arithmetical Progreſ. the ſum of thoſe amounts is $2a+\overline{n-1}\times ar\times\frac{1}{2}n$, or the amount (m) of the annuity (a) at the nth years end. Whence we have theſe four Theorems.

1. $m = \overline{2a+\overline{n-1}\times ar}\times\frac{1}{2}n$, when a, n, r, are given.

Note, By this Theorem, Table III. was conſtructed.

2. $a = 2m\div\overline{2+nr-r}\times n$, when m, n, r, are given.

3. $n = \sqrt{\overline{2m\div ra},+zz}\;-z$; (putting $\frac{1}{r}-\frac{1}{2}=z$), when m, r, a, are given.

4. $r = 2m-2na\div an^2-an$, when m, n, a, are given.

2. *Allowing Compound Intereſt.*

Let a, n, r, m, repreſent the ſame as before, and let $x=1+r$, the amount of 1l. for a year. Now ſince the amount of 1l. in n years is x^n, its intereſt for that time will be x^n-1; but for a ſingle year is $x-1=r$. Therefore ſay, as the intereſt of 1l. for a year : is to its intereſt for the given time :: ſo is the annuity : to the amount. Thus, as $x-1 : x^n-1 :: a : m$. Therefore, by ×ing means and extreams, $m\times\overline{x-1} = \overline{x^n-1}\times a$. Whence we have theſe four Theorems.

1. $m=\overline{ax^n-a}\div r$; when a, n, r, are given. (For $r+1=x$.)

Note, By this Theorem Table IV. was conſtructed.

2. $a=m\times r\div\overline{x^n-1}$, when m, r, n, are given.

3. $x^n=\overline{m\times r\div a,+1}$; or $n=\log.\overline{\overline{rm+a}}-\log.a\div\log.x$, when m, r, a, are given.

4. $\overline{m\div a,\times x}-x^n=\overline{m-a}\div a$, when m, a, n, are given.

From the *equation* above, x may be found, and then r. But this muſt be done by an Algebraic proceſs. Or, by help of Table. IV.

QUESTIONS *to exerciſe Theorem 1ſt: both of ſimple and compound Intereſt.*

1. If 60l. yearly rent (a), be forborn 10 years (n), what will be in arrear (m) at that time, at 4½ per cent. ſimple and compound intereſt?

1. $m=\overline{60\times2+\overline{9\times60\times,045\times5}}=721,5$l. the amount at ſimple intereſt.

Or, thus $\overline{n^2-n},\div2,\times a\times r=\overline{10\times10-10}\div2\times1\times,045=2,025$l. the ſimple intereſt of 1l annuity for 10 years, to which add 10l. the 10 years rent, and the ſum is 12,025l. the amount of 1l. annuity for 10 years at 4½ per cent. as in Table III. which was in this manner conſtructed. Then $12,025\times60=721,5$l. $=721$l. 10s. the amount.

2dly. $r=,045$ and $x=1,045$, whose 10th power is 1,55297

Then $m=\overline{ax^n-a}\div r=\overline{1,55297\times60-60}\div,045=737,2933$l. the arrear ſought, at compound intereſt

Or thus, $\overline{1,55297\times1-1}\div,045=12,28821$l. the amount of 1l. annuity for 10 years at 4½ per cent. compound intereſt, the ſame as againſt 10 years under 4½ per cent. in Table IV. which in this manner was conſtructed. Then $12,28821\times60=737,2926$l. $=m$, the arrear as before, *nearly*.

2. If 125l. yearly rent be forborn 16 years, what will be in arrear at that time, at $3\frac{1}{2}$ per cent. ſimple and compound intereſt?

By Table III. the amount of 1l. annuity for 16 years is 20,2l.; then $125\times20,2=2525$l. the arrear ſought, at ſimple intereſt.

By Table IV. the amount of 1l. annuity is 20,97103; therefore $20,97103\times125=2621,37875$l. the amount at compound intereſt.

3. If 320l yearly penſion be forborn 24 years, what will be in arrear at that time, at 5 per cent ſimple and compound intereſt? Anſw. 12096l. at ſimple intereſt, and 14240,64l. at compound.

4. What will an annuity of 50l. a year amount to in 30 years at 3 per cent. ſimple and compound intereſt? Anſw. 2152,5l at ſimple, and 2378,7705l. at common intereſt.

Note, If the rent or penſion, is payable half yearly or quarterly, the method of proceeding (in all the 4 caſes) will ſtill be the ſame, provided n be always taken to expreſs the number of payments, and r the intereſt of 1l for the time in which the firſt payment becomes due. The Tables being calculated for yearly payments, are of no uſe for half yearly and quarterly: and as the buſineſs of theſe payments at compound intereſt, is beſt performed by the Logarithms, I ſhall only give a few examples, in the ſeveral caſes, at ſimple intereſt.

5. If 30l. (a) half yearly rent be forborn 10 years, what will be in arrear at that time, at $4\frac{1}{2}$ per cent. ſimple intereſt?

Here $n=20$ the number of payments, and $r=,0225$ [$\frac{1}{2}$ of ,045],

Then, $m=\overline{30\times2+\overline{19\times30\times.0225}}\times10=728,25$ l. $=728$l 5s. the arrear ſought.

6. If 15l (a) quarterly rent be forborn 10 years, what will be in arrear at that time, at 4 per cent. ſimple intereſt?

Here $n=40$, and $r=,01125$ ($\frac{1}{4}$ of ,045), then

$m=\overline{15\times 2+\overline{39\times 15\times ,01125}}\times 20=731$l. 12s. 6d. the arrear ſought.

7. If 300l. half yearly penſion be forborn 5 years, what will it amount to at 4 per cent. ſimple intereſt?
Anſw. 3270l.

8. If 150l. quarterly penſion be forborn 5 years, what will it amount to at 4 per cent. ſimple intereſt?
Anſw. 3285l.

QUESTIONS *to exerciſe Theorem 2d, of ſimple and compound Intereſt.*

1. What yearly rent being forborn 10 years (n), will amount to 721,5l at ſimple intereſt, and to 737,2933l. at compound, at 4½ per cent?

1ſt. $a=721,5\times 2\div\overline{2+10\times ,045-,045}\times 10=60$l. the yearly rent, by ſimple intereſt. Or by Tab. III. The amount of 1l. annuity is 12,025; then $a=721,5\div 12,025=60$l. the rent as before.

2dly, $a=737,2933\times ,045\div\overline{1,045}\rceil^{10}-1=60$l. the rent, by compound intereſt. Or by Table IV. The amt. of 1l. annuity is 12,28821; then $737,2926\div 12,28821=60$ the rent, as before.

2. What yearly rent being forborn 16 years at 3½ per cent will amount to 2525l at ſimple intereſt, and to 2621,37875l. at compound? Anſw. 125l yearly rent.

3. What yearly penſion being forborn 24 years, at 5 per cent. will amount to 12096l. at ſimple intereſt, and to 14240,64l. at compound? Anſw. 320l. yearly.

4. What half yearly rent forborn 10 years, at 4½ per cent. will amount to 728,25l at ſimple intereſt?

$a=728.25\times 2\div\overline{2+20\times .0225-.0225}\times 40=30$l. the half yearly's rent.

5. What quarterly penſion forborn 10 years will amount to 731,675l. at 4½ per cent. ſimple intereſt?

$a = 731,675 \times 2 \div \overline{2 + 40 \times ,01125 - ,01125} \times 40 = 15$l. the quarterly penſion ſought.

6. What half yearly penſion forborn 5 years, will amount to 3270l. at 4 per cent. ſimple intereſt? Anſwer 300l. half yearly.

7. What quarterly annuity forborn 5 years, will amount to 3285l. at 4 per cent. ſimple intereſt? Anſw. 150l. quarterly.

Questions *to exerciſe Theorem 3d. of both ſimple and compound intereſt.*

1. In what time (n) will 60l. (a) yearly rent, at 4½d. per cent. amount to 721,5l (m) at ſimple intereſt, and to 737,2933l. (m) at compound?

First $z = \frac{1}{r} - \frac{1}{2} = 21,72$ &c. $zz = 471,845284$; then $n = \sqrt{\overline{721,5 \times 2 \div ,045 \times 60}, + 471,845284} - 21,72 =$ 10 years the time ſought: by ſimple intereſt. Or thus, ſay,

rent. amount rent amount

As 60l. : 721,5 :: 1 : 12,025l. the amount of 1l. annuity; which being found in col. 4½ per cent. Tab. III. ſtands over againſt 10 years the time ſought.

2dly $x^n = \overline{m \times r \div a}, + 1 = \overline{737,2933 \times ,045 \div 60}, + 1 = 1,55297$, which is ſuch a power of 1,045 (x), whoſe index is the number of years. Therefore 1,55297 (x^n) being continually divided by 1,045 (x) till nothing remains, the number of thoſe diviſions will be n = 10, the years ſought; by compound intereſt. Or, ſince 1,55297 is the amount of 1l. for the required time, at compound intereſt, ſeek it in Tab. II. col 4½ per C. and it ſtands over againſt 10 years the time ſought.

2. In what time will 125l. yearly rent. at 3½ per cent. amount to 2525l. at ſimple intereſt and to 2622,37875l. at compound intereſt? Anſw. In 16 years.

3. In what time will 50l. yearly penſion at 3 per cent. amount to 2152,5l. at ſimple intereſt, and to 2378,7705l. at compound ? Anſw. In 30 years.

4. In what time will 300l. (a) half yearly penſion, amount to 3270l (m), at 4 per cent. ſimple intereſt ?

Firſt r=,02, and $z = \frac{1}{r} - \frac{1}{2} = 49{,}5$, then per Theo. $n = \sqrt{\overline{\overline{3270 \times 2 \div 300 \times ,02,} + 49{,}5 \times 49{,}5}} - 49{,}5 = 10$ half years, =5 years the time ſought.

5. In what time will 150l. quarterly rent amount to 3285l. at 4 per cent. ſimple intereſt ?

Here r=.01 and $z = \frac{1}{r} - \frac{1}{2} = 99{,}5$ then per Theorem.

$n = \sqrt{\overline{\overline{3285 \times 2 \div 150 \times ,01,} + 99{,}5 \times 99{,}5}} - 99.5 =$ 20 quarters=5 years the time ſought.

6. In what time will 30l. half yearly penſion, amount to 728,25l. at $4\frac{1}{2}$ per cent. ſimple intereſt ? Anſw. in 10 years.

7. In what time will 15l. quarterly rent amount to 731,675l. at $4\frac{1}{2}$ per cent. ſimple intereſt. Anſw. in 10 years.

QUESTIONS *to exerciſe Theorem 4th, both ſimple and compound intereſt.*

1. At what rate will 60l. (a) rent, in 10 (n) years amt. to 721.5l (m) at ſimple intereſt, and to 737.2933l. (m) at compound ?

Firſt $r = \overline{721.5 \times 2 - 60 \times 10 \times 2} \div \overline{10 \times 10 - 1}, \times 00 =$,045, then $100 \times ,045 = 4\frac{1}{2}$ per cent. By ſimple intereſt.

Or ſay. as 60 rent : 721,5 amt. :: 1 rent : 12,025 the amount of 1l. annual rent Seek this number over-againſt 10 years in Tab. III. and it is found under $4\frac{1}{2}$ per C.

2dly. Say as 60 rent : 737,2933 amt. :: 1 rent : 12,28821 the amount of 1l annual rent at compound intereſt. Seek this number over-againſt 10 years in Table IV. and it ſtands under $4\frac{1}{2}$ per C. the rate ſought.

2. At what rate will 125l. rent amount to 2525l. at ſimple intereſt, and to 2621,37875l. at compound, in 16 years? Anſw. At $3\frac{1}{2}$ per cent.

III. *The preſent value of Annuities, &c.* Or

RULES for finding the diſcount, &c. in buying and ſelling of Annuities, Penſions and Leaſes in Reverſion, &c.

1. *Computed at ſimple Intereſt.*

Let $v =$ preſent value, $a =$ annuity, $n =$ time, $r =$ intereſt of 1l. Hence have theſe four Theorems; which ſolves the queſtions appertaining to this head.

1. $v = na + \overline{\tfrac{1}{2}n^2 - n} \times ar \div rn + 1$, when a, r, n, are given.

2. $a = \overline{2 + 2nr} \times v \div \overline{2 + nr - r} \times n$, when v, r, n, are given.

3. $n = \sqrt{\overline{2v \div ar}, + z^2} - z$; (putting $z = \frac{1}{r} - \frac{v}{a} - \frac{1}{2}$) when v, r, a, are given.

4. $r = 2na - 2v \div \overline{2v + a - na} \times n$, when v, n, a, are given.

2. *Computed at compound Intereſt.*

Put $x = r + 1$ the amount of the annuity for a year. Hence the four following Theorems will ſolve the queſtions appertaining to this head.

1. $v = \frac{a - \overline{a \div x^n},}{r}$ when a, x, n, are given.

2. $a = vr \div \overline{1 - 1 \div x^n}$, when, v, x, n, are given.

3. $x^n = a \div \overline{a - vr}$, or $n = \log.a - \log.\overline{a - vr} \div \log.x$, when v, x, a, are given.

4. $x^n + {}_{r}x^n - x^{n} + 1 = \frac{a}{v}$, when v, a, n, are given, from which equation x and r may be found. But this cannot be done but by an Algebraic proceſs; or by help of the Tables.

To find, by the Tables, the preſent worth of 1l. annuity to continue a certain number of years, at a given rate either ſimple or compound Intereſt.

Look into the Tables I. and III. for ſimple intereſt; or II. and IV. for compound intereſt. And under the given rate, and againſt the number of years (if within the limit of the Tables) in both Tables, you'll find two numbers, which take out and divide the latter by the former, for the preſent worth.

Ex. 1. What is the preſent worth of 1l. annuity to continue 14 years, at 5 per cent ſimple and compound intereſt?

Numb: Tab. I. is 1.7 Numb. Tab. III. is 18,55; then 18.55÷1.7=10.91176, the preſent worth at ſimple intereſt.

Numb. Tab. II. is 1.97993, Numb. Tab. IV. is 19,59863, then 19,59863÷1,97993=9,89865 the preſent worth at comp. int.

Now having found the preſent worth of 1l. annuity for any number of years at any given rate per cent. the preſent worth of any other annuity (whether ſimple or compound) may be found, thus;

As 1l. annuity : is to its preſent worth :: ſo is any other annuity : to its preſent worth.

QUESTIONS *to exercife Theorem 1ſt. ſimple and compound.*

1. A field is let upon leaſe for 18 (n) years, at 4l. (a) per annum, and the leſſee is deſirous to make preſent payment, provided the leſſer will allow him 5 per cent. how much muſt be paid down, at ſimple or compound intereſt?

1ſt. $v=\overline{4\times18+\tfrac{1}{2}\overline{18\times18-18}\times4\times,05}\div\overline{18\times,05+1}$ $=54$l. at ſimple intereſt.

Or thus, numb. in Tab. I. is 1, 9. numb. in Tab. III. is 25,65, then $25,65\div1,9=13{\cdot}51$ the preſent worth of 1l annuity for 18 years at 5 per cent. Now as 1l. : 13,5 :: 4l. : 54l, the preſent worth at ſimple intereſt as before.

2dly. $v=\overline{4-4\div2,40662}\div,05=46,7583$l. $=46$l. 15s. 2d the preſent worth at compound intereſt. Or thus, number in Tab. II is, 2,40662, numb. in Tab. IV. is 28,13238d then $28,13238\div2,40662=11,689581$ the preſent worth; of 1l annuity for the given time and rate at compound intereſt. Now as 1l. : 11,68958 :: 4l. : 46,75832l. the preſent worth as above.

2. What is the preſent worth of 100l. to continue 7 years, at 4 per cent. ſimple and compound intereſt?

Anſw. At ſimple intereſt 612l. 10s. and at compound intereſt 600l 4s.

3. How much preſent money muſt be paid for a field let for 18 years at 4l. per annum, half yearly payments, allowing 5 per cent. ſimple intereſt?

Here $a=2$l. $n=36$ payments, and $=,025$ then

$v=\overline{2\times36+\tfrac{1}{2}\overline{36\times36-36}\times2,\times025}\div\overline{36\times,025+1}=$ $54\tfrac{9}{19}$l. the preſent worth required.

4. What is the preſent worth of 600l. per annum, payable quarterly, for 5 years at 4 per cent. ſimple intereſt?

Anſw. 2737l. 10s.

Questions *to exerciſe Theorem 2d, ſimple and compound.*

1. What annuity to continue 18 years (n), will 54l. (v) ready money purchaſe at 5 per cent. ſimple intereſt, and 46·75832l. (v) at compound intereſt?

1ſt. $a=\overline{2+18\times 2}\times{,}05\times 54\div\overline{2+18\times{,}05-{,}05}\times 18=4$l. the year's rent required. Or by the Tables I. and III. find the preſent worth of 1l. annuity, which is 13,5l. then as 13,5l. : 1 :: 54l. : 4l. as above.

2dly. $r={,}05$, $x=1{,}05$ and $x^{18}=2{,}40662$ (ſee Tab. II. under 5 per C. and againſt 7 years), and $1-\overline{1\div 2{,}40662}={,}5845$ then $a=46{,}75832\times{,}05\div{,}5845=4$l. the yearly rent required, by compound intereſt.

Or by the Tab. II. and IV. find the preſent worth of 1l. annuity, which is 11,68958l.

pr. w. ann. pr. w.
Then as 11,68958 : 1 :: 46,75832 : 4 annuity.

2. What annuity (or yearly rent) to continue 7 years at 4 per cent. will 612,5l. ready money purchaſe, allowing ſimple intereſt, and 600,2l. allowing compound intereſt? Anſw. 100l. a year.

3. There is a field let upon leaſe for 18 years to come, payable half yearly, what is the half year's rent, the preſent worth at 5 per cent. ſimple intereſt being $54\frac{2}{19}$ l.? Anſw. 2l. the half year's rent.

4. What quarterly penſion to continue 5 years, will 2737l. 10s. ready money purchaſe, at 4 per cent. ſimple intereſt? Anſw. 150l. quarterly penſion.

QUESTIONS *to exerciſe Theorem 3d. ſimple and compound.*

1. How long may one have a leaſe of 100l. (a) a year, for 612,5l. (v) ready money at 4 per cent. ſimple intereſt, or for 600,2l. (v) at compound intereſt?

Here $r={,}04$, and $z=\frac{1}{r}-\frac{v}{a}-\frac{1}{2}=18{,}375$, then per Theo. $n=\sqrt{\overline{612{,}5\times 2\div 100\times{,}04}+18{,}375\times 18{,}375}-18{,}375=7$ years, the time required.

rent pre. w. rent
Or thus, as 100 : 612,5 :: 1 : 6,125l. the preſent worth of 1l. annuity for an unknown time. Then take

ſome year by gueſs, and find the amount by Tab. III. and the preſent worth of that amount by Tab. I If this agree not with 6,125l. try again, and by a few eaſy trials you'll come to the truth. In ſhort thus, ſet down the correſponding numbers in Tab. III. and I fractionwiſe, in order to approach continually to 6,125l. which at laſt you'll obtain.

Suppoſe 3 years $\frac{3{,}12}{1{,}12} = 2{,}6$ &c. too little.

5 years $\frac{5{,}4}{1{,}2} = 4{,}5$ too little.

7 years $\frac{7{,}84}{1{,}28} = 6{,}125$ juſt. So 7 years is the time required.

And in like manner you may proceed with Tab. IV. and II. for compound intereſt. Or as follows, by the Theorem.

$x^n = 100 \div \overline{100 - 600{,}2 \times {,}04} = 1{,}31593$ the amount of 1l. at compound intereſt. Seek this number in Tab. II. col. 4 per C. and you'll find it againſt 7 years, the time ſought.

2. If an annuity of 4l. a year, at 5 per cent. be ſold for 54l. ready money, at ſimple intereſt, or for 46,7583l. at compound intereſt, what is the time of its continuance?
Anſw. 18 years.

3. A leaſe of 4l. per an. payable halfyearly is ſold for $54\frac{2}{19}$l. at 5 per cent. ſimple intereſt, what is the time of continuance, or number of payments? Anſw. 18 years.

4. An annuity of 600l. per annum payable quarterly, is ſold for 2737l 16s. ready money; allowing 4 per cent. ſimple intereſt, what was the time of continuance?
Anſw. 5 years.

QUESTIONS *to exerciſe Theorem 4th. ſimple and compound.*

1. At what rate per cent. will an annuity of 100l. (a) a year produce 612,5l (v) preſent worth, at ſimple intereſt, and 600,2l. at compound, in 7 years (n)?

$r = \overline{100 \times 7 - 612.5} \times 2 \div \overline{612{,}5 \times 2 + 100 - 100 \times 7}| \times 7 = {,}04$, therefore the rate is 4 per cent. ſimple intereſt.

Again, ſay, as 100 ann. : 600,2 pr. :: 1 ann. : 6,002, the preſent worth of 1l. annuity, at an unknown rate of compound intereſt. Take ſome rate of intereſt by gueſs and find the amount for 7 years by Tab. IV. and the preſent worth of that amount by Tab. II. repeat this work with other rates, till the reſult be 6,002. Or in ſhort thus, ſet down the correſpondent numbers in Tab. IV. and II. fractionwiſe, and you'll approach to the rate ſought by a few trials. Thus,

Suppoſe 3 per cent. $\frac{7\ 6}{1{,}2} = 6{,}3$, too great.

$3\frac{1}{2}$ per cent. $\frac{7{,}77}{1{,}27} = 6{,}1$, too great.

4 per cent. $\frac{7{,}89829}{1{,}31593} = 6{,}002$, juſt.

Therfeore 4 per cent. is the rate required, by compound intereſt.

2. At what rate per cent. will 4l. yearly penſion produce 54l preſent worth at ſimple intereſt, and 46l. 15s. 2d. at compound intereſt, for 18 years? Anſw. At 5 per cent.

3. An annuity of 4l. per annum payable half yearly, having 18 years to come, is ſold for 54$\frac{2}{9}$l. what is the rate per cent ſimple intereſt? Anſw. 5 per cent.

Note, The queſtions relating to annuities, leaſes, &c. taken in reverſion, are ſolved (in all the 4 caſes) by the application of the Theorems relating to the intereſt of money, and thoſe concerning the preſent worth of annuities, both ſimple and compound. As for example (in the firſt caſe).

What is the preſent value of the reverſion of a leaſe of 100l. *per annum to continue* 7 *years, but not to commence till the end of* 5 *years, allowing* 4 *per cent. ſimple or compound intereſt?* By Theorem 1ſt. page 269.

$v = \overline{100\times7+\overline{7\times7-7}\div2\times100\times,04}\div1,+7\times,04 =$ 612,5l. the preſent worth of the annuity for 7 years, viz. the time of its continuance.

And by Theo. 2. (page 258) $p = 612,5\div\overline{5\times,04+1} =$ 510,416l. &c. the preſent value required; by ſimple intereſt.

Again, $r = ,04$; $x = 1,04$; $x^n = x^7 = 1,31593$; and $x^5 = 1,21665$; as found in Table II. Then by Theorem 1. page 269.

$v = \overline{100-100\div1,31593;}\div,04 = 600,2$l. the preſent value of the leaſe for 7 years, the time of its continuance. Now find what ſum put out for 5 years at 4 per cent. will amount to 600,2l. By Theo 2. (page 259.) $p = 600,2\div1,21665 = 493,346481$. the preſent value required; by compound intereſt.

I think it needleſs to give examples in the other three caſes, as it would but only be treading the old path over again, as may be ſeen by the example above.

IV. *The valuation of Freehold Eſtates.*

RULE.

WHEN Freehold Eſtates are to be valued; divide 1 by (r) the rate of 1l. the quotient ſhews how many years purchaſe it is worth, at compound intereſt.

Or if the annuity or rent be required; multiply the purchaſe money by (r) the rate of 1l. for the annuity.

EXAMPLES.

1. What is an eſtate of 60l. a year worth at 5 per cent?

Here $1\div,05 =$ 20 years purchaſe.

Or $60\times20 = 1200$l. the purchaſe money.

2. What annuity can I buy for 1200l. at 5 per cent? Here 1200, × ,05 = 60l. the annuity.

3. If 60l. (a) a year can be bought for 1200l. (v); what's the rate of intereſt (r)? Here $r = a \div v = 60 \div 1200 = {,}05$ and $,05 \times 100 = 5$, the rate per cent.

To find the value of a Freehold Eſtate, in Reverſion, at compound Intereſt.

Theorem 1. $v = \frac{a}{r} \div x^n$, when a, r, n, are given.

2. $a = v \times r \times x^n$, when v, r, and n, are given.

3. $x^n = a \div vr$, and $n = x^n \div\!\div x$. when r, a, v, are given.

4. $x^{n+1} - x^n = a \div v$. when n, a, v, are given. From which equation x may be found and then r: but this cannot be done, but by an algebraic proceſs.

Queſtion 1. If a freehold eſtate of 60,5l. (a) per annum, to commence 10 years hence, is to be ſold, what is it worth allowing the purchaſer 5 per cent. for preſent payment?

Here x^n or $x^{10} = 1{,}6289$, and $n = 10$, then per Theo. 1.

$$v = \frac{60{,}5}{{,}05} \div 1{,}62889 = 742{,}83715\text{l.}$$ the value required.

Queſtion 2. If a freehold eſtate to commence 10 years hence (a) is ſold for 742,83715l. (v), allowing the purchaſer 5 per cent. What is the yearly rent?

Theo. 2. $a = 742{,}83715 \times {,}05 \times 1{,}62889 = 60{,}5\text{l.}$ the yearly rent.

3. A freehold eſtate of 60,5l (a) per annum, being in reverſion, is ſold for 742,83715l. (v), at 5 per cent; how long muſt the purchaſer be before he enters upon it?

$x^n = 60{,}5 \div \overline{742{,}83715 \times {,}05} = 1{,}62889$, being ſuch a power of $x = 1{,}05$; whoſe index is the number of years: Therefore $x^n \div\!\div x = 10$ years, the time required.

SCHOLIUM.

It is contrary to law to let out money at compound intereſt. Yet in the valuation of annuities, it is always the cuſtom to allow compound intereſt; for by ſimple intereſt they would be over-valued.

TAB.

TABLE I.

A Table of the amount of 1l. for Years, at ſimple intereſt.

Years.	3. per C.	3½ per C.	4 per C.	4½ perC	5 per C.
1	1,03	1,035	1,04	1,045	1,05
2	1,06	1,070	1,08	1,090	1,10
3	1,09	1,105	1,12	1,135	1,15
4	1,12	1,140	1,16	1,180	1,20
5	1,15	1,175	1,20	1,225	1,25
6	1,18	1,210	1,24	1,270	1,30
7	1,21	1,245	1,28	1,315	1,35
8	1,24	1,280	1,32	1,360	1,40
9	1,27	1,315	1,36	1,405	1,45
10	1,30	1,350	1,40	1,450	1,50
11	1,33	1,385	1,44	1,495	1,55
12	1,36	1,420	1.48	1,540	1,60
13	1,39	1,455	1,52	1,585	1,65
14	1,42	1,490	1,56	1,630	1,70
15	1,45	1,525	1,60	1,675	1,75
16	1,48	1,560	1,64	1.720	1,80
17	1,51	1,595	1,68	1,765	1,85
18	1,54	1,630	1,72	1,810	1,90
19	1,57	1,665	1,76	1,855	1,95
20	1,60	1,700	1,80	1,900	2,00
21	1,63	1,735	1,84	1,945	2,05
22	1,66	1,770	1,88	1,990	2,10
23	1,69	1,805	1,92	2.035	2,15
24	1,72	1,840	1,96	2,080	2,20
25	1,75	1,875	2,00	2,125	2,25
26	1,78	1,910	2,04	2,170	2,30
27	1,81	1,945	2,08	2,215	2,35
28	1,84	1,980	2,12	2,260	2,40
29	1,87	2 015	2,16	2.305	2,45
30	1,90	2,050	2,20	2,350	2,50

TABLE II:

A Table of the amount of 1l. for years, at compound intereſt.

Years.	3 per C.	3½ per C.	4 per C.	4½ per C.	5 per C.
1	1,03000	1,03500	1,04000	1,04500	1,05000
2	1,06090	1,07122	1,08160	1,09202	1,10250
3	1,09273	1,10872	1,12486	1,14116	1,15762
4	1,12551	1,14752	1,16986	1,19252	1,21550
5	1,15927	1,18769	1,21665	1,24618	1,27628
6	1,19405	1,22925	1,26532	1,30226	1,34009
7	1,22987	1,27228	1,31593	1,36086	1,40710
8	1,26677	1,31681	1,36857	1,42210	1,47745
9	1,30477	1,36290	1,42331	1,48609	1,55132
10	1,34391	1,41060	1,48024	1,55297	1,62889
11	1,38423	1,45997	1,53945	1,62285	1,71034
12	1,42576	1,51107	1,60103	1,69588	1,79585
13	1,46853	1,56395	1,66507	1,77219	1,88565
14	1,51259	1,61869	1,73167	1,85194	1,97993
15	1,55797	1,67535	1,80094	1,93528	2,07893
16	1,60470	1,73398	1,87298	2,02237	2,18287
17	1,65285	1,79467	1,94790	2,11338	2,29202
18	1,70243	1,85749	2,02582	2,20848	2,40662
19	1,75350	1,92250	2,10685	2,30786	2,52695
20	1,80611	1,98979	2,19112	2,41171	2,65330
21	1,86029	2,05943	2,27877	2,52024	2,78596
22	1,91610	2,13151	2,36992	2,63365	2,92526
23	1,97359	2,20611	2,46471	2,75216	3,07152
24	2,03279	2,28333	2,56330	2,87601	3,22510
25	2,09378	2,36324	2,66583	3,00543	3,38635
26	2,15659	2,44596	2,77247	3,14068	3,55567
27	2,22129	2,53157	2,88337	3,28201	3,73345
28	2,28793	2,62017	2,99870	3,42970	3,92013
29	2,35656	2,71188	3,11865	3,58403	4,11613
30	2,42726	2,80679	3,24340	3,74532	4,32194

TAB-

TABLE III.

A Table of the amount of 1 l. annuity for years, at ſimp. intereſt.

Years	3 per C	3½ per C.	4 per C	4½ per C.	per C.
1	1,00	1,000	1,00	1,000	1,00
2	2,03	2,035	2,04	2,045	2,05
3	3,09	4,105	3,12	3,135	3,15
4	4,18	3,210	4,24	4,270	4,30
5	5,30	5,350	5,40	5,450	5,50
6	6,45	6,525	6,60	6,675	6,75
7	7,63	7,735	7,84	7,945	8,0
8	8,84	8,980	9,12	9,260	9,40
9	10,08	10,260	10,44	10,620	10,80
10	11,35	11,575	11,80	12,025	12,25
11	12,65	12,925	13,20	13,475	13,75
12	13,98	14,310	14,64	14,970	15,30
13	15,34	15,730	16,12	16,510	16,90
14	16,73	17,185	17,64	18,095	18,55
15	18,15	18,675	19,20	19,725	20,25
16	19,60	20,200	20,80	21,400	22,00
17	21,08	21,750	22,44	23,120	23,80
18	22,59	23,355	24,12	24,885	25,65
19	24,13	24 985	25,84	26,695	27,55
20	25,70	26,650	27,60	28,550	29,50
21	27,30	28,350	29,40	30,450	31,50
22	28,93	30,085	31,24	32 395	33,55
23	30,59	31,855	33,12	34,385	35,65
24	32,28	33,660	35,04	36,420	37,80
25	34,00	35,500	37,00	38,500	40,00
26	35,75	37,375	39,00	40,625	42,25
27	37,53	39,285	41,04	42,795	44,55
28	39,34	41,230	43,12	45,010	46,90
29	41,18	43,210	45,24	47,270	49,30
30	43,05	45,225	47,40	49,575	51,75

TABLE IV.

A Table of the amount of 1l. annuity for years, at comp. intereſt.

Ys	3 per C.	3½ per C.	4 per C.	4½ per C.	5 per C
1	1,00000	1,00000	1,00000	1,00000	1,00000
2	2,03000	2,03500	2,04000	2,04500	2,05000
3	3,09090	3,10622	4,12160	3,13702	3,15250
4	4 18363	4,21494	3,24646	4,27819	4,31012
5	5,30913	5,36246	5,41632	5,47071	5,52563
6	6,46841	6,55015	6,63297	6,71689	6,80191
7	7,66242	7,77941	7,89829	8,01915	8,14201
8	8,89233	9,05169	9,21422	9,38001	9,54911
9	10,15910	10,36849	10,58279	10,80211	11,02656
10	11,46388	11,73139	12,00611	12,28821	12,57789
11	12,80779	13,14199	13,48635	13,84118	14,20679
12	14,19203	14,60196	15,02580	15,46403	15,91713
13	15,61779	16,11303	16.62684	17,15991	17,71298
14	17,08632	17,67698	18,29191	18,93211	19,59863
15	18,59891	19,29568	20,02359	20,78405	21,57856
16	20,15688	20,97103	21,82453	22.71934	23,65749
17	21,76159	22,70502	23,69751	24,74171	25,84036
18	23,41443	24,49969	25,64541	26,85508	28,13238
19	25,11687	26,35718	27,67123	29,06356	30,53900
20	26,87037	28,27968	29,77808	31,37142	33,06595
21	28,67648	30,26947	31,96920	33,78314	35,71925
22	30,53678	32,32890	34,24797	36,30338	38,50521
23	32,45288	34,46041	36.61789	38,93703	41,43047
24	34,42647	36,66653	39.08260	41,68919	44,50200
25	36,45926	38,94986	41,64591	44,56521	47,72710
26	38,55304	41,31310	44,31174	47,57064	1,11345
27	40,70963	43,75906	47,08421	50,71132	54,66912
28	42,93092	46,29063	49,96758	53,99333	58,40258
29	45,21885	48,91080	52,96628	57,42303	52,32271
30	47,57541	51,62268	56,08494	61,00707	66,43885

§ XXVIII.

§ XXVIII. *The Rule of Falſe Poſition.*

THIS rule takes its name from the method made uſe of to diſcover the true anſwer to the queſtion propounded, viz from gueſſing, or the making a ſuppoſition of falſe numbers as if they were the true ones, and by their means diſcovering the true numbers ſought.

This rule conſiſts of two parts, ſingle and double: of which I ſhall treat diſtinctly, and

I. OF SINGLE POSITION.

Where one falſe poſition ſufficiently enables us to reſolve the queſtion propoſed by the following RULE.

Suppoſe ſome fit number, at pleaſure, and try according to the tenor of the queſtion whether your ſuppoſed number be true or falſe: if true no more's to be done; but if falſe, you muſt obſerve what the falſe reſult or concluſion is. Then ſay by the rule of Three. As the falſe concluſion : Is to the true number given :: So is the whole or any part of the falſe number : To the whole or reſpective part of the true number ſought.

Thus, let a=the abſolute number given, x= any ſuppoſed number; s= the ſum produced by x, when ordered according to the tenor of the queſtion.

Then s : a :: x : ax÷s=the number ſought.

QUESTIONS.

1. A man dying left 1000l. amongſt his three ſons A, B, and C; in ſuch proportion, that A's part ſhou'd be one ſixth of B's part, and B's part two thirds of C's part, what is the ſhare of each?

Suppoſe A to have 100l. then B muſt have 600l. and C 900l. by the tenor of the queſtion. Now 100+600+900 =1600l. the falſe concluſion.

			l.	s.	
Then 1600 : 1000 ::	100	:	62	10	A's ſhare.
	600	:	375	0	B's ſhare.
	900	:	562	10	C's ſhare.
	Proof		1000.		

2. There were 50 corporals, 40 ſerjeants, 36 enſigns, and 24 adjutants, who ſpent 380 ſhillings at a meeting; to which reckoning 5 corporals paid as much as 4 ſerjeants, 12 ſerjeants as much as 9 enſigns, and 6 enſigns as much as 8 adjutants; how much did each company pay?

Find 4 numbers to expreſs the above proportions.

Thus, 12 ſerj. : 9 enſ. :: 4 ſerj. : 3 enſ. = 5 corporals.
6 enſ. : 8 adj. :: 3 enſ. : 4 adj = 5 corp.

That is, 5 corporals paid as much as 4 ſerjeants, or 3 enſigns, or 4 adjutants. Suppoſe each company paid 1 ſhilling,

then 1 man in each company will pay $\frac{1}{5}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{4}$
which ×ed by the number of men 50, 40, 36, 24

produces 10, 10, 12, 6

and 10+10+12+6=38; then it will be

$$\begin{array}{l} \text{As } 38 : 380 :: \\ \text{Or } 1 : 10 :: \end{array} \left\{ \begin{array}{l} 10 : 100\text{s for the corporals.} \\ 10 : 100\text{s.} \text{——— ſerjeants.} \\ 12 : 120 \text{ ——— enſigns.} \\ 6 : 60 \text{ ——— adjutants.} \end{array} \right.$$

proof 380

3. What number is that which ×ed by 8, and the product ÷ed by 5, will leave the quote 48? Anſw. 30.

4. A man being asked what number of guineas he had about him, reply'd, if I had as many, half as many, and one quarter as many more, I ſhould have 220; how many had he? Anſw. 80.

5. A poulterer laid out 1l. 12s 6d. in fowls conſiſting of turkeys, geeſe and ducks; he gave 2s. 6d. a piece for the turkeys, 1s. 8d. a piece for the geeſe, and od. a piece for the ducks; moreover he had twice ſo many geeſe as turkeys, and thrice ſo many ducks as geeſe; how many had he of each ſort? Anſw. 3 turkeys, 6 geeſe, and 18 ducks.

II.

II. *Of Double Poſition.*

IN this rule we make two ſuppoſitions of falſe numbers, in order to find out the true number which anſwers the queſtion.

RULES.

1. Suppoſe *two* fit numbers, and proceed with each of them (as with the true one) according as the conditions of the queſtion directs; if either of the numbers thus arbitrarily taken, happens to ſolve the queſton, there needs no more to be done, but if not; find the errors, that is, find how much the reſults are different from the reſult in the queſtion, and obſerve whether they are in exceſs or defect.

When the errors are in exceſs, mark them with +, when in defect with —

Multiply alternately the firſt ſuppoſed number by the ſecond error, and the ſecond ſuppoſed number by the firſt error. And divide the ſum of the products by the ſum of the errors, when the errors are of different kinds, (that is, when one is greater and the other leſs than the truth;) or ÷ the difference of the products by the difference of the errors, when both errors are of one kind, (that is both too great, or both too little) and the quote will be the anſwer.

Thus, let P, p, be the two ſuppoſitions, E, e, their reſpective errors, when ordered according to the tenor of the queſtion; $\backsim$ the difference or exceſs; and $n=$ the number ſought. Then, $n=\frac{Pe+pE}{E+e}$, when the errors are of different kinds; or $n=\frac{Pe\backsim pE}{E\backsim e}$, when both errors are of one kind.

2. Or, having found the errors, ſay, as the ſum of the errors, if they are unlike, or the diff. if like ſigns : is to the diff. of the ſuppoſed numbers :: ſo is the leaſt error : to the correction of the ſuppoſition belonging to this error; which muſt be added to, or ſubtracted from it, according to the following directions.

Obſerve whether this be the leſs or greater ſuppoſed number, as alſo whether the errors have like or unlike ſigns.

If it is the leſs number, and the errors have like ſigns, ſubtract the correction; if unlike ſigns, add it.

If it be the greater number, and the errors have like ſigns, add the correction; if unlike ſigns, ſubtract it; and the ſum, or difference, will be the true number ſought.

Note. It is often of advantage to make 1 and 0 the two ſuppoſitions.

QUESTIONS.

1. What number is that which being ×ed by 21, the product increaſed by 28, and that ſum divided by 7, will leave the quote 120?

Firſt, ſuppoſe $40=P$, to be the number ſought; then $\overline{40\times21+28}\div7=124$, the reſult; but it ought to have been 120; therefore the error is $+4=E$, in exceſs.

Again, ſuppoſe $30=p$, to be the number ſought; then $\overline{30\times21+28}\div7=94$, the reſult; but it ought to have been 120; therefore the error is $-26=e$, in defect; the errors alſo are of different kinds.

Whence by 1ſt Rule, $n=\frac{Pe+pE}{E+e}=\frac{40\times26+30\times4}{4+26}$ $=38\frac{2}{3}$ the number ſought.

Or, 2dly thus, $26+4=30$ the ſum of the errors, $40-30=10$ the difference of the ſuppoſitions; and 40 is the greater ſuppoſed number, to which the leſs error (4) belongs; alſo the errors have unlike ſigns:

Then, as $30:10::4:1\frac{1}{3}$ the correction; therefore $40-1\frac{1}{3}=38\frac{2}{3}$ the number ſought, as before.

But to work this by the Note, ſuppoſe, firſt, $0=P$; then $\overline{0\times21+28}\div7=4$, the reſult; but ought to be 120; therefore $120-4=-116=E$, the error, in defect. Again, ſuppoſe $1=p$; then $\overline{1\times21+28}\div7=7$, the reſult;

ſult; but it ſhould have been 120; therefore the error is 113=e, in defect alſo, and the errors have like ſigns. Whence, by the firſt Rule,

$$n=\frac{Pe \backsim pE}{E \backsim e}=\frac{0\times113\backsim116}{113\backsim116}=\frac{116}{3}=38\tfrac{2}{3},$$ the number ſought.

And by the 2d. Rule, $116-113 : 1-0 :: 113 : 37\tfrac{2}{3}$ the correction; then $37\tfrac{2}{3}+1=38\tfrac{2}{3}$, the number ſought.

2. A labourer was hired for 90 days, on condition that he ſhould have 12d. for every day he wrought, and forfeit 8d. for every day he idled. At laſt he received 3l. 5s.=780d. for his labour. How many days did he work, and how many was he idle?

		d.
1. Suppoſe he wrought 50 (P) days.	$50\times12=$	600
Then he idled 40 days.	$40\times8=$	320
received but		280
inſtead of 3l. 5s.		780
1ſt. error (E) in defect.		—500
2. Suppoſe he wrought 80 (p) days,	$80\times12=$	960
Then he idled 10.	$10\times8=$	80
then he would receive		880
inſtead of		780
2d. error (e) in exceſs		+100

1. Rule. $$n=\frac{P\times e+p\times E}{E+e}=\frac{50\times100+80\times500}{50+100}=75$$ days he wrought.

2. Rule. As $500+100 : 80-50 :: 100 : 5$, the correction; then $80-5=75$ days he wrought, conſequently he idled 15 days.

3. Let a hogſhead be filled with two ſorts of wine; the one worth 6s. and the other 5s. per gallon; I demand how much of each ſort of wine the hogſhead was filled with, it being (when full) worth 16l. 14s.?

Anſw. 19 gallons at 6s. and 44 at 5s.

4. A gentleman would pay a debt of 480l. with guineas, each 21s. and jacobuſes, each 23s. and would pay 5 times ſo many guineas as jacobuſes; how many then of each ſort muſt he pay? Anſw. 375 gui. and 75 jacobuſes.

5. Two perſons A and B engage at play; A has 72 guineas and B 52 before they begin; and after a certain number of games won and loſt between them, A riſes with three times as many guineas as B: I demand how many guineas A won of B? Anſw. 21.

6. A man meeting a company of beggars, gave to each 4 pence, and has 16 pence over; but if he would have given them 6 pence a piece, he would have wanted 12 pence for that purpoſe: I demand the number of perſons?

Anſw. 14.

7. Two perſons A and B were talking of their money, ſays A to B, give me 5s. of your money, and I ſhall have juſt as much as you will have left; ſays B to A, rather give me 5s. of your money, and I ſhall have juſt three times as much as you will have left: how much money had each? Anſw. A had 15s. and B 25s.

8. A man being at play, loſt $\frac{1}{4}$ of his money, and then won 3s. after which he loſt $\frac{1}{3}$ of what he then had, and won 2s. laſtly, he loſt $\frac{1}{7}$ of what he then had; this done, he had but 12s. left: what had he at firſt? Anſw. 20s.

9. A trader maintained himſelf for 3 years at the expence of 50l. a year; and in each of thoſe years augmented that part of his ſtock, which was not ſo expended by $\frac{1}{3}$ thereof; at the end of the third year his original ſtock was doubled: what had he at firſt? Anſw. 740l.

10. A and B play at cards; A ſtakes B 8s. to 6s. every game. After 28 games they leave off play, and find that neither of them are winners: how many games did each win? Anſw. A won 16 games, and B 12.

11. Required a number, which ſeverally multiplied by 14, and 19; the difference of the two products may be 60? Anſw. 12.

12. If the number 18 be divided into two ſuch parts, that the difference of their ſquares may be 10 times the ſaid number 18, what are the parts required?
Anſw. 14 and 4.

13. A governor of a fort ſends out *one third* of his ſoldiers, and 25 over, and has then left ½ his ſoldiers and 100 over: how many ſoldiers had he at firſt? Anſw. 750.

CHAP. III.

§ I. *Geometrical Definitions.*

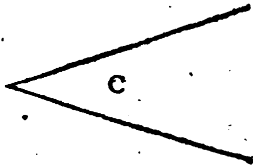
1. BY Geometry we learn how to compare things extended with one another, or to determine how one is more or leſs, or as much extended, as another. Or it is that ſcience wherein we conſider the properties of bodies as they conſiſt of three dimenſions, namely, length, breadth, and thickneſs; which may all be conceived to take their riſe from a point.

2. *A Point* is that in which we conſider neither length, breadth, nor thickneſs; and is abſolutely indiviſible.

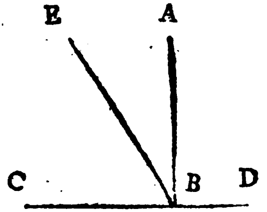
3. *A right line* (or ſtraight line) is length without breadth, being the neareſt diſtance between two points, or places, as AB.

4. *A curve line*, is that which lies bending between thoſe points which limit its length, as CD. There are various ſorts of curves. A line (whether ſtraight or curved) is generated by the motion of a point.

5. *A right lined angle* is that which is formed by the inclination of two right lines, meeting each other in a point, as C.

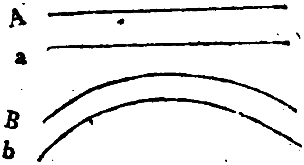


6. There are three ſorts of right lined angles.—1. When one right line A B ſtands any where upon another C D, ſo as to incline no more towards one end than the other, making the angles on both ſides A B equal, then thoſe angles are called right angles; and the two right lines, A B and C D, are then ſaid to be perpendicular to each other.—2. When the angle (EBD), is greater than a right angle (ABD), it is called an Obtuſe-angle.—3. If the angle (EBC) is leſs than a right angle (ABC), it is called an Acute-angle.



Note. When an angle is denoted by three letters (ABC), the middle letter ſtands at the angular point, and the other two at the extremeties of the lines which form the angle: Thus in the preceding definition, the letter B is the angular point of the right, obtuſe, and acute angles, there ſpecified.

7. *Parallel lines*, whether ſtraight or curved, are ſuch as are equally diſtant in all their parts, though infinitely extended; as Aa, or Bb.



OF PLANES, and SURFACES.

8. A *figure* is either a ſurface (viz. a ſuperficies) or a ſolid.

9. A *Plane ſurface* is any figure which lies evenly between its extreams, or bounds; and if thoſe extreams, or bounds, are right-lines, the figure is called a Rectineal (or right-lined) plane; but if the extreams, or bounds, of a plane, are crooked, or curve-lined, the figure is then called a Curvilineal ſurface, or plane. A ſurface may be conceived to be generated by the motion of a line.

OF SOLIDS.

10. A Solid is that which has three dimenſions, viz. length, breadth, and thickneſs:—The figure of a ſolid may be conceived to be generated either by the motion of a plane (or ſurface) in ſome certain direction, or by the revolution of a plane round ſome line as an axis.

11. The bounds or extreams of a ſolid, are either plain or curved ſurfaces.

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§ II. MENSURATION.

TO write a complete Treatiſe of Menſuration would make a Volume of itſelf: therefore my intent here, is to treat of ſuch things as are generally uſeful in common Buſineſs, with a deſign to give the Learner a ſmall inſight only, into this Affair, and to excite his curioſity to look into Authors that have treated this uſeful and delightful Subject more at large.

I. OF SUPERFICIAL MEASURE.

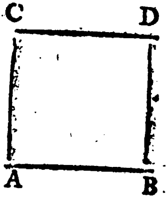
The Area, or Meaſure, of any plane ſurface, geometrically conſidered, is the whole Space contained, under the bounds of the figure, without any regard to thickneſs; as in the Menſuration of Land, Painters Work, &c. This Area, or Superficial Content of the ſpace, is computed from another ſpace, of a determinate form and magnitude; i. e. from a ſquare whoſe ſide is one inch, foot, yard, chain, &c. called the Meaſuring Unit, and the number of ſuch ſquares, or units (and parts of an unit), that are contained in any plane figure, is called the content or meaſure of that figure.

PROBLEMS.

I. *To meaſure a ſquare* ABCD.

Definition. A geometrical ſquare is a figure conſiſting of four equal ſides, and as many right angles.

RULE. Multiply the ſide of the ſquare by itſelf, and the product will be the area in the ſame meaſure the dimenſions were taken in.



Ex. 1. Suppoſe the ſide of the ſquare AB=50 inches; required its area in feet.

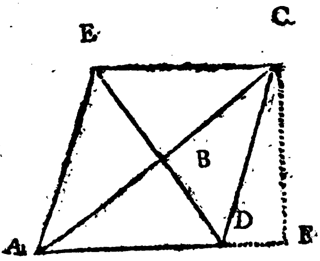
Operation. 50×50=2500 the area in inches; then 2500÷144=17,36 the area in feet.

2. Suppoſe the ſide of a ſquare be 21,269 inches, what is its area in feet? Anſw. 3.1413 feet.

2. *To meaſure a Rhombus*, ADCE.

Definition. A Rhombus has four equal ſides; and four angles, the oppoſite angles equal; two being obtuſe, and two acute.

Rule. Multiply the two diagonals (AC and DE) together, and half the product will be the area. Or × the ſide by a perpendicular let fall from an angle thereon (if within the figure, or on the ſide produced if without) and the product will be the area.



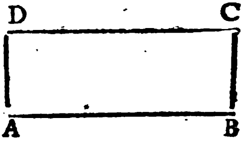
Ex. Suppoſe the diagonal AC=200, and ED=120 yards; required the area of the Rhombus.

Operation. 120×200÷2=12000 the area Or, ſuppoſe the ſide AD=114, and perpend. CF=105,2632;

then 105,2632×114=12000,0048 the area as before, nearly.

3. *To meaſure a Parallelogram.*

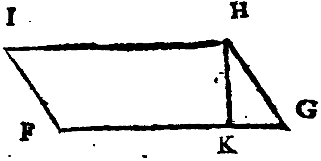
Definition. A parallelogram is a four ſided figure, having its oppoſite ſides equal and parallel.



Rule. If it be a rectangled parallelogram, as ABCD: multiply the longeſt ſide by the ſhorteſt; if otherwiſe as FGHI (called by ſome a Rhomboides); then × the longeſt ſide FG, by the perpendicular HK, and the product will be the area.

Ex. Suppoſe the longeſt ſide AB (or FG) =180, the ſhorteſt ſide BC (or perpend. HK) =60; required the area.

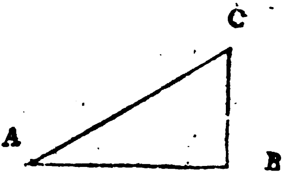
Oper. 180×60=10800 the area required.



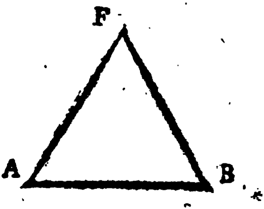
4. *To meaſure a Triangle.*

Definition. All three ſided figures are called triangles but admit of the following diſtinctions;

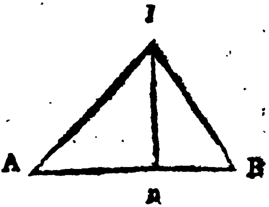
1. If it has a right angle it is called a right angled triangle as ABC, the two ſides AB, BC, containing the right angle, are called legs; the ſide AC oppoſite the right angle is called the hypothenuſe.



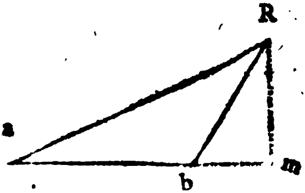
2. If the three ſides are equal it is called an equilateral triangle, as ABF.



3. If only two ſides are equal it is called an Iſoſceles triangle, as ABI.



4. If the three ſides are all unequal, it is called a ſcalene triangle, as abR. They are all meaſured by the ſame rule.



Rule. From any one of the angles let fall a perpendicular upon the ſide oppoſite (produced if needful); multiply the ſide and perpend. together, and half the product will be the area.

Ex. The baſe AB of the triangle ABC (or AIB or abR) is 90, and perpend. CB (or, In Rm) is 60; what is the area? Operation 90×60÷2=2700 the area required.

To find the area of a plane triangle by having the three ſides given. This is of great ſervice in Land Surveying.

RULE. From half the ſum of the three ſides, ſubtract each ſide ſeverally, let the ſaid half ſum, and the three differences, be ×ed continually; the ſquare root of the product will be the area required.

Ex.

Examp. If the ſides of a triangular garden be 41, 29, and 56 yards: what is the content of the garden?

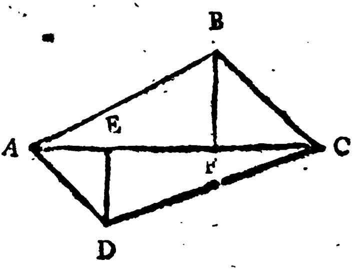
Operation. $41+29+56=126$ the ſum; $\frac{1}{2}$ ſum $=63$. And $63-41=22$, 1ſt difference, $63-29=34$, 2d diff. $63-56=7$, 3d diff. then $63\times22\times34\times7=329868$, the product, whoſe ſquare root is 574,34, the area required.

5. *To meaſure a Trapezium.*

Definition. A Trapezium is any figure that has 4 unequal ſides, as ABCD, in which the line AC is called a *diagonal*, BF and DE are perpendiculars. This Trapezium in fact is no more than two triangles ABC, and ADC, made by the diagonal AC.

Rule. Multiply the diagonal by the ſum of the perpendiculars, and half the product will be the area.

Ex. Suppoſe the diagonal $AC=120$, and the perpendiculars DE and BF to be 40 and 50 reſpectively; required the area.



Operation $\overline{50+40}\times120\div2=4590$ the area, required.

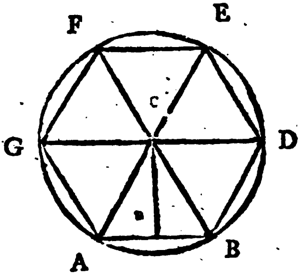
Note. The meaſure of any irregular figure (or polygon) may be eaſily found. Divide the whole figure into triangles, trapeziums, and curves; then find the area of each; add the areas together, and the ſum will be the area of the whole figure (or polygon). To find the area of a curved-ſpace. See page 55.

6. *To meaſure any regular Polygon.*

Definition. Any plane figure bounded by more than four right lines, is called a polygon; and is named according to the number of ſides it contains: Thus if it has 5 ſides it is called a pentagon; if ſix ſides, a hexagon; if ſeven, a heptagon, &c. If all the ſides and angles are equal, as in the figure (ABCDEFG), it is called a regular polygon; if otherwiſe, it is called an irregular polygon. Every regular polygon is compoſed of ſo many iſoſceles triangles as the polygon contains ſides.

Rule. Find the area of one of the triangles (by prob. 4), and multiply that area by the number of triangles: Or × half the ſum of the ſides by a line drawn from the middle of any one of the ſides to the center of the polygon, and the product will be the area.

Ex. In the hexagon ABCDEFG, if one of its ſides AB (BD &c.) be 56, the perpendicular Cn=48,49; what is the area?



Operation. 48,49 × 56 ÷ 2 = 1357,72 the area of one triangle; then 1357,72 × 6 = 8146,32 the area of the polygon.

Note. The radius of a circle inſcribed in any regular polygon is equal to the perpendicular of it, let fall from the center to the middle of the ſide of the polygon. Now by plane Trigonometry a Table may be calculated for the more ready meaſuring of regular polygons; which Table I ſhall here ſubjoin; expreſſing the angles at the center, length of the perpendicular (or radius of the circle inſcribed in it) and area.

The names of the Polygons.	Number of ſides.	The angle at the center	The perpendicular.	The area. The ſide of the Polygon being 1.
Pentagon	5	72 00	0,6881910	1,7204774
Hexagon	6	60 00	0,8660254	2,5980762
Heptagon	7	51 26	1,0382617	3,6339124
Octagon	8	45 00	1,2071068	4,8284271
Nonagon	9	40 00	1,3737387	6,1818242
Decagon	10	36 00	1,5388418	7,6942088
Undecagon	11	32 44	1,7028437	9,3656404
Duodecagon	12	30 00	1,8660254	11,1961524

The uſe of the preceding Table.

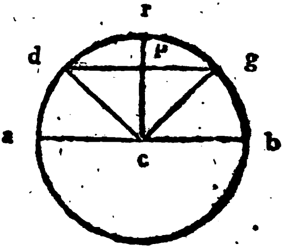
It is well known to Geometricians, that the areas of ſimilar (or like) plane figures are in proportion to one another, as the ſquare of their correſponding ſides: Therefore multiply the ſquare of the ſide of a given regular Polygon by ſuch a number taken out of the above table, as is agreeable to the name of the Polygon, and the product will be the area thereof, in the ſame denomination as the given ſide.

Ex. If the ſide of a regular Octagon be 12 feet; what is its area?

Operation. 4.8284271 × 12 × 12 = 695.2935024 the area.

7. *To Meaſure a Circle.*

Definition. A Circle is a plane figure, bounded by one continued line, called the circumference, or periphery; every part of which is equally diſtant from a point within the Circle, called its Center; from which, any right line (ac, cd, &c.) drawn to the circumference, is called the radius, or ſemi-diameter of the Circle; any right line ab, drawn thro' the center, terminating each way at the circumference, is called a diameter; a right-line dg, leſs than the diameter, meeting the circumference, in two points, is called a chord, or ſubtenſe; and the perpendicular diſtance rp, from the middle of the chord to the circumference is called a verſed-ſine.



Before I proceed to find the area of a Circle, it will be neceſſary to ſhew the learner, how to find the circumference of a Circle, by having its diameter given, and the contrary.

It is now looked upon, even by Mathematicians of the firſt rank, as abſolutely impoſſible to determine the exact proportion of the diameter and circumference of a Circle.

That great Geometer Archimedes, about 2000 years ago, firſt diſcovered this proportion to be nearly as 7 to 22 ; that is, if the diameter of a Circle be 7, its circumference will be 22, very nearly. But 22 is too much.

Metius Snellius diſcovered the proportion to be nearly as 113 to 335 ; which is nearer the truth than Archimedes, but 335 is too great.

Since Archimedes's and Metius's time, various methods have been invented, whereby the ſaid proportion may be approximated to a very great degree of exactneſs.

Van Ceulen (a Dutchman) found by incredible pains, that if the diameter of a Circle be repreſented by 1, the circumference thereof will be

3,14159265358979323846264338327950288, extreamly near. This laſt number was not only confirmed, but was extended to double the number of decimal places, by that ingenious and moſt indefatigable Mathematician, the late Mr. Abraham Sharp, of Little Horton, near Bradford, in Yorkſhire. And Mr. John Machin, Profeſſor of Aſtronomy in *Greſham College*, and Secretary to the Royal Society, has carried it to 100 places. But in the ordinary practice of meaſuring, it will be unneceſſary to take any more than 3,14159 (or 3,1416) : Hence it is evident, that if the diameter of any circle be multiplied by 3,1416, the product will be the circumference of that circle, very nearly.

Ex. What is the circumference of a circle, whoſe diameter is 50 ?

Operation. 3,1416×50=157,08 the circumference.

It is evident, from this example that if the circumference of a circle be divided by 3,1416, the quotient will be the diameter. For 157,08÷3,1416=50 the diameter.

To find the area of a circle, by having the diameter and circumference given.

Rule. Multiply half the circumference by half the diameter, and the product will be the area. For every circle is equal to a rectangled parallelogram contained under half the circumference and half the diameter.

Ex. What is the area of a circle whoſe diameter is 1, and circumference 3,1416?

Operation. ½ the circumference is 1,5708, and ½ the diameter is ,5 Then 1,5708×,5=,7854 the area of a circle whoſe diameter is 1, nearly.

Therefore if the ſquare of the diameter of any circle be multiplied by ,7854, the product will be the area, or meaſure of the circle in that denomination whereby the diameter was expreſſed, whether inches, feet, yards, &c.

As for inſtance, ſuppoſe the diameter of a circle be 50 feet, the ſquare whereof is 2500; then ,7854×2500= 1963,5 the area ſought, nearly.

The circumference of a circle being given to find the area.

Rule. Multiply the ſquare of the circumference by 0,0795776 ($=1\div 3{,}1416\times 4$) and the product will be the area.

Ex. If the circumference be 30, what is the area?

Operation. 0,0795776×900=71,61984 the area.

The area of a circle being given to find the diameter.

Rule. Multiply the ſquare root of the area by 1,12837 ($=2\sqrt{1\div 3{,}1416}$) and the product is the diameter.

Ex. What is the diameter of a circle whoſe area is 2500?

Operation. 1,12837×√2500=56,4185 the diameter.

The diameter of a circle being given to find the ſide of the greateſt inſcribed ſquare.

Rule. Multiply the given diameter by ,707 ($=\sqrt{\tfrac{1}{2}}$), and the product will be the ſide of the ſquare required.

Ex. If the diameter of a circle be 50; what is the ſide of the inſcribed ſquare? Anſw. 35,35

8. *To meaſure the ſector of a Circle.*

Definition. A Sector of a Circle *drgc*, is a figure, contained by an arch (or arc) thereof, and two radius's; when theſe two radius's form a right angle, or the arch becomes ¼th of the circumference, the figure is called a quadrant, as $adrc$, (or $ergb$, ſee the laſt figure).

Rule. Multiply half the length of the arch, by the radius, and the product will be the area.

Ex. Let $drgc$ repreſent a ſector (or quadrant) of a circle, whoſe radius dc (or gc) is 45 feet, and length of the arch drg, 70,686 feet; required its area.

Operation. $45\times\frac{1}{2}70,686=1590,435$, the area.

By this propoſition the area of the ſegment of a circle may be found: For if the area of the triangle dgc be ſubtracted from that of the ſector $drgc$, there will remain the area of the ſegment $drgd$. As for inſtance, ſuppoſe the chord dg =63,63 feet, and the perpendicular cp = 31,815. Then $63,63\times31,815\div2=1012,194225$ the area of the triangle dcg; which taken from 1590,435 (the area of the ſector) leaves 578,240775 the area of the ſegment ($drgd$). Or the area of the ſegment of a circle may be found by having the chord and verſed ſine given, by the following general rule.

Rule. Multiply the verſed ſine (or height) by ,626; to the ſquare of the product add the ſquare of half the chord: multiply twice the ſquare root of the ſum, by two thirds of the verſed ſine, and the product will be the area.

Ex. Required the area of the ſegment of a circle, whoſe chord is 48, and verſed ſine 18.

Opera. $18\times,626=11,268$, whoſe ſquare is 126,967824 which added to the ſquare of half the chord (576), makes the ſum 702,96782; twice the ſquare root of which, is 53,026; which ×ed by two thirds of the height (12), the product is 636,312 the area.

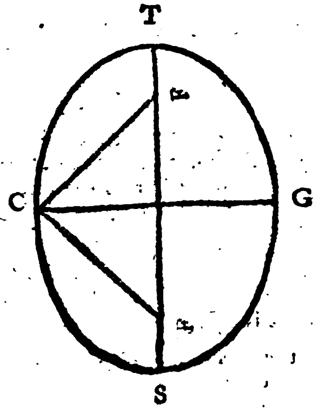
If the ſum of the ſquares of the ſemi-chord and verſed ſine be divided by the verſed ſine, the quote will be the diameter of the circle, to which that ſegment correſponds. As for inſtance, ſuppoſe the chord be 24, and the verſed ſine 8. Then $12\times12+8\times8\div8=26$ the diameter required.

By having the chord and verſed ſine you may find the length of the circular arch: thus, find the diameter (as above), divide two 3ds of the verſed ſine, by the diameter leſſened by ,82 of the verſed ſine; to the quote add 1, and × that ſum by the chord, the product will be the length of the arch very nearly. For inſtance, ſuppoſe the chord of a circular arch be 40, and the verſed ſine 10. Then $\overline{20\times20+10\times10}\div10=50$ the diameter, now $10\times,82=8,2$, which taken from 50 leaves 41,8 the diviſor; by which divide 6,666 &c, (two thirds of 10) and the quote is ,15948; to which add 1, and the ſum is 1,15948; this ×ed by 40 gives 46,3792 the length required.

9. *To meaſure an Ellipſis, or Oval.*

Definition. An Ellipſis, commonly called an Oval, is a plane figure bounded by one continued curve line which returns into itſelf like the circle, but has two different diameters, one longer than the other. The longer diameter (TS) is called the Tranſverſe, the ſhorter (CG) the Conjugate.

Rule. Multiply the tranſverſe by the conjugate, and × that product by ,7854; the laſt product will be the area.

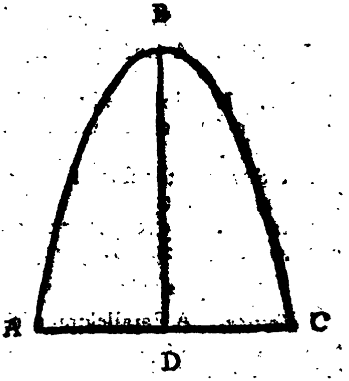


Exam. Suppoſe the tranſverſe (TS) = 70, and the conjugate (CG) = 50, what is the area?

Operation. $70\times50\times,7854=2748,9$, the area required.

10. *To meaſure a Parabola.*

Definition. A Parabola is a figure made by cutting a cone (ſee the definition of a cone farther on) by a plane parallel to one of the ſides: but the figure will give the learner a better idea than any definition.



Rule. Multiply the baſe (or ordinate AC) by the perpendicular (or abſciſſa BD); and two thirds of the product will be the area. For every conical parabola, is ⅔d of its circumſcribing parallelogram.

Ex. Let the baſe of the parabola, AC = 30, and perpendicular BD = 40; required the area.

Operation. 30 × 40 = 1200, two thirds of which is 800, the area.

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§ III. *Of* SOLID MEASURE.

THE meaſure of every ſolid figure is computed from another ſolid, of a determinate form and magnitude; namely, from a cube whoſe ſide is one inch, foot, yard, &c. called the meaſuring unit; and the number of ſuch cubes, or units, (and parts of an unit) that any ſolid is found to contain, is called the meaſure, or content, of the ſolid: therefore when the meaſure of any ſolid figure in any denomination is known, it may be changed into any other denomination by the rules in reduction.

Since all ſolid figures are but obſcurely delineated upon a plane-ſuperficies, I ſhall not lay down any ſchemes here to re-

preſent them; but rather adviſe the Maſter (who may have occaſion to teach this) to get each ſolid made of wood in its true form, to delineate and explain the figures intended by each problem, in order to aſſiſt his Pupil in a true conception of what he is about.

PROBLEMS.

1. *To meaſure a Cube (or Die).*

Definition. A Cube (or Die) is a ſolid having ſix equal ſides.

Rule. Multiply the length, breadth, and depth (which are all equal) into one another, and the product will be the ſolidity.

Ex. Suppoſe the ſide of a cube be 15 inches; required its ſolidity in feet.

Operation. $15\times15\times15=3375$ cubic inches: then $3375.00000\div1728=1,953125$ cubic foot.

2. *To find the ſolidity of a Parallelopipedon.*

Definition. A Parallelopipedon is a ſolid figure contained under ſix parallelograms, the oppoſite ſides of which are equal and parallel.

Rule. Multiply the length by the breadth, and that product by the depth (or altitude), the laſt product will be the ſolidity.

Exam. 1. The length of rectangled parallelopipedon is 72 feet, the breadth 30, and depth 20: required the ſolidity.

Operation. $72\times30\times20=43200$ the ſolidity required.

Exam. 2. What is the ſolidity of a piece of timber whoſe length is 30½ feet, breadth 18 inches, and depth 15 inches?

Operation. $30,5\times1,5\times1,25=57,1875$ feet the ſolidity.

3. *To find the ſolidity of a Priſm.*

Definition. A Priſm is a ſolid figure contained under ſeveral planes, its baſes for the moſt part are regular polygons, (moſt commonly triangular,) and is of an equal thickneſs thro' out.

Rule. Find the area of the baſe, or end, according to its form, multiply that area into the length, and the product is the ſolidity.

Exam. 1. What is the ſolidity of a triangular priſm, whoſe length is 28 feet, and one ſide of the equilateral end is 1½ foot?

Operation. 1,5 × 3 = 4,5 ſum of the ſides; ½ ſum 2,25 and 2,25 — 1,5 = ,75 the 1ſt diff. and becauſe the ſides are equal, the other two will be the ſame; then 2,25 × ,75 × ,75 × ,75 = ,94921875, whoſe ſquare root is ,9742 the area of the end or baſe; and ,9742 × 28 = 27,2776 feet the ſolidity required.

4. *To find the ſolidity of a Cylinder.*

Definition. A Cylinder is a round body, like a round column, or a rolling ſtone of a garden, whoſe baſes are equal circles.

Rule. Multiply the area of the circular baſe (or end) by the length, and the product will be the area.

Exam. What is the ſolidity of a Cylinder, whoſe length is 3¾ feet, and the diameter of the end is 2½ feet?

Operation. 2,5 × 2,5 × ,7854 = 4,90865 the area of the end, and 4,90865 × 3,75 = 18,4074375 feet the area.

5. *To find the ſolidity of a Pyramid.*

Definition. A Pyramid is a ſolid, which from a triangular or other figured baſe, diminiſhes gradually in thickneſs till it ends in a point, called the vertex.

Rule. Multiply the area of the baſe, or end, by the altitude, or length, and ⅓ of the product will be the area. For every Pyramid is equal one third of its circumſcribing priſm.

Exam. 1. What is the ſolidity of a pyramid, whoſe height is 15 feet, and one ſide of its hexagonal baſe is 1½ foot?

Operation. The area of an hexagon, whoſe ſide is 1, is found to be 2,598076 (by the Tab. of polygons). And 1,5 × 1,5 × 2,598076 = 5,845671, the area of the baſe. Then 5,845671 × 15 ÷ 3 = 29,228355, the ſolidity.

Exam. 2. What is the ſolidity of a pyramid, whoſe height is 72 feet; and the ſide of its ſquare baſe 18 feet?

Anſw. 8100 feet.

6. *To find the ſolidity of a Cone.*

Defin. A Cone is a ſolid, which, from a circular baſe, diminiſhes gradually in thickneſs till it ends in a point at top (or vertex) in the form of a round ſpire. A Cone may be conceived to be generated by the rotation of a right-angled triangle round one of its perpendicular legs.

Rule. Find the area of the cone's baſe (whether the diam. or circum. be given), multiply this area by the perpendicular altitude, and one third of the product will be the area. For every cone is equal to one third of its circumſcribing cylinder.

Exam. 1. What is the ſolidity of a cone, the diameter of whoſe baſe is 20 feet, and the altitude 60 feet?

Operation. 20 × 20 × ,7854 = 314,16 the area of the baſe. And 314,16 × 60 ÷ 3 = 6283,2 feet the ſolidity required.

Exam. 2. If the circumference of the baſe of a cone be 30 feet, and the height 50 feet: what is the ſolidity?

Operation. 30 × 30 × ,0795776 = 71,61984 the area of the baſe. And 71,61984 × 50 ÷ 3 = 119,3664 feet the ſolidity.

7. *To find the convex ſuperficies of a right cylinder, by having its length, and the diameter, or circumference given.*

Rule. Multiply the diameter by 3,1416, and the product by the length, will give the convex ſurface. Or, × the circumference by the length and the product will be the convex ſuperficies.

Exam. 1. What is the convex ſuperficies of a right cylinder whoſe diameter is 50 inches, and length 90 inches?

Operation. $3{,}1416 \times 50 \times 90 = 14137{,}2$ the convex ſuperficies.

Exam. 2. A cylinder is painted at 6d. per foot, whoſe length is 20 feet, and circumference 10 feet, what does it come to?

Operation. $20 \times 10 = 200$ the curve ſuperficies.
And $10 \times 10 \times {,}07958 \times 2 = 15{,}916$ the area of both ends.
Then, $200 + 15{,}916 = 215{,}916$ feet, the whole ſuperficies; which at 6d. per foot, amounts to 5l. 7s. 11½d. the Anſw.

8. *The length of the ſlant ſide and the diameter, or circumference of the baſe of a right cone, being given, to find the convex ſuperficies.*

Rule multiply the $\left\{\begin{array}{l}\text{diam. by } 1{,}5738\ (\tfrac{1}{2}\text{of } 3{,}1416)\\ \text{circum. by } 0{,}5\end{array}\right\}$ and the product ×ed by the length of the ſide, gives the convex ſuperficies.

Exam. 1. What is the convex ſuperficies of a right cone, the length of the ſlant ſide being 60, and the diameter of the baſe 22?

Operation. $1{,}5708 \times 22 \times 60 = 2073{,}456$, the convex ſuperficies.

Exam. 2. What is the convex ſuperficies of a right cone, whoſe length of the ſlant ſide is 50 feet, and the circumference of whoſe baſe is 60 feet?

Operation. $60 \times {,}5 \times 50 = 1500$ feet the convex ſuperficies.

9. *To find the ſolidity of the Fruſtum of a Cone (or pyramid of any kind.)*

Definition. The Fruſtum of a Cone, or pyramid, is what remains (towards the baſe) when the top is cut off by a plain parallel to the baſe.

Rule. To the ſum of the areas of the two ends of the fruſtum, add a geometrical mean between thoſe arrears, viz.

(the ſquare root of their product, ſee page 231), multiply this ſum by one third of the altitude of the Fraſtum, the product will give the ſolidity.

Note. This rule holds good, if the ends of the fruſtum be of any other figure, than that of a regular polygon.

Exam. 1. Suppoſe a ſquare pyramid's fruſtum, the ſide of whoſe greater baſe is 18 feet, the ſide of whoſe leſs is 6, and whoſe altitude is 60: What is the fruſtums ſolidity?

Operation. 6×6+18×18=360 the ſum of the areas of the two ends; and 36×324=11664, the product of theſe areas, whoſe ſquare root is 108, a geometrical mean; then 360+108=468 the ſum, which being ×ed by one third of the altitude (20) the product is 9360, the ſolidity required.

Exam. 2. What is the ſolidity of an hexagonal pyramid's fruſtum, the ſide of whoſe greater end is 3 feet, that of the leſs end 2 feet, and the length 12 feet.

$$\left.\begin{matrix}3\times3\\2\times2\end{matrix}\right\}\times2{,}598076\left\{\begin{matrix}=23{,}382684\text{ area of the greater end}\\=10{,}392304\text{ area of the leſs end.}\end{matrix}\right.$$

$$\sqrt{23{,}382\times10{,}392}=15{,}588000\text{ geom. mean}$$

$$49{,}362988\text{ ſum}$$

$$4=\tfrac{1}{3}\text{d the height.}$$

ſolidity required is 197,451952

The ſolidity of the fruſtum of a cone, may be found by the above general rule, but more expeditiouſly by the following rule.

When the diameter or circumference at each end, and the altitude are given.

Rule 1. Multiply the $\left\{\begin{matrix}\text{diam.}\\\text{circum.}\end{matrix}\right\}$ at one end, by that at the other.

2. To the product add the ſquare of thoſe $\left\{\begin{matrix}\text{diameters.}\\\text{circumfer.}\end{matrix}\right.$

3. Multiply the ſum by the given length.

4. The product ×ed by the No. $\begin{cases} 0{,}2618 \; (\tfrac{1}{3} \text{ of } ,7854) \\ 0{,}02656 (\tfrac{1}{3} \text{ of } ,0795775 \end{cases}$ will give the ſolidity.

Exam. 1. What is the ſolidity of the fruſtum of a cone, the diameter of the greater baſe being 20, that of the leſs 10, and the altitude 30?

Operation. $\overline{20\times10+20\times20+10\times10}\times30\times0{,}2618=$ 5497,8 the ſolidity required.

Exam. 2. What is the ſolidity of the fruſtum of a cone, the circumference of the greater end being 40, that of the leſs end 20, and the length or height 50?

Operation. $\overline{40\times20+40\times40+20\times20}\times50\times.026526$ $=3713{,}64$ the ſolidity required.

Exam. 3 There is a fruſtum of a cone (by ſome called a cylindroid), one end of which is a circle whoſe diameter is 26; the other parallel end is an ellipſis whoſe tranverſe diam. is 44, and conjugate 14; the altitude of the fruſtum is 9; required its ſolidity.

Operation. First reduce the ellipſis to a circle, and then proceed according to the rule; thus, $\sqrt{44\times14}=24{,}82$ nearly, the diam. of a circle,

Then $\overline{24{,}82\times26+26\times20+24{,}82\times24{,}82}\times9{,}2618=$ 4564,68981458 the ſolidity required.

10. *The ſolidity of the fruſtum of a rectangular pyramid (called a priſmoid), whoſe parallel ends are any diſſimilar rectangles, and whoſe ſides are four plane ſurfaces.*

The ſolidity of this figure may be obtained by the firſt of the preceding rules, or by the following.

Rule. To the longeſt (or ſhorteſt) ſide of the rectangle at either end, add that ſide at the other end (whether it be the length or breadth) which is parallel to it; multiply this ſum by the ſum of the other two parallel dimenſions (viz one at the top, and the other at the bottom), and to the product add the areas of the two ends; this total being multiplied by the height, one ſixth of the product will be the ſolidity.

Ex. Suppoſe there is a fruſtum of a pyramid, whoſe parallel ends are rectangles, the length at one end being 40 inches, and breadth 34; the length and breadth of the other end (which in this caſe, are reſpectively parallel to thoſe above) being 50 and 46 inches, and the length 60 inches; required its ſolidity.

Operation. $40+50=90$; $34+46=80$; and $90\times80=7200$ the product; $32\times40=1360$ the area of the leſs baſe, $46\times50=2300$ the area of the greater baſe, and $7200+1360+2300=10860$ the total;

then $10860\times60\div6=108600$ cubic inches.
1728)108600,00(84,97 feet.
282)108600,00(385,1 ale gallons.
231)108600,00(470,12 wine gallons.

11. *To find the convex ſurface of the fruſtum of a right cone, the length of the ſlant ſide, and the diameters or circumferences of the parallel ends being given.*

Rule. Multiply the ſum of the diameters by 1,5708, or the ſum of the circumferences by 0,5, and that product × by the length of the ſide, and the laſt product will be the convex ſurface.

Exam. 1. What is the convex ſurface of a right cone, the diameters of the ends being 8 and 4; and the length of the ſide 20?

Operation. $8+4=12$ the ſum of the diameters; and $12\times1{,}5708\times20=376{,}992$ the convex ſuperficies.

Exam. 2. What is the convex ſurface of the fruſtum of a right cone; the circumference of the greater end being 30 feet, that of the leſs end 10 feet, and the length of the ſlant ſide 20 feet?

Operation. $\overline{30+10}\times{,}5\times20=400$ feet, the convex ſurface.

12. *The dimenſion of a Pyramid or Cone being given, to find what length from the vertex will anſwer to a given part of the ſolidity.*

Rule. Say as the ſolidity of the whole pyramid or cone : is to the cube of the altitude :: ſo is any given part of the ſolidity : *to the cube of its altitude, reckoned from the vertex downwards; whoſe cube root is the length.*

Examp. There is a conical peice of timber, the diameter of whoſe baſe is 1,5 feet, and the length 12 feet; what diſtance from the vertex muſt a ſaw be applied, to cut it into two pieces of equal ſolidity.

Operation. 1,5 × 1,5 × 12 × ,2618 = 7,0686 feet the ſolidity; half whereof is 3,5343. And 12 × 12 × 12 = 1728 the cube of the length. Then as 7,0686 : 1728 :: 3,5343 : 864, whoſe cube root is 9,5244; and ſo far from the vertex muſt the ſaw be applied.

13. *To find the ſolidity of the Hoofs of the fruſtum of a pyramid, the linear meaſures of the end, and the length being known.*

Definition. If the fruſtum of a pyramid be cut through the extremities of both baſes by a diagonal plane, each part (or Section) is called a hoof.

If the fruſtum be of any other form than a ſquare pyramid, reduce it to ſuch, by finding the ſide of a ſquare of equal area; then

Rule. To the ſquare of the ſide of one baſe, add one half the product of the ſides of the two baſes; this ſum × by the height, and one third of the product will be the ſolidity.

Examp. Given x = one ſide of the greater baſe of a ſquare pyramid's fruſtum, y = one ſide of the leſs baſe, and a = the altitude, required the ſolidity of each hoof.

Operation. $\overline{xx+\frac{1}{2}xy}\times a\div 3=$ the ſolidity of the greater hoof.

And $\overline{yy+\frac{1}{2}yx}\times a\div 3=$ the ſolidity of the leſs hoof.

Now ſuppoſe $x=1,5$; $y=1,25$, and $a=9$: then $1,5\times1,5=2,25=xx$ the ſquare of the greater ſide. And $1,5\times1,25\div2=0,9375=\frac{1}{2}xy$ half the product of the ſides: Then $\overline{2,25+0,9375}\times9\div3=9,5625$ the ſolidity of the greater hoof. Again, $1,25\times1,25=1,5625=yy$, then $\overline{1,5625+0,9375}\times9\div3=7,5$ the ſolidity of the leſs hoof.

Note, The ſolidity of the greater or leſs hoof will be obtained according to the end of the fruſtum uſed; as evidently appears by the example above.

14. *In a conical fruſtum, the diameters, and diſtance of the two parallel ends being given, to find the ſolidity of each hoof, when the fruſtum is cut by a diagonal plane.*

1. To find the greater hoof (S).

Rule. From the cube of the greater diameter (x), take the ſquare root of the product of the cubes of the two diameters. Divide the remainder by the difference of two diameters. Multiply the quote by the altitude (a). The product multiplied by 0,2618 (n) (one 12th of 3,1416), will give the ſolidity.

Theo. $S=\frac{x^3-\sqrt{x^3\times y^3}}{x-y}\times na$, putting $y=$ the leſs diameter.

2. To find the leſs hoof (s).

Rule. From the ſquare root of the products of the cubes of the two diameters, take the cube of the leſs diameter. Divide the remainder by the difference of the two diameters. Multiply the quote by the height. The product multiplied by 0,2618 will give the ſolidity.

Theo. $s=\frac{\sqrt{x^3\times y^3}-y^3}{x-y}\times a\,n$

Examp. There is a conical fruſtum, the diameter of the greater end is 4; that of the leſs end 2; and the altitude 9; what is the ſolidity of each hoof?

Operation. $4\times4\times4=64$ the cube of the greater diameter, and $2\times2\times2=8$ the cube of the leſ. diameter. Alſo $64\times8=512$; whoſe ſquare root is 22,6272.

Then $\frac{64-22,6272}{4-2}\times9\times0,2618=48,7413$ the ſolidity of the greater hoof.

And $\frac{22,6272-8}{4-2}\times9\times,2618=17,2323$ the ſolidity of the leſs hoof.

15. *Of a Sphere (or Globe).*

1. *The diameter of a Globe being given to find its ſuperficies.*

Rule. Multiply the ſquare of the diameter by 3,1416 (=p) and the product is the ſuperficies.

Exam. What is the ſuperficies of a ſphere whoſe diameter is 3?

Operation. $3\times3\times3,1416=28,2744$ the ſuperficies.

2. *The circumference of a circle biſecting the ſphere being given, to find its ſuperficies.*

Rule. Multiply the ſquare of the circumference by 0,31832, (1÷p) and the product is the ſuperficies.

Exam. What is the ſuperficies of a ſphere whoſe circumference is 9,4248?

Operation. $9,4248\times9,4248\times0,31832=28,2753645$ the ſuperficies.

3. *The diameter and circumference of a ſphere being given, to find its ſuperficies.*

Rule. Multiply the diameter by the circumference, and the product is the ſuperficies.

Exam. What is the ſuperficies of a ſphere, whoſe diameter is 3, and the circumference 9,4248?

Operation. $9,4248\times3=28,2744$ the ſuperficies.

4. *The diameter of a ſphere being given to find its ſolidity*

Rule.

Rule. Multiply the cube of the diameter by ,5236 $(=p\div 6)$ and the product is the ſolidity.

Exam. What is the ſolidity of a ſphere whoſe diameter is 3?

Operation. $3\times 3\times 3\times 0{,}5236=14{,}1372$ the ſolidity.

5. *The circumference of a circle biſecting the ſphere being given, to find its ſolidity.*

Rule. Multiply the cube of the circumference by 0,016887 $(=1\div 6pp)$ and the product is the ſolidity.

Exam. What is the ſolidity of a ſphere whoſe circumference is 9,4248?

Operation. $\overline{9{,}4248}\rceil^{3}\times 0{,}016887=14{,}1373$ &c. the ſolidity.

6. *The ſuperficies and diameter of a ſphere being given, to find the ſolidity.*

Rule. Multiply the ſuperficies by one ſixth of the diameter, and the product is the ſolidity.

Exam. What is the ſolidity of a ſphere, whoſe diameter is 3, and the ſuperficies 28,2744?

Operation. $28{,}2744\times 3\div 6=14{,}1372$ the ſolidity.

7. *If the ſuperficies and circumference of a ſphere be given to find the diameter.*

Rule. Multiply the ſuperficies by the circumference, and the product ×ed by 0,05305 $(=p\div 6)$ gives the ſolidity.

Exam. What is the ſolidity of a ſphere whoſe circumference is 9,4248, and the ſuperficies 28,2744?

Operation. $28{,}2744\times 9{,}4248\times 0{,}05305=14{,}1372$ the ſolidity.

Note. The ſolidity of a ſphere, is equal to two thirds of its circumſcribing cylinder.

8. *The ſolidity of a ſphere being given, to find the diameter.*

Rule. The cube root of the ſolidity ×ed by 1,2407 ($=\sqrt[3]{6 \div p}$) will give the diameter.

Exam. What is the diameter of that ſphere whoſe ſolidity is 14,1372?

Operation. $\sqrt[3]{14{,}1372} \times 1{,}2407 = 3$ the diameter.

16. *To find the ſolidity of the ſegment of a ſphere, the diameter of its baſe (2x) and altitude (a) being given.*

Definition The ſegment of a ſphere (or globe) is a part of it cut off by a plane; and therefore the baſe of ſuch a ſegment muſt be a circle; and its convex ſurface a part of the ſurface of the ſphere.

Rule. To thrice the ſquare of the ſemidiameter of the baſe, add the ſquare of the altitude, multiply the ſum by the height; then the product ×ed by 0,5236 (n) will give the ſolidity (s).

Theorem. $s = \overline{3xx + aa} \times a \times n$.

Exam. What is the ſolidity of a ſpherical ſegment, the diameter of whoſe baſe is 26, (=2x); and its height 12 (a)?

Operation. $s = \overline{13 \times 13 \times 3 + 12 \times 12} \times 12 \times 0{,}5236 =$ 4090,3632 the ſolidity.

17. *The diameter of the baſe (x) and the altitude (a) of ſpherical ſegment, being given, to find its convex ſuperficies (s)*

Rule. To the ſquare of the diameter of the baſe, add the ſquare of twice the height, the ſum ×ed by 0,7854 (n) will give the ſuperficies required.

Theorem. $s = \overline{xx + 4aa} \times n$.

Exam. What is the convex ſurface of that ſpherical ſegment, the diameter of whoſe baſe is 18 (=x), and whoſe altitude is $4\frac{1}{2}$ (=a)?

Operation. $18\times18=324$; and $\overline{4,5\times2}\rceil^2=81$ the ſquare of twice the height. Then $\overline{324+81}\times0,7854=318,087$ the ſuperficies required.

18. *To find the ſolidity of a ſpheroid, the axis of rotion, and the revolving axis being given,*

Definition. A ſpheroid is a ſolid figure, formed by the rotation of an ellipſis round either of its axis. If the rotation be about the tranſverſe axis the ſolid generated is called a prolate (or oblong) ſpheroid reſembling an egg, only both its ends are the ſame. But if the rotation be about the conjugate axis, the ſolid generated is called an oblate ſpheroid. Such a form the fig. of the earth is ſaid to be.

Rule Multiply the fixed axis by the ſquare of the revolving axis, the product ×ed by 0,5236 ($=p\div6$) will give the ſolidity.

Note. A ſpheroid (as well as the ſphere) is two thirds of its circumſcribing cylinder.

Exam. 1 What is the ſolidity of a prolate ſpheroid, whoſe tranſverſe axis is 100, and its conjugate is 60 ?

Operation. $100\times60\times60\times0.5236=188496$ the ſolidity.

Ex. 2. What is the ſolidity of an oblate ſpheroid, whoſe longeſt axis is 100, and ſhorteſt axis 60 ?

Operation. $60\times100\times100\times0.5236=314160$ the ſolidity.

19. *To find the ſolidity of a parabolic conoid, the diameter of its baſe, and altitude being given.*

Definition. A parabolic conoid is a ſolid formed by the rotation of a ſemi-parabola about its axis. It is equal to half its circumſcribing cylinder.

Rule. Multiply the ſquare of the diameter of its baſe by .7854; then the product ×ed by half the height will give the ſolidity.

Ex. What is the ſolidity of a parabolic conoid, whoſe height is 50, and the diameter of whoſe baſe is 60 ?

Operation. $60\times60\times,7854\times50\div2=70686$ the ſolidity.

20. To

20. *To find the ſolidity of a Parabolic-Spindle.*

Definition. A parabolic-ſpindle is a ſolid generated by the rotation of a parabola about its ordinate; which ordinate will be the ſpindle's length, and the abſciſs of the parabola will be the radius of its greateſt circle (or its ½ breadth).

Rule. Multiply the ſquare of the diameter (d) by the length (b), the product ×ed by 0,41838 (=n) will give the ſolidity (s). Theorem. $s=dd\times l\times n$.

Ex. Suppoſe the length of a parabolic-ſpindle, to be 60 (l). and the greateſt diameter 40 (d): Required the ſolidity.

Operation. $40\times 40\times 60\times 0.41888=40212.48$ the ſolidity.

Note. The uſual method of meaſuring irregular ſolids, ſuch as craggy ſtone, lumps of metal, buſhes of ſhrubs, &c. is to fill a veſſel with water, and then to put in the ſolid, and meaſure the water that flows over; as ſuppoſe you immerſe a ſolid in a regular veſſel full of water, and by ſuch immerſion 5 ale gallons run over the top of the veſſel, then the content of this immerſed ſolid is 282 multiplied by 5 is equal to 1410 ſolid inches. Or if you put the ſolid into a regular veſſel, and fill the veſſel up with water, then take out the ſolid and meaſure the empty part of the veſſel, it will give the ſame content.

§ IV. *Of* BOARD MEASURE.

BOARDS are meaſured by ſuperficial meaſure, the thickneſs not being regarded: So that a board is moſtly a parallelogram (or may be reduced to ſuch), the meaſuring of which is ſhewn in Sect. 2. Prob. 3: The uſual method of meaſuring boards, is to take the length along the middle between the two ſides, and breadth in the middle between the two ends. Then,

Rule. Multiply the length by the breadth and the product will be feet, if both the dimenſions were taken in feet, or feet and decimal parts of a foot: but if the length was taken in feet and breadth in inches (which is moſtly the caſe), divide the ſaid product by 12. Or if both was taken in inches, divide the product by 144, and the quote will be flat feet.

Note. In meaſuring a quantity of boards that have the ſame length, but different breadths, the beſt way is to take (with a ſtring) all the breadths in one dimenſion, and multiply that by the length of one board. And in meaſuring a ſtock of boards that have the ſame length and breadth, find the content of one board and multiply that by the number of boards.

EXAMPLES.

1. If a board be 20 feet (or 240 inches) long, and 18 inches ($=1{,}5$ feet) broad; how many ſquare or flat feet doth it contain?

$$\textit{Operation}\left\{\begin{array}{ll} 20\times1{,}5 & = \\ 20\times18\div12 & = \\ 240\times18\div144 & = \end{array}\right\}\text{30 feet, the content required.}$$

2. There is a parcel of boards, the ſum of whoſe breadths, taken together, is 87,5 feet, and the length of each is 8,75 feet; how many ſquare feet do they all contain?

Operation. $87{,}5\times8.75=765.625$ ſquare feet.

3. Suppoſe a ſtock of tapering boards, 25 in number, the length of each to be 16.6feet, and breadth taken in the middle, 10½ inches; how many ſquare feet does the ſaid ſtock of boards contain?

Operation. $16.6\times10.5\div12=14{,}525$ feet, the content of one board. Then $14{,}525\times25=363{,}125$, feet the content of the whole.

Note. If a board be irregular, that is, broader in ſome places than in others, take breadths at the broadeſt and narroweſt places, add all thoſe breadths together, and divide the ſum by the number of breadths, and the quotient will be a mean breadth; which will reduce the board to the form of a parallelogram.

4. There is a board whoſe length is 15½ feet, but irregular in breadth; therefore five breadths are taken which

are as follows: 1ſt. 11,5 inches, 2d. 11 inches; 3d. 11,25 inches; 4th. 13,5 inches, and 5th. 14.25 inches; how many ſquare feet doth it contain?

Operation. 11,5 + 11 + 11.25 + 13,5 + 14 25 ÷ 5 = 12,3, the mean breadth.

Then 15,5 × 12,3 = 190.65 ſquare feet, the anſwer.

5. If a board be five inches broad, how much in length will make a ſuperficial foot?

Note. Since the product of the length and breadth gives the area, its evident that if the area be divided by one of thoſe dimenſions, the quotient will be the other.

Operation 144 ÷ 5 = 28.8 inches in length.

6. If a board be 18 inches broad, how much in length will make a ſquare foot? Anſw. 8 inches.

§ V. *Of the* Meaſuring *of* TIMBER.

THERE are various ſorts and forms of timber. If it is hewed, and the ends are ſquares or rectangles, and all of a thickneſs, it is a parallelopipedon. If it is thicker at one end than the other, it is the fruſtom of a pyramid. If it be round and all of the ſame thickneſs it is a cylendar; but if it be thicker at one end than the other, it is the fruſtum of a cone For the meaſuring of each ſee Section III. But there is a cuſtomary way of meaſuring both ſquare and round timber, which, tho' it be erroneous, is uſed by all dealers therein. Workmen, and meauſurers, uſually gird the tree in the middle with a cord, and take the 4th part of that compaſs or circumference, and eſtimate it as though it was the ſide of a ſquare, whoſe area is equal to the ſection of the tree, at the place it was girded.

It is a cuſtomary allowance to the buyer, to take the girt where he pleaſes, between the greater end and middle of the tree.

PROBLEM I.

To compute the ſolidity (or content) of round timber, by having the length and circumference given.

CASE I. *When the tree is ſtreight, and the ends are equal, or nearly ſo.*

Rule. Multiply the ſquare of one fourth the circumference by the length, and the product is the content in feet, if both the dimenſions be expreſſed in feet and decimal parts thereof; which is the beſt and eaſieſt way. But if one dimenſion be feet and the other inches, divide the ſaid product by 144, and the quote will be the ſolidity; or if both dimenſions are inches, divide the product by 1728, and the quote will be the ſolidity in feet.

EXAMPLES.

1. What is the ſolidity of a tree, whoſe compaſs is 36 inches, and length 20 feet?

Operation. $36 \div 4 = 9$ inches $= {,}75$ foot;

Then, ${,}75 \times {,}75 \times 20 = 11{,}25 = 11\frac{1}{4}$ feet the content required. Or $9 \times 9 \times 20 \div 144 = 11\frac{1}{4}$ feet.

2. What is the ſolidity of a tree whoſe length is $33\frac{1}{2}$ feet, and $\frac{1}{4}$ of the compaſs 17 inches?

Operation. $17 \times 17 \times 33{,}5 \div 144 = 67{,}23$ feet the content required.

3. What is the ſolid content of a piece of timber whoſe length is 204,5 inches; and $\frac{1}{4}$ of the circumference 20,5 inches?

Operation. $20{,}5 \times 20{,}5 \times 204{,}5 \div 1728 = 49{,}734$ feet the content required.

4. What is the ſolidity of a piece of timber, whoſe length is $9\frac{3}{4}$ feet, and $\frac{1}{4}$ the circumference 39 inches?

Anſw. 102,984 feet.

Note. If the tree is crooked, its length muſt not be meaſured on either the convex, or concave ſide of the curve.

CASE II. *When the tree is unequally thick, or taper.*

Rule. Gird the tree in as many places as are thought neceſſary: add the ſeveral girds together and divide that ſum by the number of girds, the quote will be (as thought by the workmen) a mean compaſs; the fourth of this mean compaſs ſquared and multiplied by the length, gives the ſolidity.

5. A tapering tree 20 feet long, is girded in four places; in the firſt it is 72 inches, in the ſecond 60, in the third 52, and in the fourth 48 : what is the ſolidity in feet ?

Operation. $72+60+52+48\div4=58$ a mean compaſs, and $58\div4=14{,}5$ inches; then $14{,}5\times14{,}5\times20\div144=29{,}2$ feet, the content required.

CASE III. *Any piece of wood that is 24 inches in compaſs (or 6 inches ſquare) is eſteemed timber; therefore if any branches or boughs meaſure 24 in compaſs, their ſolidity muſt be found and added to that of the tree.*

Note. So much of the trunk, as meaſures leſs than 24 inches compaſs, is not eſteemed timber.

6. Required the ſolidity of a tree 40 feet long; 27 inches quarter compaſs; one branch 14 feet long by 28 inches circumference, and another 10 feet long, by 24 inches compaſs.

Operation. Firſt, $27\div12=22{,}5$ feet, $28\div4=7$ in. $=,583$ foot, and $24\div4=6$ in. $=,5$ foot: then $2{,}25\times2{,}25\times40=202{,}5$ feet in the tree, $,583\times,583\times14=3{,}35846$ feet in one branch, $,5\times,5\times10=2.5$ feet in the other branch. And $202{,}5+3{,}35846+2{,}5=208{,}35846$ the ſolidity required.

CASE IV. *When the trees have their bark on.*

In meaſuring ſuch timber for ſale, it is common to make an allowance to the buyer on account of the bark; thus, in oak, one tenth or one twelfth part of the circumference is deducted; but the allowance for the bark of aſh, beach, elm, &c. is leſs.

Rule. From the given circumference, ſubtract the allowance for bark; and with the remaining compaſs, find the ſolidity, as before.

7. Required the ſolid content of an oak tree whoſe length is 44½ feet, and 60 inches quarter compaſs; allowing one tenth for bark?

Operation. $60\div10=6$ the allowance, then $60-6=54=4{,}5$ feet, ¼th of which is 1,125 foot;
Then $1{,}125\times1{,}125\times44{,}5=56{,}32$ feet, the ſolidity.

PROBLEM II.

To compute the ſolidity of ſquare or flat timber, ſuch as balks, or plank, &c.

Rule. Take the breadth and depth in the middle, if both ends be nearly of a thickneſs; but in tapering ſquare (or flat) timber, take as many breadths and depths, as are thought neceſſary; from which find a mean breadth and depth as taught in Caſe II. Prob. I. Then multiply length, breadth and depth together, and the product will be the true ſolidity.

EXAMPLES.

1. If a piece of timber be 13.2 feet long, and 1,52 foot ſquare, how many ſolid feet doth it contain?

Operation. 1,52 × 3,52 × 13.2 = 30,49728 feet, the anſw.

2. If a piece of timber be 30 feet long, 20 inches broad, and $15\frac{1}{2}$ inches deep, how many feet doth it contain?

Operation. 20 × 15,5 × 30 ÷ 144 = 64,583 feet the anſw.

3. There are 5 planks, the length of each is 20,5 feet, breadth 1,2 foot, and depth 0,4 of a foot; how many ſolid feet of plank do they contain?

Operation. 1,2 × 0.4 × 20,5 = 9,84 feet, the ſolidity of 1 plank; then 9,84 × 5 = 49,2 feet the anſw.

Note. Workmen, and meaſurers, reckon 40 feet of unhewn, or rough timber, and 50 feet of hewn timber to a load, ſuppoſed to weigh a ton, or 20 cwt. For ſay they, hewn timber is meaſured by the ſquare, and is very near exact; but rough timber by the girt, (or quarter compaſs,) which is about one fifth leſs than exact: therefore divide the feet in rough timber by 40, and in hewn (or ſawn) timber by 50, and the quotient gives the loads (or tons).

To find how much in length is required to make any given ſolidity of timber. Becauſe the product of the length, breadth, and depth, gives the content; therefore, if the given ſolidity be divided by the product of any two dimenſions, the quotient will be the third.

Exam. 1. A piece of ſquare timber whoſe ſide is 6 inches; how much in length will make 2 ſolid feet?

Operation. 6×6=36, and 1728×2=3456 cubic inches in 2 feet: Then 3456÷36=96 in.=8 feet the Anſw.

Exam. 2. A piece of timber whoſe breadth is 9 inches, and depth 4; how much in length will make 1½ ſolid feet?

Operation. 9×4=36, and 2592, the cubic inches in 1½ feet: Then 2592÷36=72 inches=6 feet the Anſw.

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§ VI. *Of* ARTIFICERS WORK.

ARTIFICERS have different ways of meaſuring according to the cuſtom of each craft; ſome give the content in feet, ſome in yards, ſome in ſquares, ſome in rods, &c.

They alſo differ according to the cuſtom of the place, ſome give the content in one denomination, and ſome in another.

Artificers ſeldom uſe ought but ſuperficial or flat meaſure, and becauſe *walls*, *doors*, *floors*, *windows*, *roofs*, &c. are *ſquares* or *parallelograms*, their areas are found by Prob. I. or III. Sect. II. but if any other form come in practice, it cannot but come under ſome of the rules in the foregoing ſections.

1. *Of Bricklayers Work.*

Bricklayers commonly meaſure their work by a rod of 16½ feet: the ſquare of which is 272¼ feet: but in Practice, divide the area found by 272, the quotient of which are the rods required. In ſome places they meaſure by a rod of 18 feet; whoſe ſquare is 324 feet: and in others they meaſure by the rood, of 21 feet long, and 3 broad, containing (21×3=)63 ſquare feet; and here they do not regard the wall's thickneſs, but moderate the price accordingly.

Note. Bricklayers moſtly compute or value their work at the rate of a brick and an half thick, called ſtandard thickneſs; and if the thickneſs of the wall happen to be more or leſs than ſtandard, it muſt be reduced to ſtandard thickneſs as follows:

Multiply the area of the wall, by the number of half bricks in the thickneſs of the wall; divide the product by 3, and it gives the area.

Exam, 1. How many ſquare rods, of ſtandard thickneſs, are there in a wall $72\frac{1}{2}$ feet long, $19\frac{1}{4}$ feet high, and $9\frac{1}{2}$ bricks thick? Anſw. 18,8 rods.

First $72,5 \times 19,25 = 1395,625$ the ſuperficial content, then $1395,625 \times 11 \div 3 = 5117,291$ feet the area in ſtandard thickneſs; and $5117,291 \div 272 = 18,8$ rods the Anſw.

Exam. 2. A wall being 72 feet long, 34 feet high, and $3\frac{1}{2}$ bricks thick, how many rods of brick work, at 18 feet the rod, are contained in it?

Here $72 \times 34 = 2448$, and $2448 \times 7 \div 3 = 5712$ the area in feet at ſtandard thickneſs. Then $5712 \div 324 = 17$ rods, 204 feet, remaining; the Anſw.

Exam. 3. A wall being 62 feet, 6 inches long, 14 feet, 9 inches high, how many roods of brick work, at 63 ſquare feet or 7 ſquare yards the rood, are contained in it? Anſw. 14 roods, and 4,4305 yards.

First $14,75 \times 62,5 = 921,875$ ſquare feet,
Then $921,875 \div 9 = 102,4305$ ſquare yards, and $102 \div 7 = 14$ roods: 4,4305 yards remain.

Note. The uſual way to take dimenſions of a building is to meaſure half round it on the out-ſide, and half round it on the inſide with a chord; which gives the true compaſs of the building. If it be all of a height, meaſure its height at any place from the bottom of the foundation; but if the height be unequal, take ſeveral heights, add them together, and divide their ſum by the number of heights, and the quotient will be a mean height. The gable ends above the ſquare being triangles, are meaſured as ſuch.

Exam. 4. Suppoſe each ſide wall of an houſe be $30\frac{1}{2}$ feet long on the out ſide, each end wall $14\frac{1}{2}$ feet broad on the inſide; and the height from the foundation to the ſquare $55\frac{1}{2}$ feet; and the gable (or triangular part at top) to riſe 42 courſe of bricks (reckoning 4 courſe to a foot). Now the firſt 20 feet in height is 2½ bricks thick, the next 20 feet 2 bricks thick; 15½ feet above 1½ brick thick, and the gable-end 1 brick thick: what will it amount to at 5l. 10s. per rod, of 272 ſquare feet, ſtandard thickneſs? Alſo how many ſquare yards, not minding the thickneſs, as if it were a ſtone building?

$42\div4=10,5$ the height of the gable, at each end.
$30\frac{1}{2}+30\frac{1}{2}+14\frac{1}{2}+14\frac{1}{2}=90$ the compaſs of the building.

ſquare feet

$$\left.\begin{array}{l} 14,5\times10,5\times2\div3 \\ 90\times15,5 \\ 90\times20\times4\div3 \\ 90\times20\times5\div3 \end{array}\right| = \left|\begin{array}{l} 101,5 \text{ at } 1 \\ 1395 \text{ at } 1\frac{1}{2} \\ 2400 \text{ at } 2 \\ 3000 \text{ at } 2\frac{1}{2} \end{array}\right\}\text{bricks thick.}$$

The area of the houſe is 6896,5 ſtandard thickneſs.

feet l. feet l. l. s. d.
As $272 : 5,5 :: 6896,5 : 139,4512 = 139 : 9 : 0\frac{1}{4}$ the amount. And $6896,5\div9=766,277$ ſquare yards.

In meaſuring a chimney, the uſual way is, if it ſtands alone without leaning againſt a wall, &c. to gird it about below the mantle, and take that for length, and the height of the room or chimney ſo far as it keeps the ſame girt for the breadth; the product of theſe two is the content: but if it ſtand againſt a wall you muſt meaſure it round to the wall for the girt, and take the height as before. It is cuſtomary in moſt places to allow double meaſure for chimnies. But in Sheffield, and ſeveral other places, they allow one ſhilling a yard running meaſure for chimney pipes, meaſuring from the hearth to the chimney top.

2. *Of meaſurements by the Square, as Flooring, Partitioning, Roofing, Tyling, &c.*

In meaſuring theſe works, the dimenſions are taken by a rod of 10 feet; and therefore the reſult is in ſquares of 100 feet each. Hence divide the area found by 100, and the quote will be the number of ſquares required.

In flooring they deduct the hearth-ſtone, except it has a border round it; and then the hearth is meaſured in with the floor.

In *roofing*, *tyling* and *ſlating*, it is cuſtomary to reckon the area and half area of the flat or floor for the area of the roof; or thus, as 2 is to 3, ſo is the area of the floor to that of the roof, when the ſaid roof is of a true *pitch*.

Note 1. All roofs are ſaid to be of a true pitch, when the rafters are three-fourths of the breadth of the building.

If the roof is more or leſs than a true pitch they meaſure on the ridge, and from the eaves on one ſide to the eaves on the other with a rod or ſtring.

2. Double meaſure is commonly allowed for hips, vallies, gutters, &c. And in ſlating and tyling it is common to allow double meaſure at the eaves for ſo much as projects over the plate.

Slating is ſometimes meaſured by the ſquare yard, and ſometimes by the rod of 49 yards ſquare, &c.

Exam. 1. Suppoſe a houſe of three ſtories, beſides the ground floor, is to be floored at 5l. 10s. per ſquare, that the houſe meaſures 30 feet 6 inches, by 24 feet 9 inches; that there are 7 fire places, whoſe meaſures are, three, each of 6½ feet by four feet; three other, each of 5 feet 9 inches, by 4 feet 6 inches; and the ſeventh 6 feet 9 inches, by 5 feet; and that the well hole for the ſtairs, is 10 feet 6 inches, by 8½ feet; what will the whole come to?

Firſt 6,5 × 4 × 3 = 78 the 1ſt three fire-places.
5,75 × 4,5 × 3 = 77,625 the 2d ditto.
6,75 × 5 = 33,75 the 7th fire place.
10,5 × 8,5 × 4 = 357, the well hole upon 4 floors.

546,375 the whole of the deduction

And 30,5 × 24,75 × 4 = 3019,5 the area of the 4 floors.
546,375 the area of the deduction

100)2473,125 = area of the work.

24 ſquares 73,125 feet.

feet l. feet l.
As 100 : 5,5 :: 2473,125 : 136,021875
Or, 136l. 5¼d. the Anſwer.

Ex. 2. If the breadth of a roof with the uſual allowance at the eaves be 30,5 feet, and the length of the ridge 50 feet, how many yards, ſquares, and rods are contained in it?

First 30,5 × 50=1525 ſquare feet, or 169 yards 4 feet, or 3 rods, 22 yards, 4 feet, or 15 ſquares 25 feet.

Exam 3. If a partition between two rooms be 80,5 feet in length, and 12¼, in height; what will the workmanſhip coſt at 6s. 6d. per ſquare?

Firſt 80,5 × 12,25=986,125 ſquare feet, which ÷ed by 100=9,86125 ſquares.
Then as 1 : 6,5s. :: 9,986125 : 64,098125s.
Or 3l. 4s. 1d. the Anſw.

Exam. If a houſe meaſure within the walls 50 feet in length, and 30,5 in breadth; and the roof be of a true pitch; what will it coſt roofing at 10s. per ſquare.

Firſt 30,5 × 50=1525, the area of the flat, or floor;
Then as 2 : 3 :: 1525 : 2287,5 the area of the roof.
And 2287,5 ÷ 100=22,875 ſquares, which at 10s. per ſquare amounts to 11l. 8s. 9d. the Anſw.

There is other work about a building done by the Carpenter, that is meaſured by the foot runing meaſure, as cornices, doors and caſes, window frames, lintels, guttering, ſkirt-boards, &c.

3. *Of Meaſurements by the Yard Square.*

As Joiner's, Painter's, Plaiſterer's and Paviour's Work.

In each of theſe works, it is beſt to take the dimenſions by a decimally divided yard.

Of Joiners Work.

Joiners take their dimenſions with a ſtring, meaſuring round the room or floor for the length; and for the breadth, they meaſure from the top of the cornice to the floor, girding the ſtring over all the mouldings and ſwelling pannels; for they ſay they ought to meaſure where their plane touches.

Note. All ſtuff an inch and half thick and under, wrought on both ſides, is by Joiners reckoned at work and half work; but ſtuff of greater thickneſs wrought on both ſides, is valued at double work. They make deductions for vacancies that fall within their work; and meaſure window boards, ſofit boards, cheeks, &c. by themſelves.

Exam. 1. What will the wainſcoting a room come to at 5s. per ſquare yard; ſuppoſing the compaſs of the room is 20,52 yards, and the height (taking in the cornice and moulding) 4,09 yards; ſix window ſhutters, each 2,3 yards by 1,3 yards; and the door 2,4 yards by 1,08 yards; the ſhutters and door as they are wrought on both ſides being reckoned work and half?

Firſt $2,3 \times 1,3 \times 6 = 17,94$ = ſix ſhutters area once.
$1,08 \times 2,4 = 2,592$ = door's area once.

$2)20,532$ = ſhutters and door.

$10,266$ = half the ſhut. and door.
$20,52 \times 4,09 = 83,9268$ = { rooms area including the ſhut. & door once

$4)94,1928$ = whole work.

The Anſw. L. $23,5482$ = 23l. 10s: 11½d.

Exam. 2. If the window ſhutters about a room, or ſeveral rooms, be 70 feet broad (that is, if all their breadths in one ſum = 70 feet) and the height of each ſhutter 6,5 feet; what's the content in yards at work and half? Anſw. 75,8 yards.

Exam. 3. If the compaſs of a room be 96 feet 3 inches, and height 12 feet 9 inches; how many yards of wainſcoting doth it contain? Anſw. 136 yards, 3 ſquare feet.

Of Painters Work.

Painters work is meaſured in the ſame manner as that of Joiners, by a ſtring, becauſe the bruſh goes into the hollows and over the ſwellings: it being reaſonable they ſhould be paid for all places whereon their colour is laid. Their price is generally proportioned to the number of times they lay on their colour.

Note. There muſt always be made deductions for vacancies within the dimenſions taken.

Exam. Suppoſe the compaſs of a room is 24,94 yards, and height (taking in the cornice and mouldings) 3,86 yards; the door 2,5 yds. by 1,25 yd; five window ſhutters, each 2,22 yds. by 1,11 yd.; the window caſes 0,38 yd. deep, and 2,66 yds. high; a cloſet the height of the room, 1,16 yd. deep, 1,58 yd. in front; the ſhelves taken together, 7,5 yds. by 0,28; the fire place 2,25 yds. by 1,66 yd. (which muſt be deducted); the ſhutters, door, and ſhelves coloured on both ſides; what will the whole come to at 8d. per yard?

First $\overline{2,66\times2}+\overline{1,11\times2}=7,54$ the length of both ſides, top and bottom of one window caſe, in one ſum: And

$\overline{\overline{1,16\times2}}+\overline{\overline{1,58\times2}}=5,48$ the compaſs of the cloſet:
Then 7,54×0,38×5=14,3260 area of the 5 wind. caſes.
5,48×3,86 =21,1528 area of the cloſet.
7,5×0,28×2= 4,2000 area of the ſhelves.
2,22×1,11×5=12,3210 the ſhutters once.
2,5×1,25 = 3,1250 the door once.
24,94×3,86 =96,2684 { area of the room including the door, ſhutt. &c.

The ſum of the areas=151,3932 out of which deduct.
2,25×1,66 = 3,7350 the area of the fire place.

Remainder 147,6582 the area of the work.
Then 147,6582÷30=4,92194l.=4l. 18s. 5¼d. the Anſw.

Plaiſterers Work.

When a room is plaiſtered take its compaſs in the inſide girding in chimnies &c. where ever the work comes: and in taking over head dimenſions gird in the ſummer &c.

Ex. What will the plaiſtering a room come to at rod, per yard, whoſe compaſs is 47,2 yards, and height 4,3 yards; and the top 6,08 yards by 5,22 yards?

Firſt $47,2 \times 4,3 + 6,08 \times 5,22 = 234,6976$ yards, the area; which at 10d. per yard amounts to 9l. 15s. 7d.

Note. In meaſuring walls, deductions muſt be made for doors and windows when caſed with boards, but if plaiſtered where the caſing ſhould be, no deductions are made: Alſo, in rendering between quarters, you may deduct one fifth part for the quarters, braces, &c. but in whitening and colouring one fourth or fifth part muſt be added, where the quarters and braces project beyond the wall.

Maſons Work.

Maſons work is ſometimes meaſured by the foot ſquare, ſometimes by the runing foot (viz. only feet in length), and ſometimes by the ſolid foot, taking in all the three dimenſions, length, breadth, and thickneſs. Paving (which is meaſured by the yard-ſquare) is alſo reckoned in with maſons work.

Ex. 1. If a wall be 112,25 feet long, and 16,5 feet high, how many rods, at 63 ſquare feet to a rod, doth it contain?

$112,25 \times 16,5 = 1852,125$ feet $\div 63 = 29,4$ rods the Anſw.

Ex. 2. How many ſolid feet are contained in a wall 64,5 feet long, 20,5 feet high, and 2,25 feet thick?

$64,5 \times 20,5 \times 2,25 = 2975,0625$ ſolid feet the Anſw.

Ex. 3. A Pavement being 32 yards long, and 5,58 yards broad, how many ſquare yards doth it contain?

$5,58 \times 32 = 178,56$ yards, the Anſw.

Glaziers Work.

Glaziers meaſure by the ſquare foot, and commonly take their dimenſions nearer than any of the foregoing, for they'll go to the 8th or 10th of an inch.

Ex. 1. If there be 8 panes of glaſs each 2,23 feet high, and 1,28 feet broad, how many ſquare feet?

$2,23 \times 1,28 \times 8 = 22,8352$ ſquare feet, the Anſw.

Ex. 2. There is a houſe with three tire of windows, five in a tire; the height of the firſt is 7,5 feet, of the ſecond 6,3 feet; of the third 5,2 feet; and the breadth of each, 3,32 feet: what will the glazing come to at 15d. per foot?

Firſt $7,5+6,3+5,2=19$ the height of the windows in one row upwards that are over each other, which × by 5, the number of windows in a tire, this gives the length of all the 15 windows: thus $19\times5=95$ feet, the length of the whole.

Then $95\times3,32=315,4$ feet the area. And $315,4\div16$ =19l. 14s. 3d. the Anſw.

§ VII. *Of* LAND SURVEYING.

SURVEYING of Land, or Planometria, is the art of meaſuring all manner of plain figures, in order to know their ſuperficial content; and requires the practitioner (who would be a complete maſter of it) to be well ſkilled in numbers, both whole and broken, with the extraction of Roots, Geometry, and plain Trigonometry: But as this Treatiſe does not furniſh the learner with plain Trigonometry, I refer him to a little Treatiſe wrote by Mr. Tho. Simpſon, price 1s. 6d. Thoſe who are deſirous of ſeeing a great variety of Trigonometry both in theory and practice, plain and ſpherical, may conſult Mr. B. Martin's Young Trigonometer's Complete Guide, in two Volumes; price 10s. 6d.

In Surveying Land, the Inſtruments in common uſe are the Gunter's Chain (as deſcribed in Table 7th, page 23) for meaſuring of diſtances; the ſemicircle, plain table and theodolite for taking the angles; and a protractor, or plain ſcale, and compaſſes, for plotting or delineating the dimenſion of the field on paper, in order to reduce it to triangles, and thereby to find the true ſuperficial *content* or *area*, in *acres*, *roods*, and *poles*.

In Meaſuring with the Chain, be careful to get the ſhorteſt diſtance between any two objects; otherwiſe you will make more of the land than there really is. In order hereunto you muſt provide ten ſmall ſticks like arrows ſharp at the end to ſtick in the ground. Then having ſet up white marks at the angles of the field, and at other places where you ſee occaſion, let your aſſiſtant that leads the chain, carry all the arrows in his left hand, ſave one in the right with the chain; and when he ſticks down one of his arrows, always be ſure that it be in a right line with the white and the eye of him that follows the chain, who is alſo to gather up the ſticks. Go on thus to the diſtance you are to meaſure, whether it be a diagonal, perpendicular, or the boundary, off-ſet &c. By this method he that follows the chain and gathers up the ſticks has a ſure method of counting the number of chains, without any trouble, or fear of error. For inſtance, you find a diſtance to be 7 chains 56 links; when this is ſet down in your field book, ruled in proper columnes, for that purpoſe, it will ſtand thus 7,56. So if the diſtance is 5 chains 8 links, you enter it thus, 5,08, always minding to ſet a cypher in the place of primes in the links, if they be under ten. The reader, thus furniſhed being well acquainted with Decimal Arithmetic, and with Menſuration of Superficies, as taught in this Teatiſe, is qualified to go into the field, in order to meaſure the ſame.

How to take the Plot of a Field, by the Chain only, and to caſt up the content thereof.

In order hereunto, you muſt provide yourſelf with a ſmall inſtrument called a Croſs, which you may make of wood; thus, take a piece of Box wood turned round, and about 2 or 3 inches diameter: with a fine ſaw cut on the upper ſide two ſlits croſs ways, that ſo they may be exactly at right angles with each other; and on the under ſide of it let there be a hole for the conveniency of placing it upon a ſtaff about your own height in time of practice. Being thus prepared, go into a field, and upon a piece of paper draw a rough draught of it, as near the ſhape as you can. While you are doing this, let your aſſiſtant place pieces of white paper at the angles A, B, C, D, E, F, G, O, H, and I. Then, your aſſiſtant taking the chain and arrows, ſtand at B and direct him to go towards I; and as you go along, by help of your croſs take the perpendicular KC; that is, find ſuch a point in the line BI, that the ſtaff being pricked down, you can ſee thro' the ſlits the points I, B, and C, without moving your ſtaff, which will be the point K; put down the chains and links BK=1,50; leave a mark at K, and meaſure on to L; here, by help of your croſs looking at B and A, you will find out the point L, where the perpendicular AL doth fall; ſet down the number of chains either from B to L, or from K to L, it matters not which; leave a mark at L, and meaſure on to I, and ſet down the whole digonal BI=3,75. Go back to L and meaſure the perpendicular LA=1,40, and CK=1,78, which put down upon your rough draught. And thus the trapezium ABCI is finiſhed, which note with the figure 1.

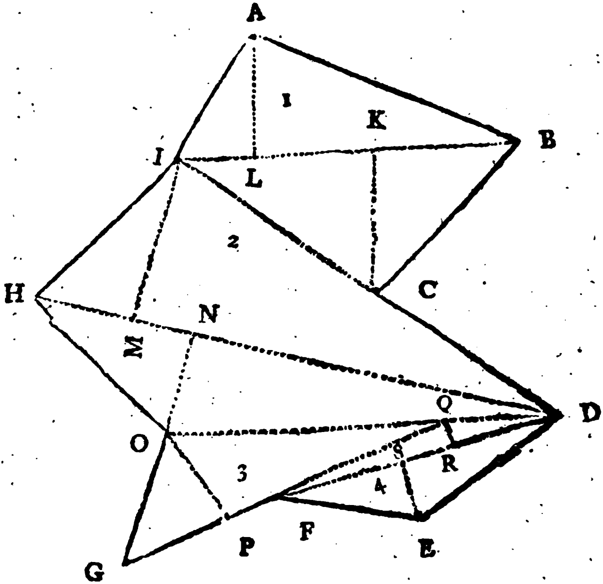
Secondly. Mark the trapezium IDOH with the figure 2. Then go to D, directing the aſſiſtant towards H, and you will obſerve the point N to fall at 3,93 chains, and the point M at 4,77 from D; where leave marks, and meaſuring on to H you will find the whole diagonal DH =5,78 chains; which inſert in your rough draught as before. Then meaſuring the perpendicular MI, you will find it to be 1,80 and NO=1,09.

Thirdly. Go into the triangle GOQ, which mark with the figure 3; and beginning at G, meaſure towards Q; and as you go along, obſerve with your croſs that the perpendicular OP falls at P, 1,11 from G; at P leave a mark, and meaſure to Q, finding GQ=3,54 chains. Then going back to P meaſure PO and you will find it to be 1,08 chains.

Laſtly. There ſtill remains the Trapezium FQDE unmeaſured, which note with the figure 4. Then coming to F, meaſure towards D, and you will ſee the perpendicular SE to fall at 1,43 chains from F, which note down in your rough draught as before; meaſuring on you will find the perpendicular RQ to fall at 1,92 chains from F, which alſo note down: And meaſuring to D, you will find FD=3,23 chains which alſo note down. Then coming back to the marks which you left at R and S, meaſure SE and you will find it = 0,60 links, and

QR =0,35 links, which ſet down in your field draught, and the meaſuring is all finiſhed.

Note. You muſt remember, when in the field, to meaſure ID and DO, they being of uſe in plotting, but not in caſting up of the dimenſions.



For the Reader's better information, I ſhall here inſert each particular meaſurement in chains and links.

	ch. links		ch links		ch links.
BK =	1, 50	DH =	5, 78	FR =	1, 72
KL =	1, 22	MI =	1, 80	FD =	3, 23
BI =	3, 75	NO =	1, 09	SE =	0, 60
AL =	1, 40	GP =	1, 11	QR =	0, 35
CK =	1, 78	GQ =	3, 54	ID =	4, 94
DN =	3, 93	PO =	1, 08	DO =	4, 15
DM =	4, 77	FS =	1, 43		

To